ELECTRONOTES

WEBNOTE 06   09/25/2012

NOTES ON THE INTUITIVE FFT

Honestly – This is the Only Way

I Really Understand It Anyway!

About 25 years ago, DSP courses likely included extensive material on FFT algorithms – not only the Cooley-Tukey algorithm, but a large variety of alternatives. Students became fluent in terms such as "decimation in time", "decimation in frequency", "common factor", "prime factor", "index mappings", "Chinese remainder theorem", and others. Each of these was perhaps its own numerical magic. (There were even "in place" algorithms where the goal was to take numbers out of memory, compute a result, and store the results in the just vacated locations. This was to save memory. Imagine the amount of memory being an issue!)

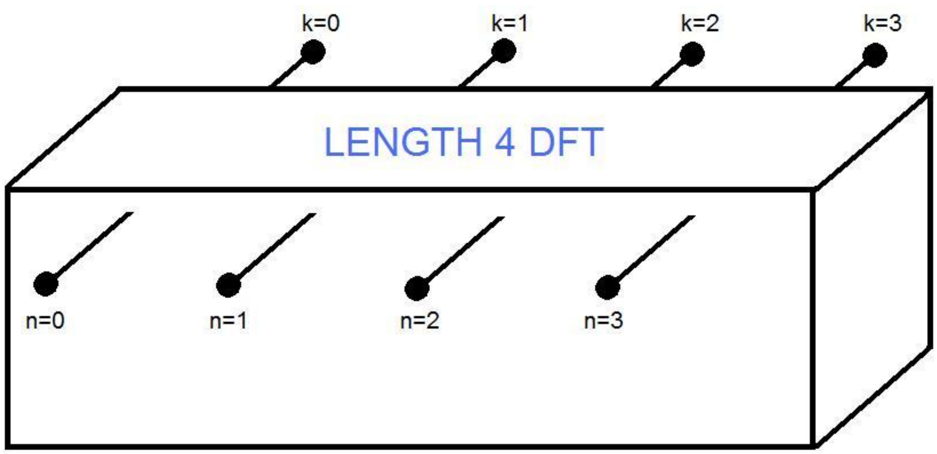
In short order there was less interest in teaching much on FFTs. Perhaps in part because there was so much else to teach in a DSP program, but certainly also because virtually no one was actually writing FFT programs. Yes – everyone was using available programs, extensively, but seldom did you need to write your own. This did not mean that it was not very useful to understand why FFTs worked. What was being exploited?

The length-N DFT is computed as:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j(2\pi/N)nk} \qquad \text{for } k=0:N-1$$

so there are N summations each of N terms. That's N^2 operations. So something like N=77 would take 5929 operations. On the other hand, suppose we could "factor" this computation into 11 length-7 DFTs and 7 length-11 DFTs. This would be a total of 1386 operations. That's the idea, but this approach (even in detail) does not lead to an intuitive understanding of WHY this works.

In the view of breaking the larger DFT into smaller ones, we thought of the DFTs as "Back Boxes" like the length-4 example here:



Often we invented some absurd scenario of needing a certain length DFT and checking the stockroom to find the shelves loaded with unused shorter length ones. This was entertaining, but no one ever had a stockroom with black-box DFTs. It's usually just software after all. So while these exercises or exam problems yielded to the numerical manipulations of the correct methods, they did not yield much understanding.

That is – until we look inside the black boxes. They are flow-graphs at least. Here I want to reference one of our older (1990) app notes – AN-312 "An Intuitive Approach to FFT Algorithms".

http://electronotes.netfirms.com/AN312.pdf

This note really just did a length-12 FFT as an example. It illustrated a methodology that I had used for many years to avoid the numerical “magic”. **The AN-312 note was recently scanned and posted on this site**. **It’s STILL GOOD.**

The AN-312 note is not at all hard to follow, and suggests a general approach to a right answer (often there are several equivalent answers). It illustrated why the FFT worked, and what was being exploited: (1) The Mod-N periodicity of the exponential factors ($e^{-j(2\pi/N)nk}$) and the fact that a required exponential factor could be achieved in combination going through the interconnected blocks.

I saw this first of all as a dodge that allowed me to avoid the number magic (which was to me tedious, subject to error, and not verifiable except through repetition). I always just said I was using an alternative check. Those faculty and students who appreciated more rigor did however note the intuitive understanding it provided. I managed to cover FFT in my own course for many years with just this example. “So that’s what it really is all about!”

Perhaps the highest compliment I received was praise for offering intuitive understandings. In the instance, it was to point out that perfect reconstruction filters and trans-multiplexers were closely related: you could find a trans-multiplexer in the middle of two PRFs in series. One of my teaching colleagues, for whom I have the highest respect, said to the students “I just write equations on the board – Bernie tells us what they mean.” I was immensely pleased. Somewhere, which I can’t seem to relocate, Richard Hamming said that a mathematical proof was not too useful until the students could see ahead of time that it had to be true.