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## Time Reversing Oscillators And Through-Zero FM:

-by Bernie Hutchins
In EN 1113 , in answer to a reader's question about through-zero FM , we raised some additional questions about the process. Here we will discuss some additional findings. It turns out that one source of difficulty is the inherent conceptual difficulty of the FM process itself, even before we get to through-zero effects. We will look at a trigonometric interpretation first, and then at a more general model worked out by Lester Ludwig.

The problem to be considered first is that of an instantaneous change from a positive to a negative frequency of the same magnitude. As we have discussed in the past, we can consider that a negative frequency is the same (somehow) as a reversal in time; as a phase factor $\omega t$ (as in $\operatorname{Sin} \omega t$ ) can take on a nagative value in two ways: $-\omega t=(-\omega) t=\omega(-t)$. This leads to the idea that a reversing oscillator provides the necessary through-zero FM function. Thus if an oscillator is reversed at a time $t_{0}$ (at a phase $\omega t_{0}$ ), the waveform effectively "mirrors" about that point. A waveform such as this is shown in the solid line of Fig. 1. Since the change is instantaneous at $t_{0}$, we do not see the slowing down and flat portion of the waveform (see EN\#113 questions). This is the way a reversing oscillator will respond to a square wave control voltage. Thus it has the time reversal interpretation, equivalent to a negative frequency effect. The question is: can we show explicitly that the new portion of the waveform corresponds to a negative frequency?

Study of Fig. 1 shows that the reversal is equivalent to a jump of phase by angle $\emptyset$ as shown, or we can say that the oscillator jumps from one waveform (A) to a second one (B) at the phase $\omega t_{0}$. If we represent waveform $A$ as $\operatorname{Sin}(\omega t)$, then waveform $B$ is $\operatorname{Sin}(\omega t-\phi)$. What we need to do is determine $\phi$ as a function of $t_{0}$, and from that, break the waveform into a sine and a cosine part. These sine and cosine parts should show negative frequencies. This developes as:

$$
\begin{align*}
& \omega t_{0}=\pi / 2+\phi / 2 \quad \text { (by inspection of Fig. 1) }  \tag{1}\\
& \phi=2 \omega t_{0}=\pi
\end{align*}
$$

hence we have:


$$
\begin{align*}
\sin (\omega t-\phi) & =\sin \left(\omega t-2 \omega t_{0}+\pi\right)  \tag{3}\\
& =\sin \left(2 \omega t_{0}-\omega t\right)  \tag{4}\\
& =\sin \left(2 \omega t_{0}\right) \cos (\omega t)-\cos \left(2 \omega t_{0}\right) \sin (\omega t)  \tag{5}\\
& =\left[\sin \left(2 \omega t_{0}\right)\right] \cdot \cos (-\omega t)+\left[\cos \left(2 \omega t_{0}\right)\right] \cdot \sin (-\omega t) \tag{6}
\end{align*}
$$

where each of the steps in the development involves a well known trig identity. Note that the final result shows a cosine component and a sine component, each with a negative value of $\omega$. The coefficients in [ ] are constants that depend on the value of $t_{0}$. Thus we have succeeded in identifying the time reversal with a description in terms of negative frequency.

Another development worked out by Lester Ludwig has the potential of handling a more general case. It is based on the idea that an FM process can be described as a rotating vector, in which case the waveform is described by the angle $A(t)$ as shown in Fig. 2. In this case, the angle $A(t)$ can be found as:

$$
\begin{equation*}
A(t)=\int_{0}^{t} d A(\tau)=\int_{0}^{t} \frac{d A(\tau)}{d \tau} d \tau \tag{7}
\end{equation*}
$$

where $d A(t) / d t$ is an instantaneous frequency which we can call winst. In a general case, winst could be any function of time, and we can get an answer if we can work the integral. In the special case of an instantaneous change of sign of a constant $\omega$, at time $t_{0}$, we need to break the integral into two parts, but first, let's look at the case where $t$ is less than $t_{0}$.

$$
\begin{equation*}
A(t)=\int_{0}^{t} \omega d \tau=\omega t \tag{8}
\end{equation*}
$$

and thus waveform $A$ of Fig. 1 would be $\sin [A(t)]=\sin (\omega t)$, just what we need. Next we look at the case of $t$ beyond the transition point $t_{0}$.

$$
\begin{align*}
A(t) & =\int_{0}^{t^{\omega} \omega_{i n s t}}(\tau) d \tau=\int_{0}^{t_{0}} \omega d \tau+\int_{t_{0}}^{t}(-\omega) d \tau  \tag{9}\\
& =\left.\omega \tau\right|_{0} ^{t_{0}}-\left.\omega \tau\right|_{t_{0}} ^{t}=2 \omega t_{0}-\omega t \tag{10}
\end{align*}
$$

Thus after $t_{0}, \sin [A(t)]$ becomes $\sin \left(2 \omega t_{0}-\omega t\right)$

which is the same as equation (4), thus establishing that the change to negative frequency does the $A$ to $B$ change of Fig. 1.

These findings make it easier for us to accept the reversing oscillator as the proper one for through-zero FM. Perhaps some readers can suggest some other interesting cases to check, or other methods of implementation.

INTRODUCTION:
I finally got around to building a full filter bank. Perhaps it was put off so long because it was a device that had been around for some time $[1,2,3]$, and the necessary design data had been established [4, 5]. A remaining problem was that it had not been well established just exactly what $Q$ should be used for the filters, making it difficult to actually go ahead and build a filter bank, not knowing if a different $Q$ would give more satisfactory results. Thus a variable-Q filter bank was an attractive idea, and a number of brute-force methods of $Q$ control were available, although they generally greatly increased the complexity of the circuitry. An important idea suggested by Lester Ludwig [6], and independently implemented by Ian Fritz [7] was to use feedback around the whole bank to vary the Q of all the filters simultaneously. This methods is the key to the present design.

Another reason that the bank has taken a long time is that I must have had in the back of my mind the idea that this would be a somewhat laborious and tedious task. As it turns out, this is basically true, and a good deal of what follows is a discussion of construction and tuning ideas and techniques. This is not a project you just build and turn on (and troubleshoot). This one you have to build, turn on and troubleshoot (troubleshooting was no problem, all 39 filters worked in my case), and then tune. If you use trim pots for tuning, you can probably get the bank tuned in a couple of hours. I did mine by hand, substituting in series resistors, and it took me about six hours total, although part of this was concerned with deriving and using the necessary incremental formulas and graphs to make this easier, and you can make use of my result. The use of fixed resistors has the advantage that it is much cheaper (than trim pots) and more stable once completed. Some tuning techniques used in a different sort of filter bank [8] can be used here.

Basically, the filter bank is well worth the effort. I find the high-Q results to be the most interesting, giving metallic and reverberant type sounds. Certainly variable-Q is an important feature, and readers with existing fixed-Q banks should consider patching up an external feedback loop to get the effect, and then very possibly making this into a permanent modification. This design has the undesirable structure of $39 \mathrm{op}-\mathrm{amps}$ feeding into a summer, with the resulting summation of the residual output noise of all 39 . This is not a problem in many applications that I have used [probably because I like to use high-0 and wide bandwidth (heavily modulated) sounds as input]. Readers greatly concerned with noise may find Ian Fritz's design [7] with transistor based filters more suitable.

## BASIC BANDPASS FILTER:

Our basic bandpass filter is of course our beloved "Deliyannis" bandpass that has proven so useful in the past. The analysis is given elsewhere [4] and need not detain us here. We will settle for giving you the basic design equations, reference some calculator programs for this filter [9], and present the data for the 39 filters in tabular form. The filters as designed are set for the center frequency specified, a Q of approximately 25 , and a gain of unity. In the design equations that follow, $R$ is the parallel equivalent of $R^{*}$ and $R^{* *}$ and is thus $R^{*} R^{* *} /\left(R^{*}+R^{* *}\right)$, and this also effectively defines B. The parameter " $a$ " is
 the attenuation of the R1, R2 divider: $a=R 2 /(R 1+R 2)$.

The design equations are:

$$
\begin{align*}
& f_{0}=1 / 2 \pi \sqrt{B R C}  \tag{1}\\
& Q=[(1-a) \sqrt{B}] /[2(1-a)-a B]  \tag{2}\\
& g=B R^{* *} /\left[\left\{R^{*}+R^{* *}\right\}\{2(1-a)-a B\}\right] \tag{3}
\end{align*}
$$

The design procedure is basically to choose freely B ( $=25$ here) and C, and using equation (1) and the desired $f_{0}$, calculate R. Next, equation (2) can be used with the desired $Q$ to get $a$, and an appropriate ratio of R1 and R2 to get this value $a$. Getting the attenuation factor $R^{* *} /\left(R^{*}+R^{* *}\right)$ from (3) is a bit roundabout, since the parallel resistance condition must be met. It is easiest to calculate $\mathrm{g}^{\prime}$, the gain that would occur if $R^{\star}=R, R^{\star *}=\infty$, which is given by:

$$
\begin{equation*}
g^{\prime}=B /[2(1-a)-a B] \tag{4}
\end{equation*}
$$

In which case, $R^{*}$ and $R^{* *}$ are given by:

$$
\begin{equation*}
R^{\star}=g^{\prime} R \quad R^{\star \star}=g^{\prime} R /\left(g^{\prime}-1\right) \tag{5,6}
\end{equation*}
$$

These equations you can use if you need to create any new data, as would be necessary for example if you were using different values of capacitors. Other than that, you need not worry about these except as they relate to the tuning process which will be described later.

All we need now is the center frequencies, and we will use the technique suggested by Burhans [5], starting with 50 Hz and spacing the peaks at the fifth root of 2.1 from then on (approximately $16 \%$ apart - by comparison, three semitones are about 19\% apart). Once these frequencies were generated, the calculator program [9] was used to calculate the component values using closest $5 \%$ resistors. It must be realized at this point that these $5 \%$ values are probably not exactly the ones you will end up with. What they are are good ballpark values. If you had a good selection of $5 \%$ resistors that you could measure and select, and if you had capacitors that were accurate to $1 \%$ or better, you could probably just put the thing together. Probably you will be working with capacitors that are 5\% or 10\% at best. Try not to use capacitors of unmarked tolerance, because they may be $20 \%$ or even much worse. I used LF13741(SL30005) op-amps for the channels below 1000 Hz and LF351 op-amps for those above 1000 Hz , although this is probably not an essential transition point, and LF13741's may go a few higher. LF351's can be used for all op-amps. See the table on page 5 for the component values.

THE STRUCTURE OF THE OVERALL BANK:
Fig. 2 shows the overall structure of the filter bank. The squares are the filters of Fig. 1, numbered 1, 2, ... 39, according to Table 1. Op-amp A40 serves as an input summer and buffer, while A41 serves to sum the output of all 39 channels. A42 offers an inverted output which can be connected with the regular output through a pot, thus providing a means of enhancing or cutting the $Q$ with positive or negative feedback. The second input resistor is useful for an external loop achieving voltage-controlled $Q$ with a VCA.

Fig. 2


| Number | Frequency | R* | $\mathrm{R}^{* *}$ | C | BR | R1 | R2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 50.0 Hz | 820k | 6800 | 0.1 mf | 150k | 15k | $1200+22$ |
| 2 | 57.7 | 750k | 5100 | 0.1 mf | 150k | 15k | 910 |
| 3 | 67.3 | 620k | 4700 | 0.1 mf | 120k | 15k | $1000+51$ |
| 4 | 78.2 | 510k | 3600 | 0.1 mf | $100 k+16 k$ | 15k | 820 |
| 5 | 90.5 | 1 M | 6800 | 0.05 mf | 180k + 3.3 k | 15k | 1000 |
| 6 | 104.8 | 820k | 6200 | 0.05 mf | 150k | 15k | $1100+15$ |
| 7 | 121.8 | 680k | 5600 | 0.05 mf | $120 k+3 k$ | 15k | $1200+30$ |
| 8 | 141.3 | 1.3 M | 10k | 0.022 mf | $240 \mathrm{k}+24 \mathrm{k}$ | 15k | $1000+12$ |
| 9 | 164.0 | 1.2M | $8.2 k+680$ | 0.022 mf | 220 k | 15k | $1000+82$ |
| 10 | 189.9 | 1 M | 7500 | 0.022mf | 180k + 15k | 15k | $1000+30$ |
| 11 | 220.1 | 910k | 6800 | 0.022 mf | 160k | 15k | $1000+51$ |
| 12 | 255.7 | 750k | 5600 | 0.022 mf | 120k + 24 k | 15k | 1000 |
| 13 | 296.7 | 680k | 5100 | 0.022 mf | $100 \mathrm{k}+18 \mathrm{k}$ | 15k | $1100+68$ |
| 14 | 334.1 | 560k | 4300 | 0.022 mf | $100 k+3.6 k$ | 15k | $1100+20$ |
| 15 | 399.0 | 470k | 3600 | 0.022 mf | $91 \mathrm{k}+1 \mathrm{l}$ | 15k | $1000+47$ |
| 16 | 462.8 | 430k | 3000 | 0.022 mf | 82 k | 15k | $910+62$ |
| 17 | 535.9 | 360k | 2700 | 0.022 mf | 68 k | 15k | $1000+68$ |
| 18 | 622.8 | 330k | 2400 | 0.022 mf | $56 \mathrm{k}+620$ | 15k | $1100+51$ |
| 19 | 721.8 | 270k | 2000 | 0.022 mf | $47 \mathrm{k}+3.6 \mathrm{k}$ | 15k | $1000+62$ |
| 20 | 838.1 | 240k | 1800 | 0.022 mf | $39 \mathrm{k}+2.7 \mathrm{k}$ | 15k | $1100+68$ |
| 21 | 972.4 | 200k | 1500 | 0.022 mf | $36 \mathrm{k}+1.2 \mathrm{k}$ | 15k | $1000+82$ |
| 22 | 1129.2 | 180k | 1300 | 0.022 mf | $30 \mathrm{k}+2.2 \mathrm{k}$ | 15k | $1000+91$ |
| 23 | 1308.3 | 150k | 1100 | 0.022 mf | $27 \mathrm{k}+1 \mathrm{k}$ | 15k | $1000+56$ |
| 24 | 1519 | 130k | 910 | 0.022 mf | $24 \mathrm{k}+1.1 \mathrm{k}$ | 15k | $910+62$ |
| 25 | 1761 | 2.4 M | 18k | 1000pf | $430 \mathrm{k}+27 \mathrm{k}$ | 15k | $1000+51$ |
| 26 | 2044 | 2 M | 16k | 1000pf | $360 k+22 k$ | 15k | $1100+27$ |
| 27 | 2369 | 1.8 M | 15k | 1000pf | $300 \mathrm{k}+3.3 \mathrm{k}$ | 15k | $1300+39$ |
| 28 | 2747 | 1.5 M | 12k | 1000pf | $270 k+12 k$ | 15k | $1100+39$ |
| 29 | 3189 | 1.3 M | 10k | 1000pf | $240 k+11 k$ | 15k | $1000+68$ |
| 30 | 3697 | 1.24 | 8200 | 1000pf | $220 \mathrm{k}+7.5 \mathrm{k}$ | 15k | $910+68$ |
| 31 | 4288 | 1M | 7500 | 1000pf | 180k + 5.1k | 15k | $1000+91$ |
| 32 | 4974 | 910k | 6200 | 1000pf | $160 \mathrm{k}+6.2 \mathrm{k}$ | 15k | $1000+4.7$ |
| 33 | 5771 | 750k | 5600 | 1000pf | $130 \mathrm{k}+6.8 \mathrm{k}$ | 15k | 1100 |
| 34 | 6694 | 620k | 4700 | 1000pf | $120 \mathrm{k}+1.2 \mathrm{k}$ | 15k | $1000+39$ |
| 35 | 7763 | 560k | 4300 | 1000pf | $91 \mathrm{k}+7.5 \mathrm{k}$ | 15k | $1100+82$ |
| 36 | 8997 | 470k | 3600 | 1000pf | $82 \mathrm{k}+5.1 \mathrm{k}$ | 15k | $1100+10$ |
| 37 | 10444 | 390k | 3000 | 1000pf | $75 k+3 k$ | 15k | $1000+30$ |
| 38 | 12122 | 360k | 2400 | 1000pf | $68 k+4.3 k$ | 15k | $820+68$ |
| 39 | 14056 | 300k | 2200 | 1000pf | $56 k+3 k$ | 15k | $1000+4.7$ |

There are a number of possible additions to the basic structure of Fig. 2 that can be useful, but the reader can plainly see that most of what needs to be done involves the bandpass filters themselves. The rest is pretty simple.

## CONSTRUCTION:

We don't usually need to talk much about construction, but it will be useful here. If you go out and buy all $1 \%$ components, you can pretty much just put the circuit together. If you prefer to use trim pots, then you must leave room for two of them per filter. The first trimmer would probably be about 500 ohms, and it, along with a fixed series 1 k resistor would form R2. The second trimmer would go with a fixed series resistor to form BR, and its value should take up about 20\% either way about the tabulated value of BR. That's two ways to go.

The third way to go, the way I did, is to work with capacitors on hand and a good supply of $5 \%$ resistors, allowing for the fact that it's going to take some time to tune these up. If you go this way, I suggest that you follow the procedure I outline here with some care. First of all, if you have never used my top of the board only construction technique [10], use it here, even if you never intend to again. You have to be able to change resistors, and flipping the board and pulling leads through holes would seem to add greatly to the nightmare. A suggested layout for the top side construction is shown in Fig. 3 for a single filter stage. Naturally you will have to duplicate this structure 39 times, so look at it carefully and perhaps work out dimensions on paper. So you work this out, paint and etch the board. In some projects, this gets you about $40 \%$ done. Here you are just getting started at about 10\%!

A couple of important points right here. You are going to have to build each filter by first trying the suggested components, and then making corrections according to the procedures given below. Secondly, you have to test and revise as you go. Thirdly, the heat of the soldering iron is going to affect the components, so once you solder in a change, you have to wait perhaps a full minute or more for the components to settle down to the ambient temperature before testing. [While you are waiting, you start putting in the components for the next stage.]

TUNING AS YOU GO:
You will note from Fig. 3 that there are seven resistors total while Fig. 1 shows only five per section. Note that in Fig. 3, BR and R2 are shown as a series combination of two resistors (in lucky cases, you only need one). Basically, BR is adjusted to get the frequency right, and then R2 is adjusted to get the gain unity. In the process of getting the gain unity, the $Q$ comes up nearly perfect, as long as $B R$ is not too far from nominal. That's the basic idea - here are the individual steps:

1. Install the op-amp, resistors $R^{*}, R^{* *}$, the two capacitors $C$, and the resistor R1 $=15 \mathrm{k}$, getting values from Table 1. These parts you will not change.
2. Install half the resistors $B R$ (the left side) and R2 (the right side). Choose for the half BR the larger portion of the combination shown (using a short jumper wire for the right portion of BR). For R2, choose the larger shown of its combination. Run a clip lead from the left lead of this trial R2 to ground.
3. Turn on the filter and find its center frequency (making sure time enough has been allowed for the resistors to cool). This value
should be high. If it is low, replace this trial BR with the next lower 5\% value. Now its high say. Take the percentage it is high, multiply this percentage by 2 , and use for the second half (right side) of $B R$ this percentage of the original value. [see further details below]. This should bring the frequency to within the expected drift range (say $1 \%$ or better).
4. With the filter now on frequency, check the gain. If it is high, reduce the trial R2 to the next lower $5 \%$ value. If it is lower than 0.5 , replace the trial value of R2 with larger $5 \%$ values until the gain is between 0.5 and 1.0 . Use the graph in Fig. 4 to estimate the needed series part of R2 to make up the needed gain. Choose this resistor, and connected it between your clip lead and ground. If the gain is not within $1 \%$ of unity, try a lower or higher $5 \%$ value to lower or raise the gain. If you are fairly close, try different individual resistors of the same value for the best fit. Solder this in, and you're done.

DISCUSSION OF THE TUNING PROCEDURE:
The procedure above may appear a bit complex, and it will take you a half-hour or so to do your first filter, but your next few will go faster, and very soon you will have the procedure well memorized. This section is intended to give you a better understanding of the procedure, and then we will look at an example.

First, with regard to the tuning of the frequency. The reason we want to get the frequency a bit high to begin with is so that we can correct it with a smaller series resistor. The first resistor gets you in the ballpark. The formula for the correction is a ballpark correction. A ballpark value plus a ballpark correction comes out to a percent or two. The ballpark correction is obtained by just taking a derivative of the frequency as a function of BR. A slight modification of equation (1) is:

$$
\begin{equation*}
f_{0}=1 / 2 \pi \sqrt{B R} \sqrt{R} C \tag{7}
\end{equation*}
$$

from which we get:

$$
\begin{equation*}
d f_{0} / d(B R)=1 / 2 \pi \sqrt{R} C(-1 / 2)(1 / B R)(1 / \sqrt{B R})=f_{0}(-1 / 2 B R) \tag{8}
\end{equation*}
$$

or:

$$
\begin{equation*}
\frac{d(B R)}{B R}=-2 \frac{d f_{0}}{f_{0}} \tag{9}
\end{equation*}
$$

which says basically that the percent change of resistance needed is twice the percent error in frequency.

A similar derivative procedure can be used for the gain, but this is a lot messier, as the derivative depends on R2 itself and on its square in a complex way. Thus we have dispensed with the formulas in favor of a graphical method (Fig. 4).


The solid curve in Fig. 4 is the calculated curve, while the small $x$ markes show some actual resistor and gain changes that I found in my bank while tuning it. The curve will get you close to the needed value.

Note two things about resistor tolerance. First, if you are close, there is a chance you can get excellent agreement if you search through your whole set of resistors for a given value. Second, 5\% resistors tend to cluster well inside 5\%, and perhaps inside $3 \%$. This means that when you are correcting the gain, you can have cases that just done seem to work out. For example, you may find that 82 ohms added gives a gain that is $4 \%$ low while 91 ohms added gives a gain $4 \%$ high. Search all your 82 and 91 ohm resistors, and you don't find any reaching to the 86 or 87 ohm value you obviously need. Solution? Well, use the 82 ohm value and hunt up a 5 ohm resistor somewhere. Or, look for some $10 \%$ tolerance units which have a greater spread. Or, change the larger valued resistor to a different one of the same nominal value. It is around 1100 ohms and is $1 i k e l y$ to be off by more than 5 ohms from the former value. Then perhaps the 82 or 91 ohms added will work, or it may be somewhere else altogether, 56 ohms say. The key is to get a reading on the performance first, and then make a correction to this observed performance.

## AN EXAMPLE:

Let's run through one example. We design for $f_{0}=1000 \mathrm{~Hz}, \mathrm{Q}=25$, and gain $=1$. We have $R 2=1000$ ohms and $B R=33000$. The performance we observe is $f_{0}=$ 1047 Hz and $\mathrm{g}=0.37$. First we correct the frequency by adjusting BR. Note that the frequency is high, the way we want it. The frequency is $4.7 \% \mathrm{high}$, so we want to add twice this, or $9.4 \%$ to BR (which is 33000). $9.4 \%$ of 33000 is 3102 , so we try adding a 3.0 k resistor to the 33 k one we have. This causes the frequency to go to 1002 Hz , well within our needs, and at the same time, the gain is up to 0.58 , since effectively we have increased B a bit. Now, the frequency is fine, and won't be changed by R2. We consult Fig. 4, seeing that we need to add ( $1-0.58$ ) $=0.42$ to our gain. We would like something like 86 ohms. We try 91 ohms, a standard value. The gain comes out 1.05 , too high, so we try 82 ohms, and get 0.97 , a bit too low. So we change to a different nominal 1 k resistor for the basic value. Say it's actually 1011 ohms if we measured it. If we add 75 ohms to this, we get 1086 total, and the gain is 1.00 , just what we need.

The above example shows what is probably the maximum bad luck that you are likely to run into. Most filters will come up fine with just the two correction resistors, and a few will come up near perfect with no corrections.

WHEN TO STOP:
You should keep in mind that at some point you are going to have to stop the tuning process - there is always the temptation to get it "just a little better." Two things to keep in mind: First, if this were just an ordinary fixed-Q filter bank, you do not have to go overboard with accuracy, particularly with regard to peak gain, as this will not change the effect much. However, this bank uses feedback to achieve higher Q's, and this means that some additional care must be given to the equalization of gains. As the $Q$ is increased, the filter with the highest gain will oscillate first (in general), so it necessary to have the gains well equalized so that very high $Q$ can be achieved without any filter oscillating. Secondly, even after the filters are constructed, you have one last shot at equalizing the gains - when you do the summing of all filters. In fact, the gains at the output of the summer are the ones that really matter. Also, it is much easier to adjust gains at the summer stage since you have just an ordinary inverting summer, and gain changes are directly proportional to resistor changes.

If we want to make some rules, let's say the frequencies should be adjusted within $1 \%$. The gains of the filters themselves should be within $3 \%$, with the idea in mind that they will be adjusted to within $1 \%$ in the final summing process.

You should also get some feeling for temperature drift. Mcasure a few center frequencies from time to time. Some may vary more than $1 \%$ (particularly with ceramic capacitors). Try to see if they are all varying proportionally. If so,
there is less to worry about, because the selection of 50 Hz as the base frequency is totally artificial anyway. What would be of more concern would be if the individual filters drifted toward each other.

COMPLETING THE CIRCUITRY:
With the 39 filters tuned up properly, all that remains is to complete the circuitry of Fig. 2. Basically, you start by just summing into A41 with 100 k resistors. Then you measure the response. It will not be the simple sum of all individual responses, because phase must be considered in the overlap. Thus each channel is reinforced to a degree by the "skirts" of side channels, but not as much as one would expect, because the contribution of an upper skirt is not in phase with that of a lower skirt. In fact, this phase difference approaches $180^{\circ}$ so the contribution is relatively minor. [The ends of the bank are a special case of course.] As a result, expect that the unity gain peaks you established individually will be a bit higher (perhaps $8 \%$ or so). This is normal, and is not too important. What is important is that the peaks be leveled off to the same value. Thus you should add series resistors to the 100 k summing resistors proportional (relative to 100 k ) to the amount of gain you want to reduce. For example, if one channel has a gain of 1.05 , you need to reduce to 1.00 , thus by 0.05 , and this is done by adding a $0.05 \times$ $100 \mathrm{k}=5 \mathrm{k}$ resistor in series with the 100 k . You can reduce the gains to unity once more, or you may just reduce to your lower values, slightly above unity gain. The important thing is that all be matched to about $1 \%$.

Fig. 5 shows the actual response of the leveled bank ( 3 volts RMS in). Actually I drew straight lines between max and min points, even though the actual peaks are rounded. Fig. 6 shows the response of an individual filter. Note that the peaks


Fig. 5 (above), Response of the Filter Bank with No Feedback Fig. 6 (right), Response of a Single Filter ( 622.8 Hz ) of

Fig. 5 showing $Q$ of 25 .
are very level, while the valleys are less uniform in depth, but this is of less concern because they represent areas of reduced response. Presumably, more exact valley levels could be obtained if the spacing were more accurately adjusted, but this is not necessary here. In fact, the leveling of the peaks is only required because we want to increase the Q of the bank with overall positive feedback, and we need a uniform approach to high $Q$ to avoid having one filter just getting up to high values while another is already oscillating. Without feedback, the Q's of the filters are about 25 . With full negative feedback, the filters go to a Q of about 14 (see Fig. 7) and with full


Fig. 7, Response with Negative Feedback $(Q=14)$


Fig. 8, Response with Positive Feedback (Q ~ 100)
positive feedback (see Fig. 8), the $Q$ goes to about 100. Between the extremes (Fig. 7 and Fig. 8), the peak-to-valley ratio changes from about $4: 1$ to about $25: 1$. The subjective effect is much much greater - one might guess 1000:1 or more. Sounds through the low-Q filters sound fairly normal, while those through the high-Q filters sound reverberant, ringy, and "tinny".

In a later issue, we will discuss applications of the filter bank, possible alterations, and the way our results relate to other reported filter banks.
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