ELECTRONOTES

WEBNOTE 39 4/27/2016

ENWN-39

CALCULATING/MEASURING THE NOTCH

Here we further continue a discussion of “The Hum” that was the subject of three previous webnotes:

[1] ENWN-31 2/13/2016 “Oh-Hum” http://electronotes.netfirms.com/ENWN31.pdf

[2] ENWN-37 4/08/2016 “More on The Hum” http://electronotes.netfirms.com/ENWN37.pdf

[3] ENWN-38, 4/11/2016 “Notching to Try to Display ‘The Hum‘ ”
http://electronotes.netfirms.com/ENWN38.pdf

No radically new findings with regard to “the Hum” are presented here. Rather the material relates more to what would normally be called “methods”.

THE TRANSFER FUNCTION, POLES/ZEROS, FREQUENCY RESPONSE

First we gave a full circuit diagram (repeated in Fig. 1 below) and told how to modify the design if different notch frequencies were desired, and cited a large number of references. Here we will give a few more details of the theory [4-6]. In particular. we need to explicitly give the transfer function, equivalently the poles and zeros, and the magnitude of the transfer function AKA frequency response. The transfer function is:

$$T(s) = \frac{\left(\frac{R'}{R_i}\right)[s^2 + \frac{1}{R_1^2C_1^2}]}{[\, s^2 + \left(\frac{R'}{R_Q}\right)\frac{s}{R_1C_1} + \frac{1}{R_1^2C_1^2}\,]} \; \frac{\left(\frac{R'}{R_i}\right)[s^2 + \frac{1}{R_2^2C_2^2}]}{[\, s^2 + \left(\frac{R'}{R_Q}\right)\frac{s}{R_2C_2} + \frac{1}{R_2^2C_2^2}\,]}$$

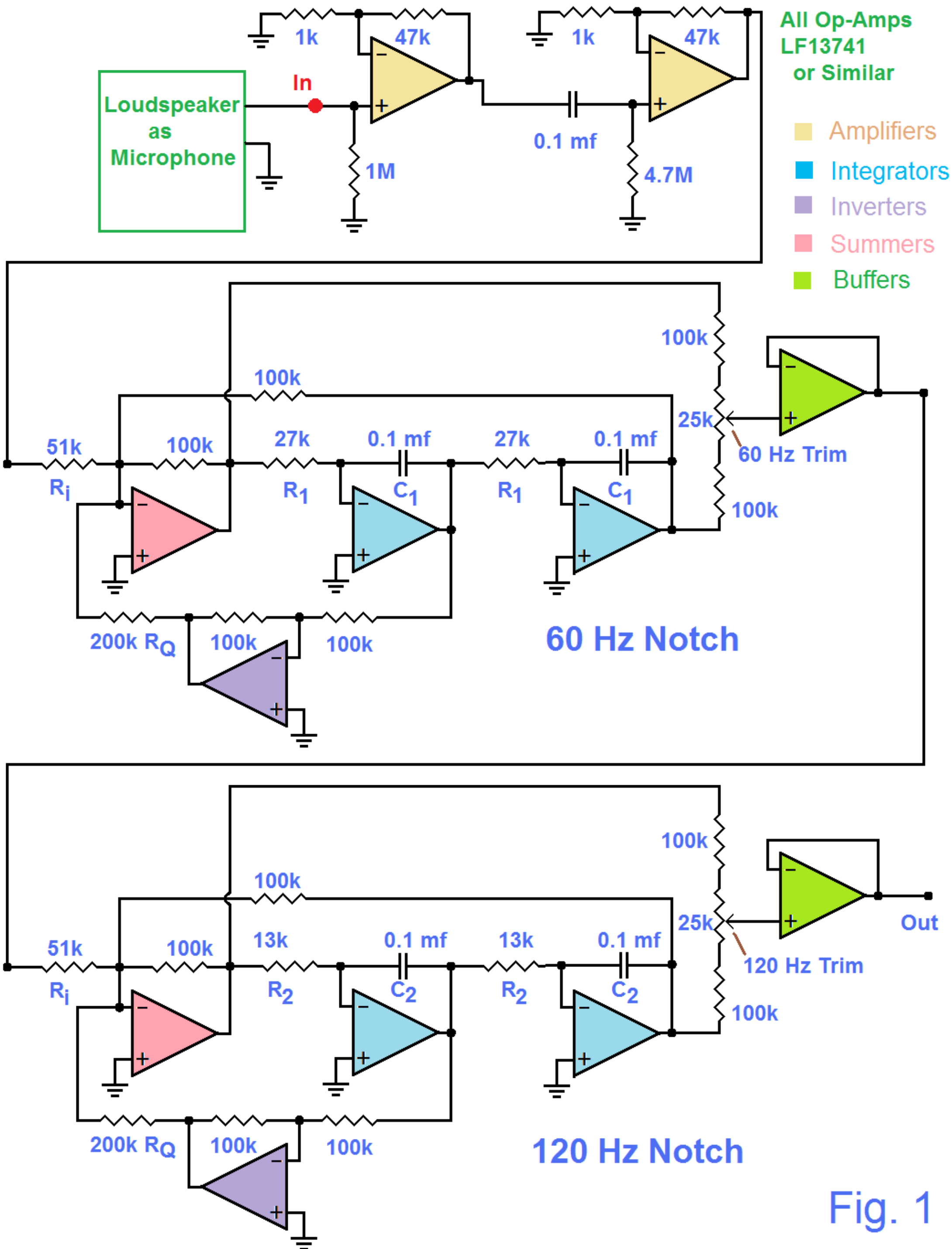


Fig. 1

which assumes perfect matching. There are four zeros of T(s) which are found by setting the numerator equal to zero (solving for s). The four corresponding poles of T(s) are found by setting the denominator equal to zero. These “singularities” are calculated here in terms of ordinary frequency f in Hz. This is done using a numerical “root finder” on the 4th order numerator and denominator polynomials that result from multiplying out the transfer function. [Incidentally, the algebraic multiplication can be done by convolving the 2nd-order coefficients.]

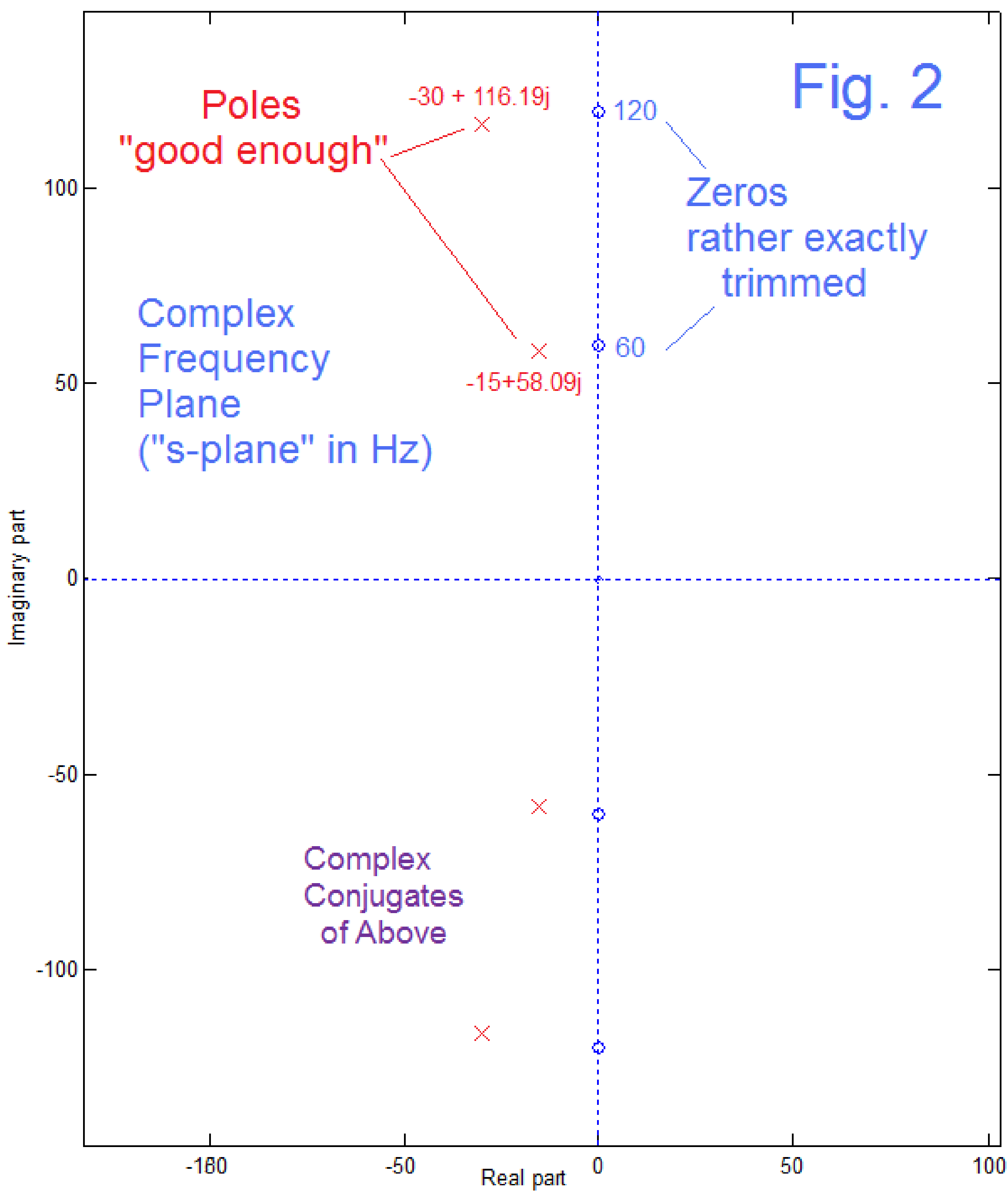
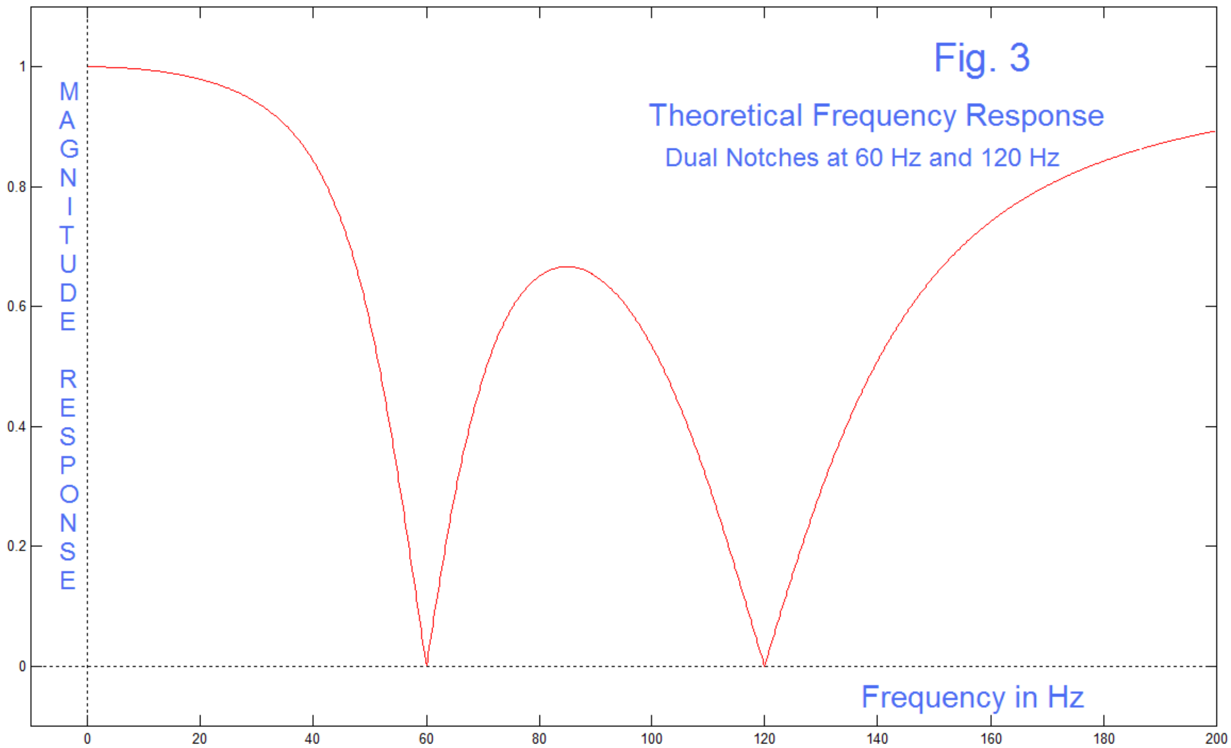


Fig. 2 is a plot of the poles/zeros in the complex frequency (Laplace variable) plane. Usually this is in the s-plane (frequency in rad/sec) but here we use ordinary frequencies (s/2π). Note that the zeros (nulls or notches) are on the imaginary axis at ±60 Hz and ±120 Hz. The poles have negative real part and are on the exact same radius as the zeros (Pythagoras). The poles tend to enhance the response in their vicinity instead of null it. Thus they "fight" with the zeros. The net result of this fight is a favorable case where the nulls are shielded by the poles until we are almost upon them. A "pothole" in the road instead of a swale.

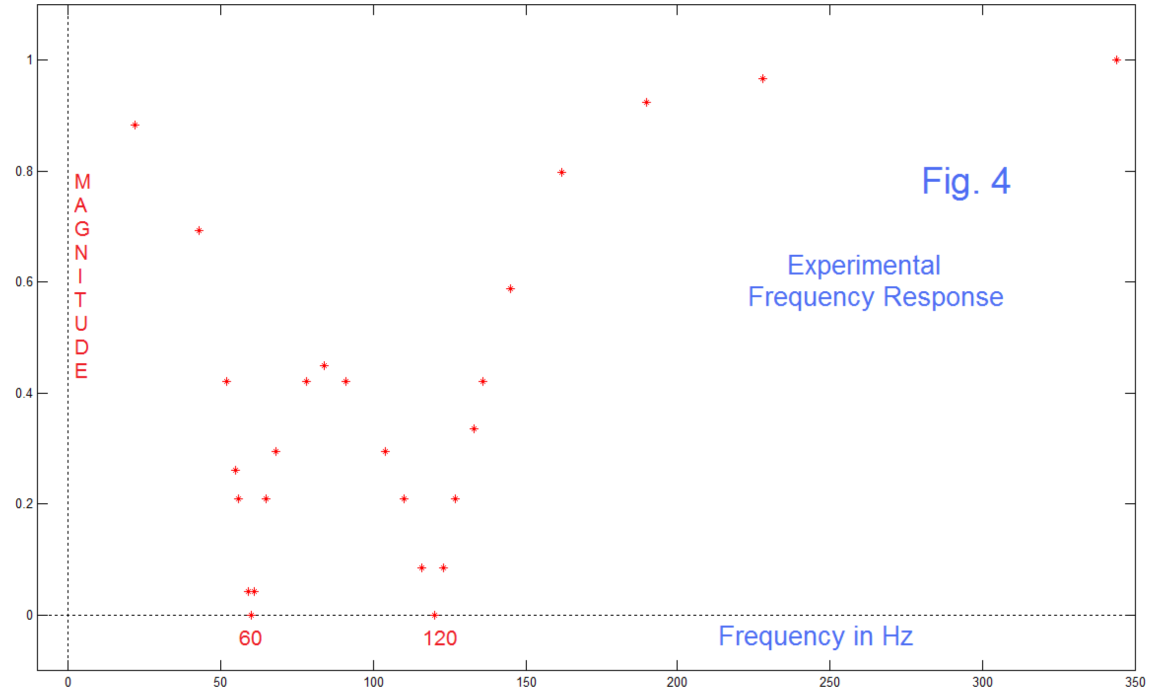


Probably poles/zeros are less familiar than the concept of a frequency response (FR) of a filter. The FR can be obtained from the transfer function or the poles/zeros (equivalently). The transfer function T(s) on the first page has just algebraic symbols. These become actual numbers by plugging in component values from Fig. 1 (an analysis problem). In the corresponding synthesis (design) problem we have in mind actual numbers (usually from famous mathematical functions) and manipulate components (Rs and Cs here) to realize the circuit. Usually there are several or many ways to do this.

The frequency response is not difficult to compute in closed form (it may be tedious) so we usually resort to available programs. Here we used ***freqs*** from Matlab, and the result is shown in Fig. 3. We see the expected nulls or notches at the frequencies 60 Hz and 120 Hz. Ignore the negative frequencies for our purposes here – it's the same result. We note that the undesired frequencies are notched out, BUT there is rejection in the vicinity of these notches. We can control this some by bringing the poles closer to the zeros, but this can cause the filter overall to "ring" more - make its own. We can do little if, for example, we had a 122 Hz desired signal with 120 Hz power line components and lots of random noise. So we look at Fig. 3 as a compromise. Our main goal was to block what we could not tolerate and we expected some degrading of a "Hi-Fi" ideal.

EXPERIMENTAL FREQUENCY RESPONSE

We built and trimmed the notch filter breadboard as in Fig. 1. Finding it worked the very first time, we went ahead and ran the experiments with the Hum. At some point we needed to get back and check the FR, and this is what we report here (Fig. 4).



One generally measures a frequency response by "sweeping" the region of interest with a sine wave from a function generator, recording the amplitude level of the output, point by point. Most conveniently one takes data by mentally zooming in on regions where something is actually going on. You do not set up a grid of target frequencies (like every 10 Hz) and read off the amplitudes after believing you have set the frequency correctly. Most commonly an output amplitude is set and the actual frequency is then read digitally from a frequency counter. In fact, this is most easily done with an analog scope. Particularly (as I was reminded here) for low frequencies where digital voltmeters tend to "wobble" too much. The points of Fig. 4 took about 4 minutes total.

The agreement between theory and experiment is not too bad. The nulls are spot on – but again – that's how we tweaked it. The only thing to gripe about is the center lobe. In theory, it was about 0.67 while the experiment has it at about 0.45. This is not particularly important to the experiment at hand. But often we do somewhat better.

What's wrong? Well, recall that we threw the filter together with the closest components available. Mostly these were 5% and 10% tolerances. This laziness was not entirely careless because we knew that we could tune the notches rather exactly with the output pots. We remarked [3] that when this was done, the knob setting were disconcertingly close to the extremes (we guessed that about 10% trim would do – and it just did).

We made no attempt to tune the poles. Recall that ideally the poles were supposed to be on the exact same radius as the final notch. Here we had “ballpark” poles at the whim of a part actually grabbed, and these were never measured or tuned. As such, the poles were not supporting the sides of the notch in an ideal manner, so both sides of the individual notches were out of balance (not “shelved”) and we are even less surprised that the common region between the notches was non-ideal. But – GOOD ENOUGH.

THE LEVEL OF AUDIBILITY

Previously [3] we observed that we could “hear” the Hum and that if we did notch out the AC line components, nothing particularly non-random appeared (some residual 120 Hz and possibly something in the range of 10-20 Hz). So the question is: given the extreme dynamic range of the ear, is it possible that there is something still vibrating the air that is not picked up and displayed on the scope. In an attempt to address this possibility, I decided the drive the second stereo speaker with something like 50 Hz and adjust the level so that it seemed comparable to what I “hear” as the Hum. This was a simple matter of connecting the function generator to the speaker. This you can do directly. The 600 ohm output impedance of the function generator means that very little voltage is impressed across the 8 ohm speaker, but it is enough, and assures low-frequency coupling. { Few know that you can drive a stereo speaker using an op-amp (!) with a 1000 ohms series resistor [7] }.

Fig. 5 shows the pickup from the first speaker (used as microphone as described). It is a very recordable signal. Here the volume was turned up enough to make it roughly as apparent as the Hum (which apparently gives way to the distracting “competition”). Keep in mind that 50 Hz is very very difficult to hear. You would not generally hear the signal of Fig. 5 (top panel, blue) due to the insensitivity of the ear (Fletcher Munson) to low frequencies. In comparison, turn up the frequency to 1000 Hz (without adjusting the amplitude) and even this low level shrieks a bit.*

The conclusion is that there is no Hum signal, hiding below what is being recorded and displayed, that is detectable to the ear. The top panel (yellow) shows the output of the notch filters. It is smaller than the input. Note (Fig. 2) that the notches at 50 Hz are down to about 0.58. This pretty much agrees with the top panel, blue to yellow. Finally we switch off the function generator and arrive at Fig. 5, bottom panel. We use the same horizontal and vertical scales on all four traces in Fig. 5. Turning off the generator removes the 50 Hz test signal and leaves pretty much the noise we saw previously.

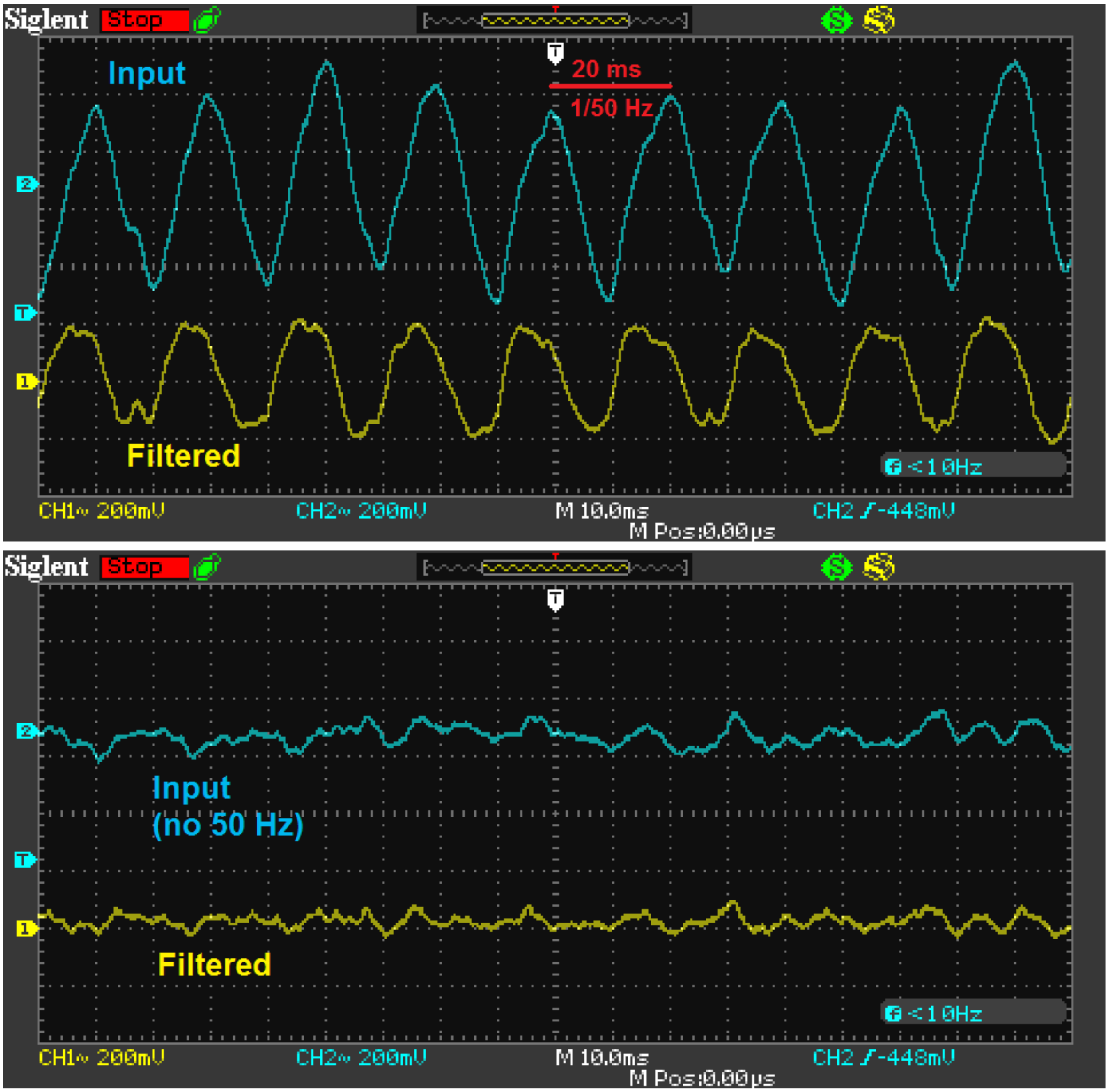
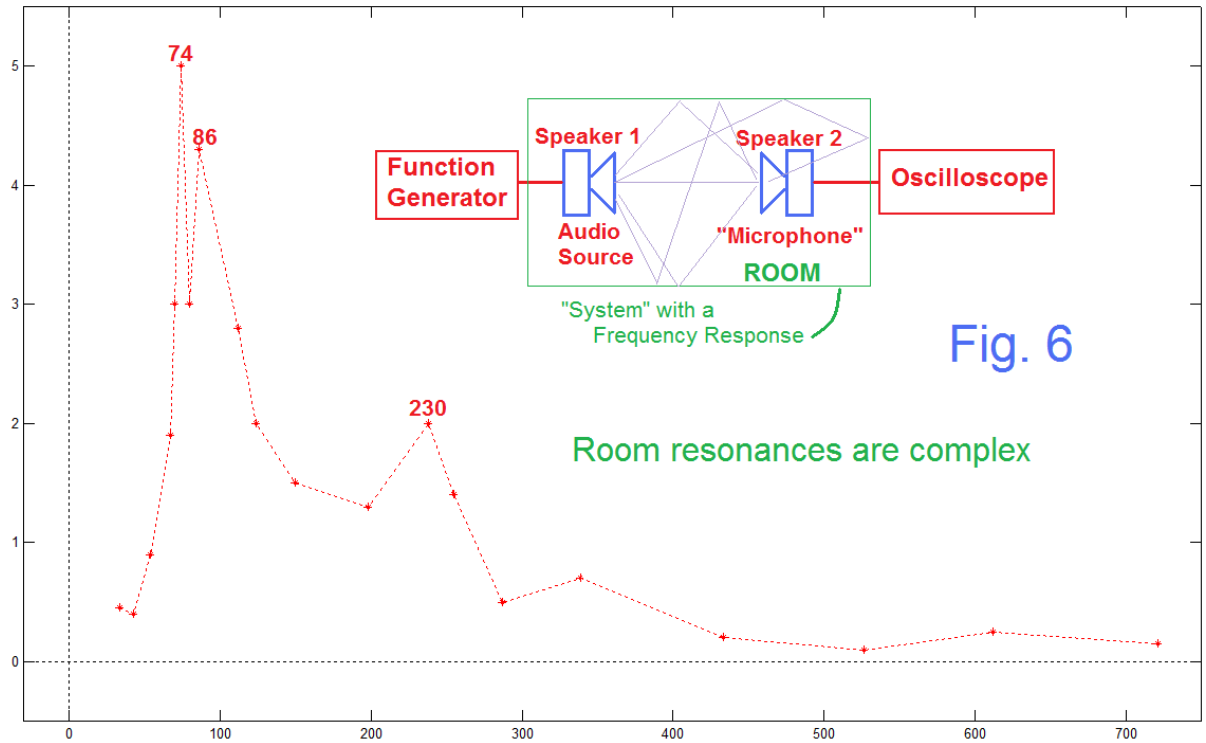


Fig. 5

ACOUSTIC PATH - RESONANCES?

Although we felt assured the notch nulls were optimized, there is of course a temptation to twist any knob provided! Then you have to recalibrate! This was a matter of disconnecting the amplifiers and connecting the function generator directly to the two filters in series. At one point, I decided to re-tweak the notches (I had messed with them!) and since I already had the speaker driver connected, I thought why not just send the test sinewaves across in acoustic form (lazy). So I tried. What a mess. Not only was there the background noise, but also some features that were not flat but



apparently unrelated to the notches. It was clear that the various room resonances were confounding the measurement. So I decided that taking a rough look at the resonances was useful. Essentially you are using the room itself as a "system" and measuring the frequency response. The result is in Fig. 6 and what is remarkable is that it is quite uneven with major resonances in the general region of the notches. We perhaps need to keep these in mind.

[As an aside, it is these resonances enhancing the feedback path between a loud-speaker of a PA system back to the microphone that is responsible for the "feedback squeal" if the volume is turned too high. It is usually the major resonance, not the distance between the speaker and the microphone, that determines the pitch of the howl.]

REFERENCES

[1] "Oh-Hum" , Electronotes Webnote ENWN-31, 2/13/2016, http://electronotes.netfirms.com/ENWN31.pdf

[2] "More on The Hum" Electronotes Webnote ENWN-37 4/08/2016 http://electronotes.netfirms.com/ENWN37.pdf

[3] "Notching to Try to Display 'The Hum' " Electronotes Webnote ENWN-38, 4/11/2016 http://electronotes.netfirms.com/ENWN38.pdf

[4] "Notch Filtering – Some Notes", Electronotes Application Note No. 413, July 30, 2014 http://electronotes.netfirms.com/AN413.pdf

[5] Analog Signal Processing, Section 4-2 (notch filtering), ***Electronotes***, Vol. 19, No. 193, March 2000, pp 6-10. http://electronotes.netfirms.com/EN193.pdf

[6] Analog Signal Processing , Chapter 6, Section 6-2. pp 20-24 ***Electronotes***, Vol. 20, No 194, April 2000 (analysis for state-variable) http://electronotes.netfirms.com/EN194.pdf

[7] "Bench Loudspeaker Amplifier with Op-Amps", Electronotes Application Note No. 216, May 11, 1981

* Note added May 1, 2016. We noted above that a sound as low in frequency as 50 Hz is quite hard to hear. In fact, it is about 40 db down (say to 1%) on the equal loudness "Fletcher Munson" curves. This means that if an acoustic signal is audible at 50 Hz, it is all the more detectable electronically.