CALCULATION OF THE POLES OF 90° PHASE DIFFERENCE NETWORKS [after Weaver]

1. Select F_ℓ and F_h, the lower and upper ends of the bandwidth.

2. Calculate:

$$k = [1 - (F_\ell/F_h)^2]^{1/2}$$

$$L = (1/2)\frac{1 - \sqrt{k}}{1 + \sqrt{k}}$$

$$Q' = L + 2L^5 + 15L^9 + \cdots$$

$$Q = e^{\pi^2/\log_e(Q')}$$

3. Select a phase error that can be allowed, and consult the graph below to determine the minimum corresponding number of poles "n".

4. Choose two networks, A and B. If n is even, there will be n/2 poles in each network. If the number of poles is odd, put an extra one in network A.

5. Compute the angles for A and the poles p_a:

$$\phi_a(r) = (45°/n)(4r - 3) \qquad \text{for } r = 1, 2, \ldots (n/2) \text{ or } [(n+1)/2]$$

$$\phi'_a(r) = \mathrm{ARCTAN}\left((Q^2-Q^6) \sin 4\phi_a(r) / [1 + (Q^2+Q^6) \cos 4\phi_a(r)]\right)$$

$$p_a(r) = [1/\sqrt{(F_\ell/F_h)}]\ \mathrm{TAN}[\phi_a(r) - \phi'_a(r)]$$

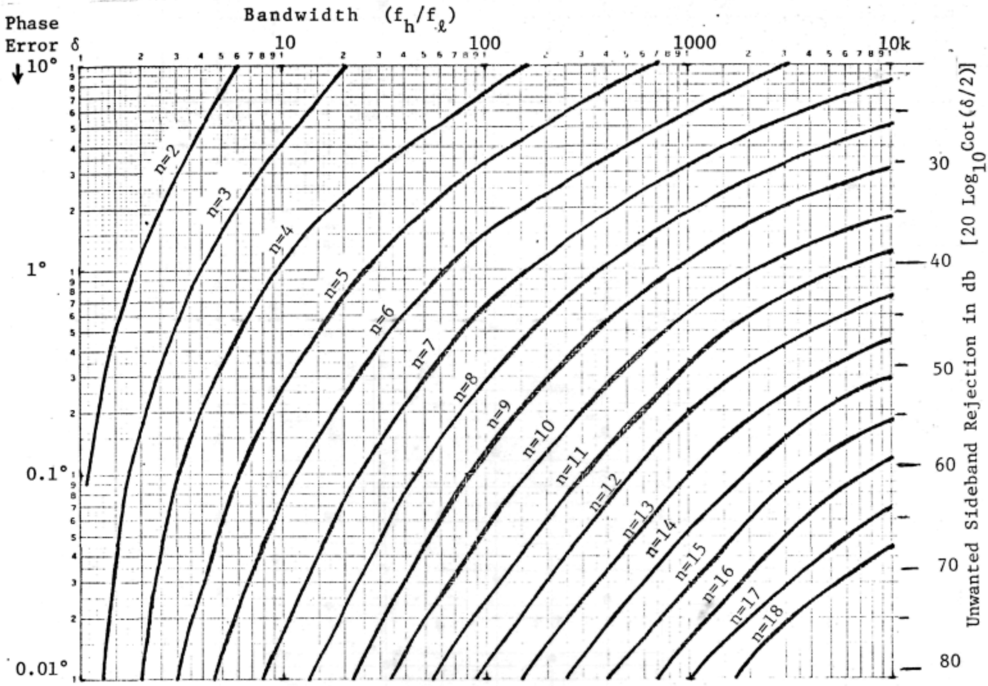
and the angles for B with its poles p_b:

$$\phi_b(r) = (45°/n)(4r - 1) \qquad \text{for } r = 1, 2, \ldots (n/2) \text{ or } [(n-1)/2]$$

$$\phi'_b(r) = \mathrm{ARCTAN}\left((Q^2-Q^6) \sin 4\phi_b(r) / [1 + (Q^2+Q^6) \cos 4\phi_b(r)]\right)$$

$$p_b(r) = [1/\sqrt{(F_\ell/F_h)}]\ \mathrm{TAN}[\phi_b(r) - \phi'_b(r)]$$

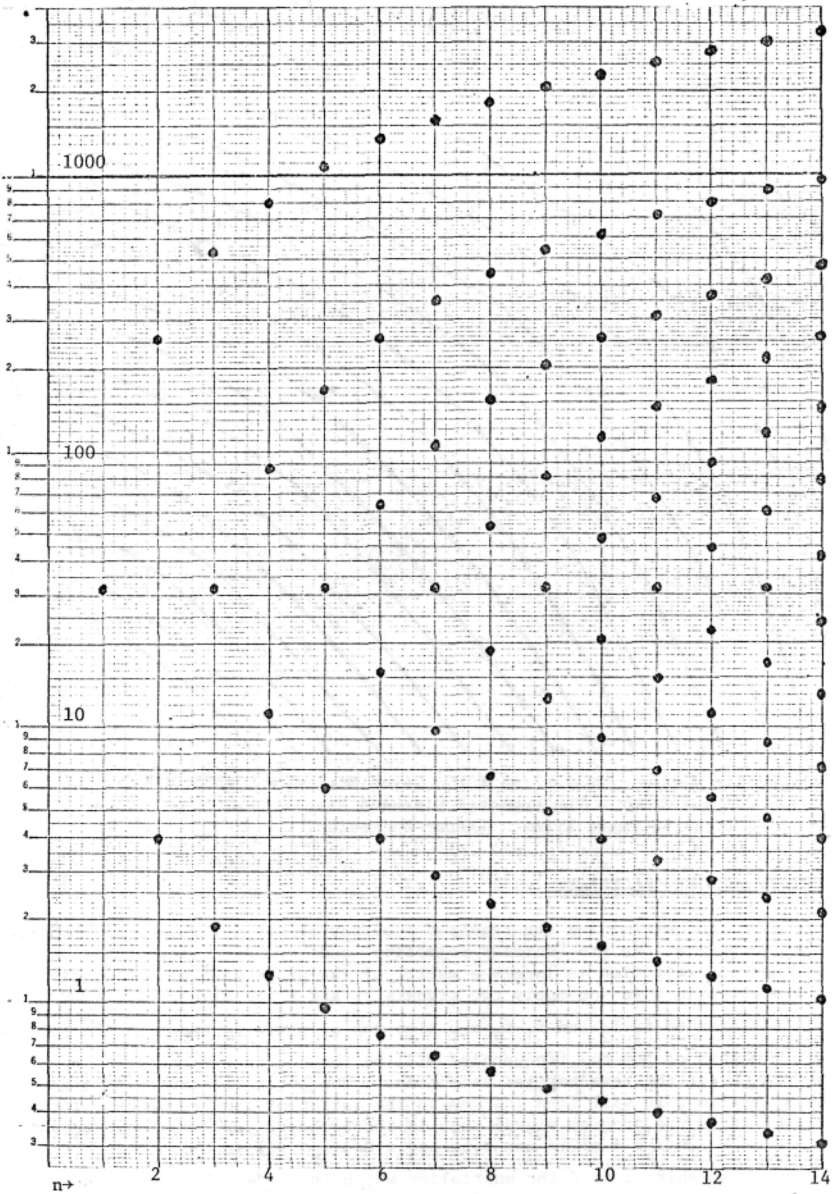
6. The above poles are normalized in terms of F_ℓ. To get the actual poles for the networks, multiply by F_ℓ.



Design Chart for Number of Poles (n) for a Given Normalized Bandwidth and Allowable Phase Error. The Unwanted Sideband Rejection is also Plotted along with the Phase Error. [After Bedrosian]

DETERMINING THE REQUIRED ACCURACY:

When it comes to determining the required accuracy of multipliers and 90°PDN's, the two major considerations are "carrier rejection" and "unwanted sideband rejection." Carrier rejection is largly a matter of the properties of the multipliers. This comes in when you apply two signals f_1 and f_2 to a multiplier and find that in addition to f_1+f_2 and f_1-f_2 you also get a little f_1 and a little f_2. Generally, the higher of the two frequencies will be the one that is noticed, and this is the reason for the term carrier rejection. The point is that if this gets through the multiplier, it will find its way to the output of the frequency shifter. For example, if it is the program signal that is the highest, this will come through the multiplier as a $\mathrm{Sin}(\omega_p t + \phi)$ in one case and a $\mathrm{Cos}(\omega_p t + \phi)$ in the other case where ϕ is the phase shift across the multiplier which is the same for either of the multipliers and for either of the quadrature signals since they are the same frequency. There is nothing that the summers do that will get rid of the program signal once it is through, since they are in quadrature, and no combination of addition or subtraction will cause them to cancel. Thus, carrier rejection is important in multiplier selection for frequency shifters.



Normalized Pole Positions for a bandwidth $f_h/f_\ell = 1000$. Note the symmetry that shows up in this plot of Log(Pole positions) vs. Linear n. The poles were calculated according to Weaver's method.