Boomslitter & Creel have shown [J. Music Theory 7 #2 (1963)] that even over relatively short passages, opinions on the exact "correct" pitch for the same written pitch can vary due to local references. This sort of result is something for the engineer to consider since often a very strict set of pitches is used, and the ones that would be "correct" are not even available.

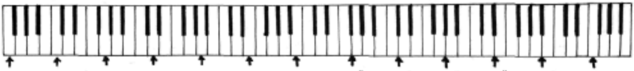
The situation on rhythm is much the same as pitch. Not only does tempo vary greatly, but the actual lengths of the notes in time may vary greatly from their written values. For example a dotted half note followed by a quarter may not be given the expected 3/4 and 1/4 time values. Instead a much larger lowest common denominator is required in many cases, e.g., 45/64 and 19/64. Again, an electronic sequencer will output rhythms with extreme precision (musically wrong?).

Probably the most important clue to an electronic sound for an average listener is the extreme precision of waveforms, stability and timing. This is not to imply that things would be better if things were made sloppy, as they just sound exactly sloppy in this case. Instead, a careful study of the implications of studies of musical accuracy should be made.

MUSICAL SCALES

It is the usual practice to divide the continuous range of pitch into discrete steps (called tones) with octave ratios (2:1) serving as the main markers. The choice of the number of tones per octave is not arbitrary since it is also the usual practice to play more than one pitch at one time, and therefore the subjective effects of various combinations must be considered. The subjective description is generally either consonant or dissonant. Dissonant intervals have long been associated with unpleasant sounds, but this is no longer true as modern music has made them sufficiently familiar to be accepted. Consonant intervals are generally associated with combinations for which the frequency ratio can be expressed by a small integer ratio. Dissonant intervals have high integer ratios or are irrational ratios. Regardless of where the individual sets the dividing line, the most useful scales are those which make possible both consonant and dissonant intervals. Thus, it is necessary to set some notes at small integer ratios.

The simplest non octave ratio is 3:2, the musical fifth. We could thus try to build a scale based on the fifth. We could start on musical F for example and build with fifths above it:



After 12 fifths, we have gone 7 octaves and arrived back at the starting note (F) and have generated a 12 tone scale. However, things are not quite right as 7 octaves is a ratio of $2^7 = 128$, while 12 fifths are $(3/2)^{12} \approx 129.74632....$, so if we want perfect octaves, the fifths will have to be a little flat. The scale based on fifths was suggested by Pythagoreas (600 BC). It turns out that scales based on other simple ratios offer no real improvement, and the thing never comes out right.

It can be demonstrated that the 12 tone per octave scale can be arrived at on the basis of the best fit to the simplest integer ratios. The procedure has been described by van Hoerner ["Universal Music" EN#35]. You first start with a prime number (5 in this case) and take all lower primes (i.e., 3 and 2 in this example). Next you consider all possible ratios of these numbers:

For prime 5:	5/4	4/5	5/3	3/5	5/2	2/5	5/1	1/5
For prime 3:	3/2	2/3	3/1	1/3				
For prime 2:	2/1	1/2						

Next, all ratios that are more than an octave are removed - leaving:

5/4	4/5	5/3	3/5
3/2	2/3		

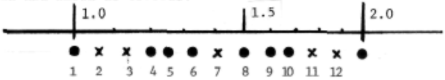
Then those ratios that are less than one are doubled (raised to one octave higher):

5/4	8/5	5/3	6/5
3/2	4/3		

These are all the possible simple integer ratios for prime numbers up through 5. These are all musical intervals named as follows:

Ratio	Name	Closest tone in C Scale
5/3 = 1.67	Major 6th	A
8/5 = 1.60	Minor 6th	G#
3/2 = 1.50	Perfect 5th	G
4/3 = 1.33	Perfect 4th	F
5/4 = 1.25	Major 3rd	E
6/5 = 1.20	Minor 3rd	D#

These are all well known consonant intervals. It is interesting to plot these ratios on a log scale (shown as dots below). Note that the spacing is pretty regular, and gaps in the scale are shown as crosses:



Thus we can have good approximations to all the small integer ratios, and if we fill in the gaps we can have a complete 12-tone scale. The various gaps can be filled by musical intervals from the table listed below. Most of these are relatively small integer ratios, but are large enough to supply dissonant intervals:

Gap Position	Ratio	Name	Comments
2	16/15 = 1.0667	Semitone	denoted by s
3	10/9 = 1.1111	Minor tone	denoted by m
3	9/8 = 1.1250	Major tome	denoted by M
7	45/32 = 1.4063	Augmented 4th	45/32 = (9/8)·(5/4)
7	64/45 = 1.4222	Diminished 5th	64/45 = (4/3)·(16/15)
11	7/4 = 1.7500	Harmonic minor 7th	
11	16/9 = 1.7777	Grave minor 7th	
12	9/5 = 1.8000	Minor 7th	
12	15/8 = 1.8750	Major 7th	

With the gaps filed, we can choose a complete 12 tone scale. This is called a just tuned scale since all the ratios are integer ratios. We will show how a just major scale can be obtained. This will also serve to show how some of the intervals that were used to fill the gaps were arrived at, since to this point, they were given somewhat arbitrarily. Successive tones of the scale will be obtained by multiplying the lower one by either a semitone (s)-16/15, a minor tome (m)-10/9, or a major tone (M)-9/8.

Just Major Scale	Ratio	Step	Interval
C	1/1		Unison
D	1·(9/8) = 9/8	M	Major tone
E	(9/8)·(10/9)= 5/4	m	Major 3rd
F	(5/4)·(16/15)= 4/3	s	Perfect 4th
G	(4/3)·(9/8) = 3/2	M	Perfect 5th
A	(3/2)·(10/9)= 5/3	m	Major 6th
B	(5/3)·(9/8)= 15/8	M	Major 7th
C	(15/8)·(16/15)= 2	s	Octave

This is a just tuned major scale. It has one major drawback. Once an instrument is tuned for one key, the intervals will be different if it is used in another key. For example, the fifth from C to D is a perfect 5th with a ratio 3/2 = 1.50. If we try to play in the key of D, the fifth from D to A would have a ratio 40/27 = 1.481... and *this is not a perfect fifth. The reason it is necessary to play* in more than one key is that much of western music "modulates" from one key to another as the piece progresses. The compromise that has been arrived at is called the "equal tempered" tuning method where all intervals are spaced by an integer number of half steps. Thus all scales are equally good in any key, and by the same token, equally bad where errors occur.

When twelve tones are spaced so that the intervals are all equal, they are spaced at the 12th root of 2 apart. The half step is thus a ratio of 1:1.059... and the full scale becomes:

Equal Tempered Scale (chromatic version)	Ratio	Interval Name	Just Interval
*C	1.0000	Unison	1.0000
C#	1.0595	Half step	1.0667
*D	1.1225	Whole step	1.1250
D#	1.1892	Minor third	1.2000
*E	1.2599	Major third	1.2500
*F	1.3348	Fourth	1.3333
F#	1.4142	Diminished 5th	1.4063 or 1.4222
*G	1.4983	Fifth	1.5000
G#	1.5874	Minor 6th	1.6000
*A	1.6818	Sixth	1.6667
A#	1.7818	Minor 7th	1.8000
*B	1.8877	Major 7th	1.8750
*C	2.0000	Octave	2.0000

*Tones for equal tempered major scale

The 12-tone equal tempered scale is a natural scale for electronic music systems since a set of equal valued resistors can be used in a voltage divider to supply all the voltages required for an exponential VCO. A one volt/octave VCO for example gets voltages that are 1/12 volt apart. It is possible to obtain many other equal tempered scales as well by scaling the VCO response up or down. Scales of 19, 31, and 52 tones are of interest because they provide tones that are good approximations to many of the just intervals.