ELECTRONOTES

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ENWN-58

Cascading Two Identical Bandpass Filters

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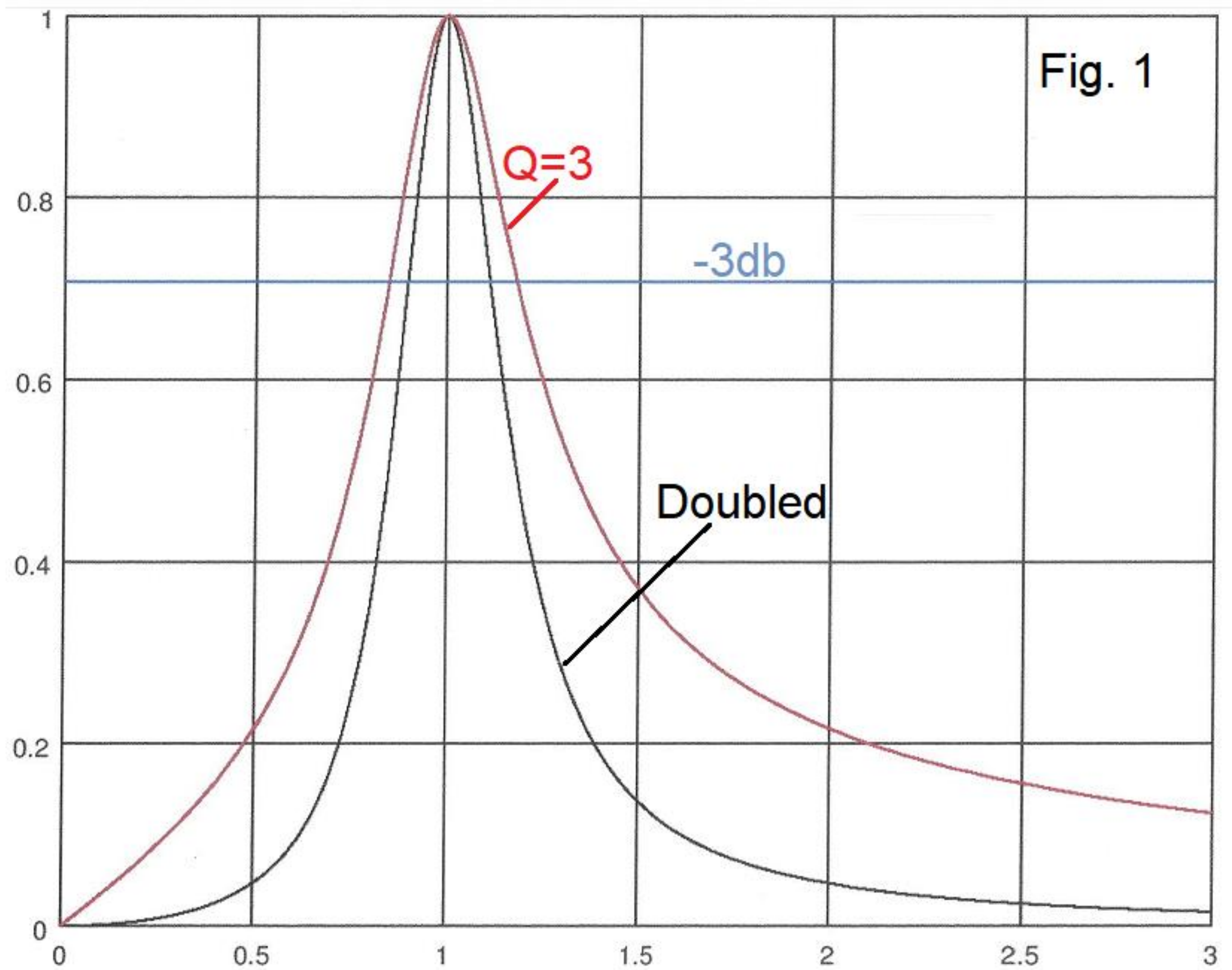
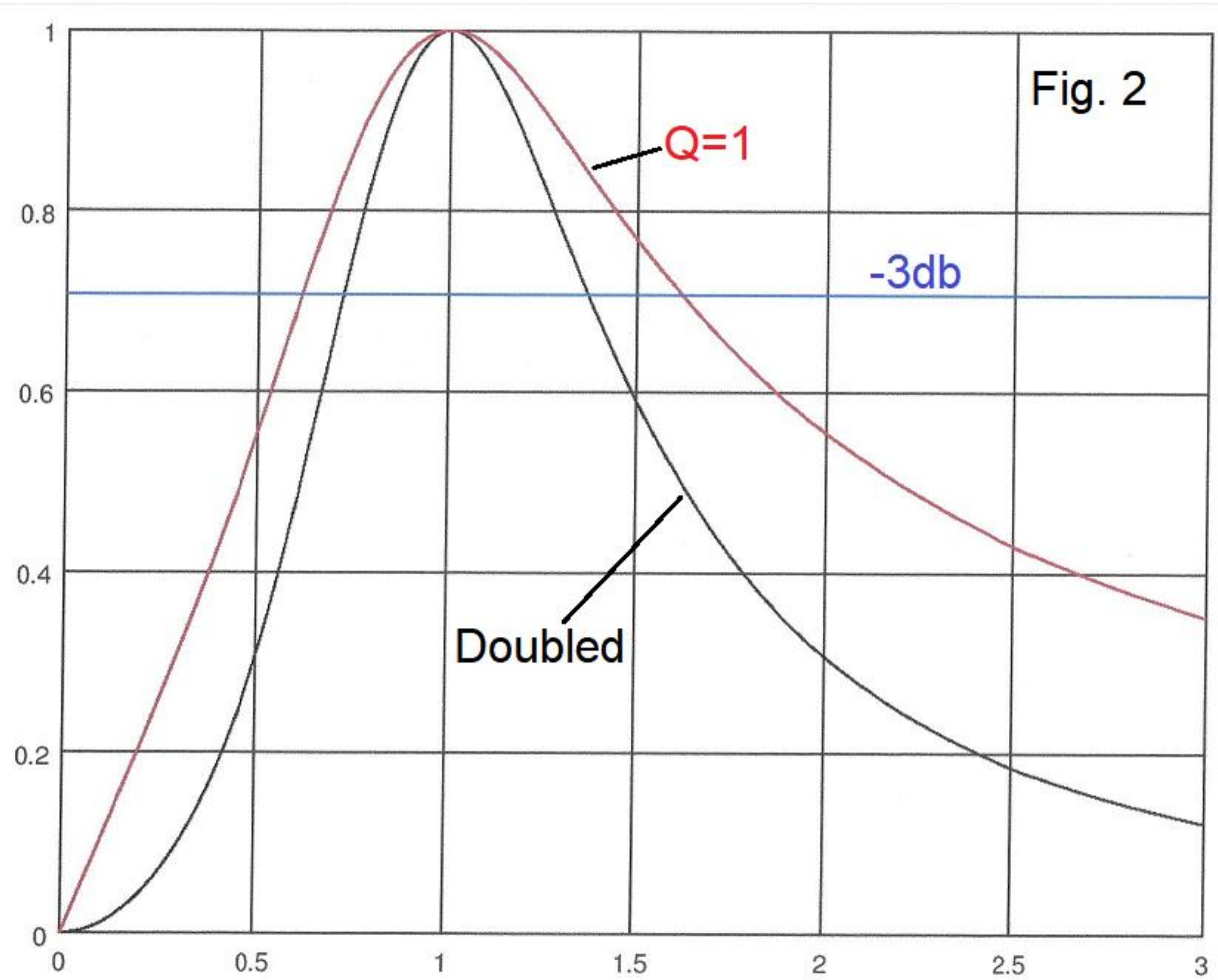
Recently there was a comment or two (about vocoders) on the SynthDIY site regarding the “sharpening” of a (2nd-order) bandpass filter by cascading the original bandpass with an identical second stage (thus 4th-order). Makes sense. Clearly this would increase the overall Q – but by how much? I remember looking at this many years back, and arriving at the conclusion that the increase in Q was quite modest – and a constant factor (on the order of 50%). Accordingly, instead of the cascade, if one wanted to sharpen the Q of a bandpass, the suggestion is that we simply try to coax a bit more Q out of the original stage. I have been so far unable to find were (if ever) I wrote this up. So here it goes again.

Let’s begin with an example – Fig 1. Here the Q=3 (red) is the result of a standard 2nd-order bandpass calculation (equations cut/pasted at end [1]). Also plotted is the 4th-order bandpass (black “Doubled”) corresponding to the cascade of the red curve with itself – hence the square of the red curve, as can be seen by comparing point-by-point for the same test frequencies. The same considerations show that the equations (at the end of his Webnote) for the black curve are easily obtained by removing the ½ exponents from the squares brackets (square both sides). How convenient.



While Fig. 1, and the other curves here are from a dense set of points calculated by computer and plotted, we intend to obtain the Q of the curves by measurements off the plot using a ruler! Specifically, we take Q to be ω_0/B where $\omega_0=1$ is represented by the measured distance from 0 to 1 on the axis of the graph and B is the distance between points where the curve intersects the -3db blue line. The ratio of these distances is the Q as "measured" and is 2.98 for the red curve (close to nominal 3) and for the black curve, it is 4.61, so has been enhanced by a factor of 1.58. What does this prove? Not much – yet.

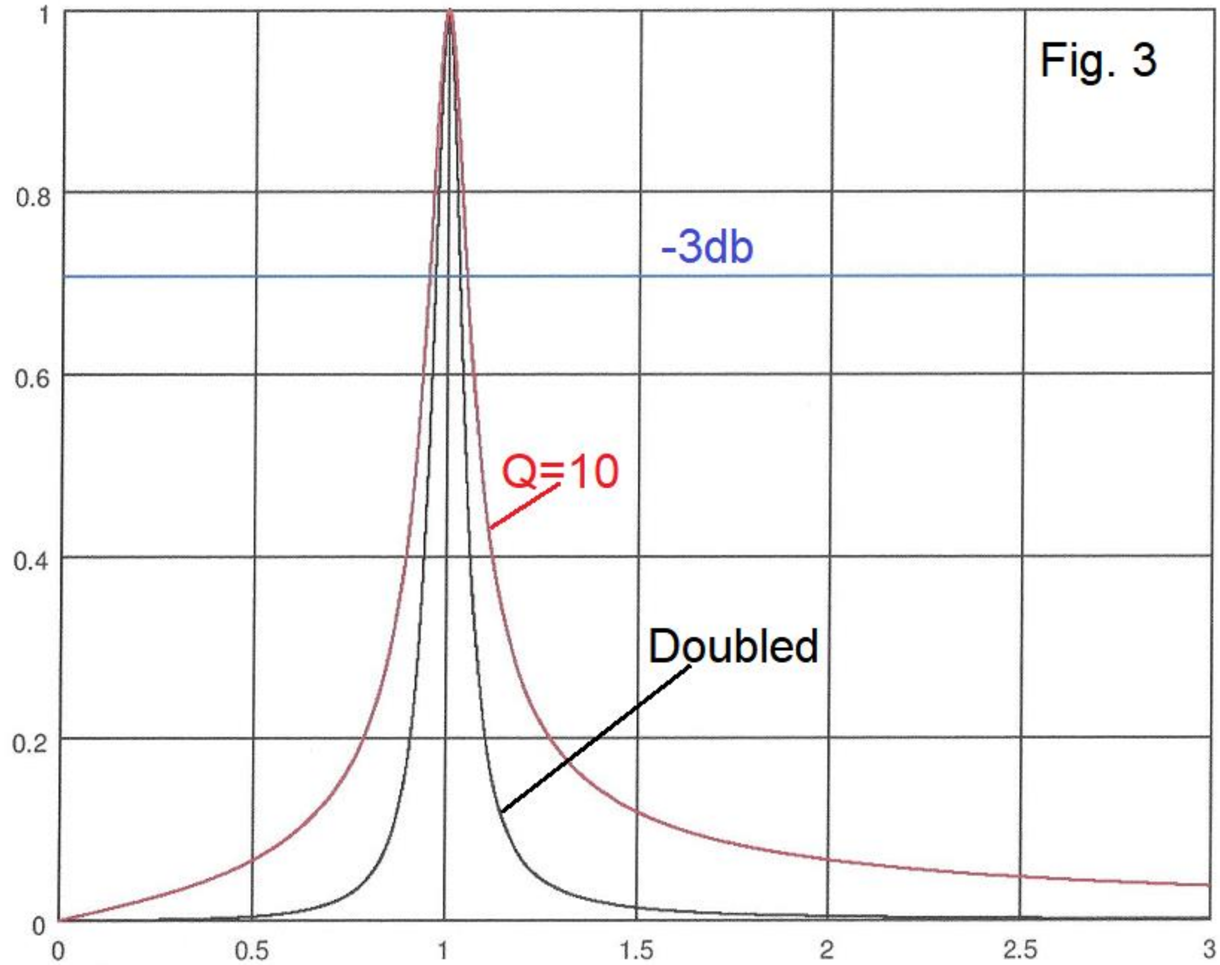
The reason for Fig. 1 is first to graphically show what is going on here. Secondly, it gives us a data point – one "right answer" to shoot for and hope to see coming out of all the algebra manipulations. Except for remembering a bit



about what I found years ago, I would not have expected anything much like the same Q-multiplication of 1.58 for other Qs. Except we do!!!

Fig. 2 shows a nominal Q of 1 enhanced to 1.54. Fig 3 shows a nominal Q of 10 enhanced to 15.6. More data points. **The theoretical multiplier is 1.5537 (as will be shown below) so the Q is enhanced by only 55% additional. The 3db bandwidth is narrowed only to 64%. - Roughly, Q is enhanced to 3/2 and bandwidth is narrowed only to 2/3.**

So, as suggested, cascading two identical 2^{nd}-order bandpass with the goal of achieving a significantly higher Q is not in the cards. It is, however a nicer filter than the 2^{nd}-order designed for the higher Q as will be seen in Fig. 4.



While we see that the Q of the response of the cascade is only modestly enhanced, it is also evident that the overall response, as a bandpass, is impressive. Viewing Figures 1, 2, and 3, we see a significant improvement of the black curves as compared to the original red curves. This is of course mainly a consequence of the sharper “skirts” falling off more rapidly (12db/octave for the cascade instead of 6db/octave of the original. It does “cost” us a second op-amp, so we might want to see if we can better characterize this improvement that is not obviously reflected by the resulting Q.

Since we are suggesting that a modest <u>enhancement in Q</u> may be more economical by “pushing the existing 2nd-order approach, perhaps we should be comparing the cascade with the 2nd-order designed for the **higher** Q, not the nominal starting Q.

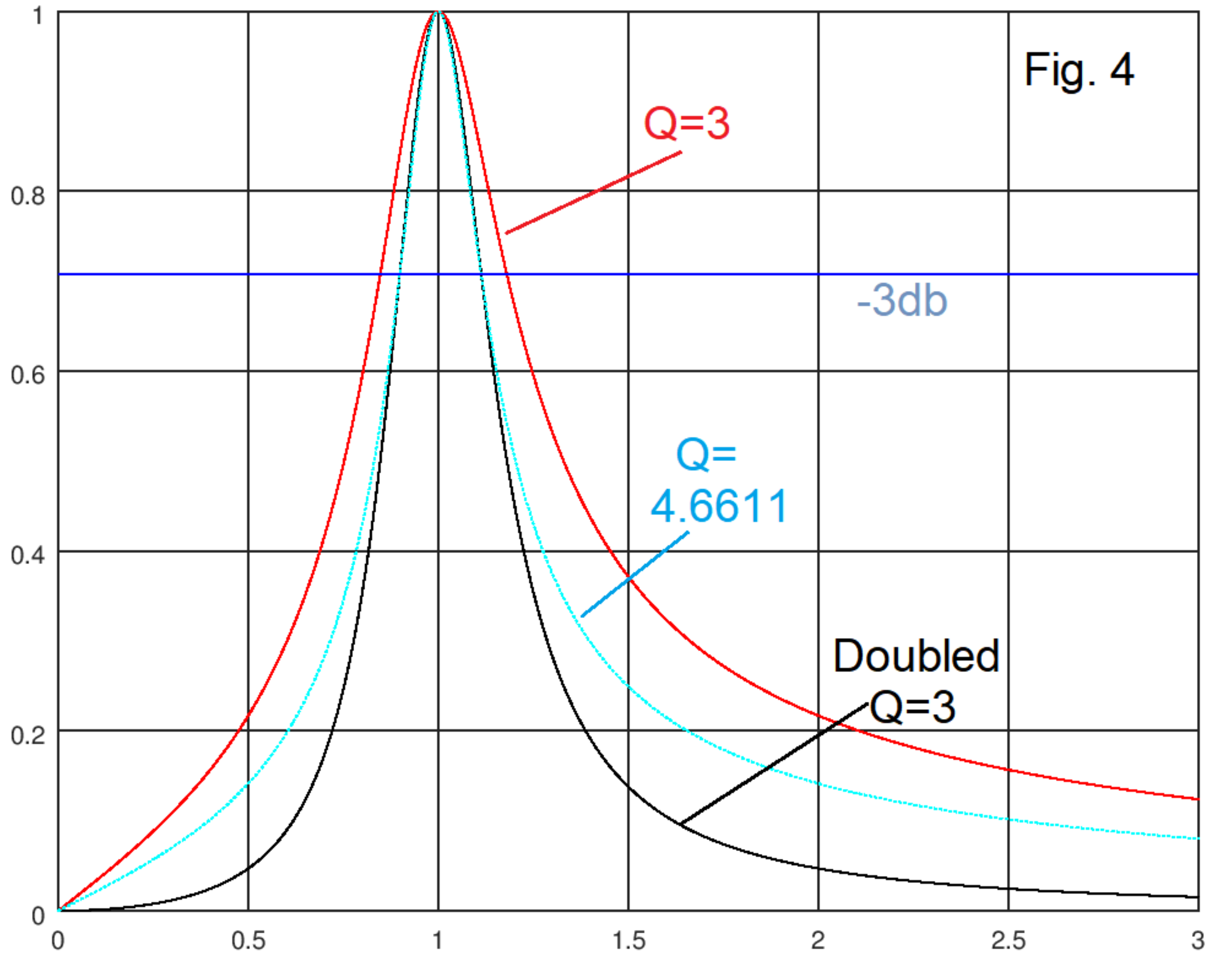


Fig. 4 shows a repeat of the Q=3 case of Fig 1 but with an additional curve (blue) of a Q = 3×1.5537 = 4.6611. We are not surprised that both the Q=4.611 and the cascade have identical 3db bandwidths – we designed it that way. Indeed, above a magnitude of about 0.6, the black curve and the blue curve overlap. Thus the payoff of the cascade is the improvement seen between the black curve over the blue, as becomes quite obvious for magnitudes below about 0.6.

THE MATH

The traditional equations for the bandpass frequency response are pasted from [1]: Equations 4 below:

$$|T(j\omega)| = \left[\frac{j\omega}{(j\omega)^2 + \frac{j\omega_o\omega}{Q} + \omega_o^2} \cdot \frac{-j\omega}{(-j\omega)^2 - \frac{j\omega_o\omega}{Q} + \omega_o^2} \right]^{1/2} \tag{4a}$$

$$= \left[\frac{\omega^2}{(\omega_o^2 - \omega^2 + \frac{j\omega_o\omega}{Q}) \cdot (\omega_o^2 - \omega^2 - \frac{j\omega_o\omega}{Q})} \right]^{1/2} \tag{4b}$$

$$= \left[\frac{\omega^2}{(\omega_o^2 - \omega^2)^2 + \frac{\omega_o^2\omega^2}{Q^2}} \right]^{1/2} \tag{4c}$$

$$= \left[\frac{Q^2/\omega_o^2}{1 + Q^2 \frac{(\omega_o^2 - \omega^2)^2}{\omega_o^2\omega^2}} \right]^{1/2} \tag{4d}$$

$$= \left[\frac{Q^2/\omega_o^2}{1 + Q^2 \left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right]^2} \right]^{1/2} \tag{4e}$$

The equations are for the magnitude of the 2nd-order response, and the various equivalent forms are useful. At the moment we want the magnitude of the 4th-order cascaded response – the square of the 2nd-order magnitude. Conveniently we just need to remove the ½ exponent from the square bracket terms. At the same time, we use the <u>square of the half-power points</u> (-3db at $\frac{1}{\sqrt{2}} = 0.707$) with $\frac{1}{\sqrt[4]{2}} = 0.841$. We also note that the final result we want is a fixed ratio, so we can choose any values of ω_0 and of Q, and the choice of 1 for both is a very good simplification. Plugging into (4c) we get:

$$\sqrt{\frac{\omega^2}{1-\omega^2+\omega^4}} = 1/\sqrt[4]{2}$$

Squaring both sides gives:

$$\frac{\omega^2}{1-\omega^2+\omega^4} = \frac{1}{\sqrt{2}}$$

Cross multiplying we get a quadradic in ω^2 :

$$\omega^4 - \left(1+\sqrt{2}\right)\omega^2 + 1 = 0$$

This yields to the quadratic formula:

$$\omega^2 = \frac{\left(1+\sqrt{2}\right)}{2} \pm \frac{1}{2}\sqrt{\left(2\sqrt{2}\right) - 1}$$

$$= \quad 1.8832\,, \quad 0.5320$$

Taking the square roots, we obtain the upper and lower -3db frequencies (for positive side of spectrum; from the ± sign of the quadratic formula):

$$\omega_u, \omega_l = 1.3723 \quad , \quad 0.7287$$

And thus:

$$B = \omega_u - \omega_l = 0.6436$$

$$Q = 1/B = 1.5537$$

Reference

[1] http://electronotes.netfirms.com/EN71.pdf