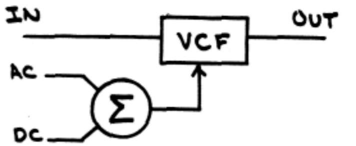
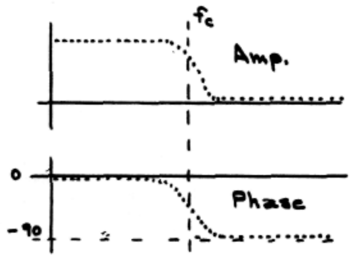
FORMANT MODULATION

Formant modulation is the alteration of tone color by means of moving the position of a filter characteristic relative to the frequency of an input signal. In practice, the type of patch shown at the right can be used. We will assume that it is the characteristic frequency of the filter that is being altered by the AC component, although other parameters may be varied at times. The effect, particularly when used with a filter response that has an amplitude peak and a low modulation frequency, is known colloquially as a "Wow-Wow." What is actually happening can be quite complex mathematically. Consider for example a fairly simple filter response - the single pole low-pass. The amplitude and phase responses are sketched at the right. When f_c is swept by a voltage, both the amplitude and phase of any spectral component in the input waveform that lies fairly close to f_c will be modulated both in amplitude and in phase at the filter output. [Moog, AES Convention, Sept. 1974 reported measuring the phase change at as much as 1000°/octave when corner peaking of the filter was used]. In addition to the combined phase and amplitude modulation problem, calculations can be expected to be further complicated by (1) The fact that the response curves are not linear. This means that even for sinusoidal modulating signals more than two sidebands will be generated for the AM part of the overall modulation, and sideband amplitudes will be altered for the phase modulation part of the modulation. (2) If an exponential VCF is used, the sideband amplitudes will be further altered. However, the output pitch will not rise as it does in the case of an exponential VCO as modulation depth increases. The filter can not alter the input frequency except by phase modulation, an effect that is either too small (slow sweep) or too rapid (fast sweep and reversing) to be heard as a pitch shift.





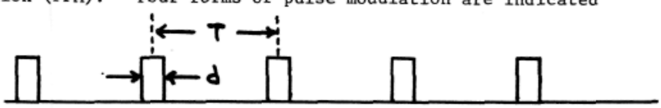
The total result can be expected to be a mathematically complex distribution of sidebands. However, only the computation of amplitudes is really complex. The sideband spacing is the normal modulation frequency, and the sidebands are centered about the original carrier. The subjective effect is in many ways similar to other phase modulations. With complex filter responses however, very rapid changes of phase and/or amplitude can produce some unique effects.

Many different parameters of a filter's response can be made voltage controlled and hence can be modulated. Voltage-controlled Q is a feature of many VCF's. This has the effect of controlling the sharpness of the resonance or of a peaked corner. Thus, the particular harmonic(s) closest to the characteristic frequency undergo amplitude modulation and generally simultaneous phase modulation.

PULSE MODULATION

There are numerous types of pulse modulation that can be used to produce complex spectra. Furthermore, most of these are easily implemented with simple circuitry or with existing synthesizer modules. The various types of pulse modulation we can consider include Pulse Amplitude Modulation (PAM), Pulse Width Modulation (PWM) of several forms, and Pulse Position Modulation (PPM). Four forms of pulse modulation are indicated below:

UNMODULATED PULSE TRAIN

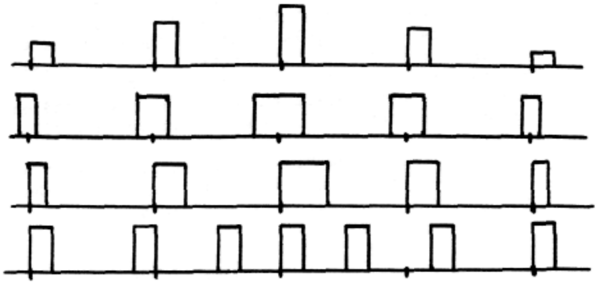


PULSE AMPLITUDE MODULATION

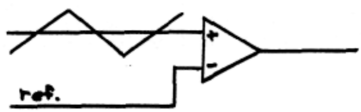
PULSE WIDTH MODULATION (Centered)

PULSE WIDTH MODULATION (Fixed Edge)

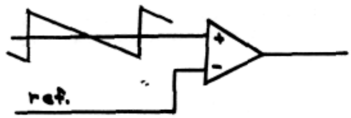
PULSE POSITION MODULATION



Pulse amplitude modulation can be implemented with a VCA, sample-and-hold, and a gating circuit. Pulse width modulation can be implemented with either a fixed center of a fixed edge by changing the basic type of driving waveform from something like a triangle to something like a sawtooth:



CENTERED PWM



FIXED EDGE PWM

Pulse position modulation is implemented by triggering a monostable from a driving waveform and varying the trigger point. For example, the monostable could be triggered when a driving sawtooth crossed a certain reference level. The reference level is then made to be the modulating signal.

Various types of pulse modulation have been studied [e.g., G. M. Russell, Modulation and Coding in Information Systems, Prentice-Hall 1962]. The basic starting point is the unmodulated pulse train represented by its Fourier series:

$$e(t) = \underbrace{Ad/T}_{\text{DC}} + \sum_{n=1}^{\infty} \underbrace{\frac{2A}{n\pi} \mathrm{Sin}[n\pi d/T]}_{\text{Fourier Coefficient}} \underbrace{\mathrm{Cos}(n\omega_o t)}_{\text{Fourier Component}} \qquad \omega_o = 2\pi/T$$

For the analysis of PAM, the term A is replaced by $A(t) = A(1 + m \,\mathrm{Cos}\, \omega_m t)$. With this plugged in, it is easy to see that the "1" gives back the original unmodulated pulse train. In addition, two terms are produced by the " $m \,\mathrm{Cos}\, \omega_m t$" term. The terms are:

$$\frac{Amd}{T} \mathrm{Cos}\, \omega_m t$$

and:

$$\sum_{n=1}^{\infty} \frac{2Am}{n\pi} \mathrm{Sin}(n\pi d/T) \; [\mathrm{Cos}\, \omega_m t \cdot \mathrm{Cos}(n\omega_o t)]$$

The first term is the modulating signal. The second term is the product of two cosines (balanced modulation) of the modulating signal and each of the Fourier components of the original unmodulated pulse train. These balanced modulation terms when combined with the Fourier components give a "carrier" for each Fourier component and two AM sidebands. The term in Cos $\omega_m t$ results from the DC term in the unmodulated pulse train. It can thus be seen that an alternative analysis could be based on the amplitude modulation of each of the Fourier components in the unmodulated pulse train. This is really no surprise.

When considering PWM, it is the term d that has to be replaced by a time varying term $d(t) = d + d_m \text{ Cos } \omega_m t$. This has assumed the centered form of PWM. Plugging this back into the unmodulated pulse train gives:

$$e(t) = Ad/T + (Ad_m/T) \text{ Cos } \omega_m t$$

$$+ \sum_{n=1}^{\infty} \frac{2A}{n\pi} [\text{Sin}(n\pi d/T + \frac{n\pi d_m \text{ Cos } \omega_m t}{T}) \text{ Cos}(n\omega_o t)]$$

The first term Ad/T is the average DC term while the second is a time varying DC term that varies with ω_m. This can be seen simply as the fact that the duty cycle changes with ω_m and this changes the DC weighting of any small time segment of the signal. The third term is the most interesting. It can be expanded using the identity for Sin(X+Y) and gives:

$$\sum_{n=1}^{\infty} \frac{2A}{n\pi} [\text{Sin}(n\pi d/T)\{\text{Cos}(\frac{n\pi d_m}{T} \text{ Cos } \omega_m t)\}] \text{ Cos}(n\omega_o t)$$
$$+ \sum_{n=1}^{\infty} \frac{2A}{n\pi} [\text{Cos}(n\pi d/T)\{\text{Sin}(\frac{n\pi d_m}{T} \text{ Cos } \omega_m t)\}] \text{ Cos}(n\omega_o t)$$

Reference to the earlier discussion of FM will show that the terms within the {} are Bessel series that can be expanded in terms of integer multiples of ω_m. The analysis closely parallels the phase modulation results. It soon becomes evident however that the "bookkeeping" becomes excessive here and that a thorough analysis would require a good deal of effort. Here we will just remark that the problem has been outlined and that it can be seen that analysis is similar to phase modulation. The one additional feature is the presence of a term at the modulating frequency which is not present in the general phase modualtion problem.

When we move on to fixed edge PWM, it can be seen that we first have to solve the PPM problem since we have to add a displacement to the term T: $T(t) = T + \tau \text{ Cos } \omega_m t$. The problem becomes very complex since the T term appears in the denominator of the pulse train expression. Russell has outlined a solution in an approximation that is not valid for the large modulation depth used in electronic music. Solution to the problem would seem to require a computer to do all the bookkeeping.

TIME SAMPLING

A system for time sampling a waveform is implemented as shown at the right. A sample-and-hold module is used to break a wavefrom into discrete steps. We will be concerned here with the case where both the input waveform and the sampling rate are periodic. We shall be interested in only the resulting spectrum of the output. We shall not be concerned with the recovery of any information.