ELECTRONOTES

WEBNOTE 08/21/2009

4-POLE EXTENSIONS

HISTORY OF THE EXTENSIONS TO THE 4-POLE LOW-PASS

On March 19th of this year, in response to a request, we posted [1] a scan of our Application Note No. 71 from January 18, 1978 "Multi-Mode Filter Based on First-Order Low-Pass." This request seems to have been in response to some January 2009 activity on Synth-DIY. Directly related, and also mentioned (but not requested since the person already had it) was [2] "Additional Design Ideas for Voltage-Controlled Filters," *Electronotes*, Vol. 10, No. 85, January 1978, a much more extensive treatment. So we seem to start out in January 1978, 31 years ago!

Reference [2] had two important findings. First, it located the poles of the four-pole as a function of feedback gain (on a square pattern as it turned out). Secondly, it showed how a weighted sum of the outputs of the cascade could be used for general 4th order transfer functions (with certain limitations). Subsequently, it was found that a more general class of "polygon" filters could be considered.

FINDING THE POLES

Bob Moog of course invented the four-pole low-pass with feedback, and he well understood how feedback could enhance resonance. But he did not know where the poles actually were all the time. When I told him I had found them, he immediately asked were – he had been wondering too. It must seem silly to many that finding poles (solving polynomials) was ever a problem. But in the 1970's, we didn't have Matlab or even a programmable calculator. Such mathematical aids existed – they just weren't something everyone had easy access to. In consequence, I had to pull a few stunts to get them to appear. Soon after, a program for finding roots for the TI-59 programmable calculator was written [3], leading to verification of previous results, and extension to polygon filters [4]. Then, much to my chagrin, Richard Bjorkman wrote to point out that a simple substitution of variable blew the problem apart [5]. I loved his characterization of my previous toils as "unnecessarily cumbersome" and have used his term several times since then when compassion seemed warranted.

SUMMATIONS FOR OTHER RESPONSES

The second finding in reference [2] was that a weighted summation of the low-pass cascade could result in frequency responses other than just the low-pass. It was only necessary to do this. It wasn't even what we could call "an idea". It was obvious to anyone who had studied the state-variable or "biquad" approach. Accordingly, reference [2] discussed high-pass and bandpass types. It also suggests a notch. {Incidentally, on page 15, 3rd line below the figure, it says "poles" where of course it should say "zeros". (corrected in link) }

Here is the issue: If we have a feedback control knob, and we turn it, how do the poles and the zeros move? This is not initially obvious – perhaps something nice happens, or perhaps something incredibly messy happens. Fig. 1 reminds us of the setup. We have four first-order low-pass sections in cascade, with a feedback gain g back to the input. In addition, we have weighted taps from the five internal points along the cascade to a summer. A bit of thought convinces us that these two are independent. One essential realization is perhaps that the cascade, along with its weighted output, "has no idea" that it is part of a feedback loop. Neither its poles nor its zeros move. It just has some input point (v') which happens to contain part of its own output. It is the poles and zeros from V_{in} to the output V_{out}

that matter. The poles move with changes of g exactly as they did without the summation, and as for the zeros: – well – once they are set by the weights, they do not move at all as g changes.

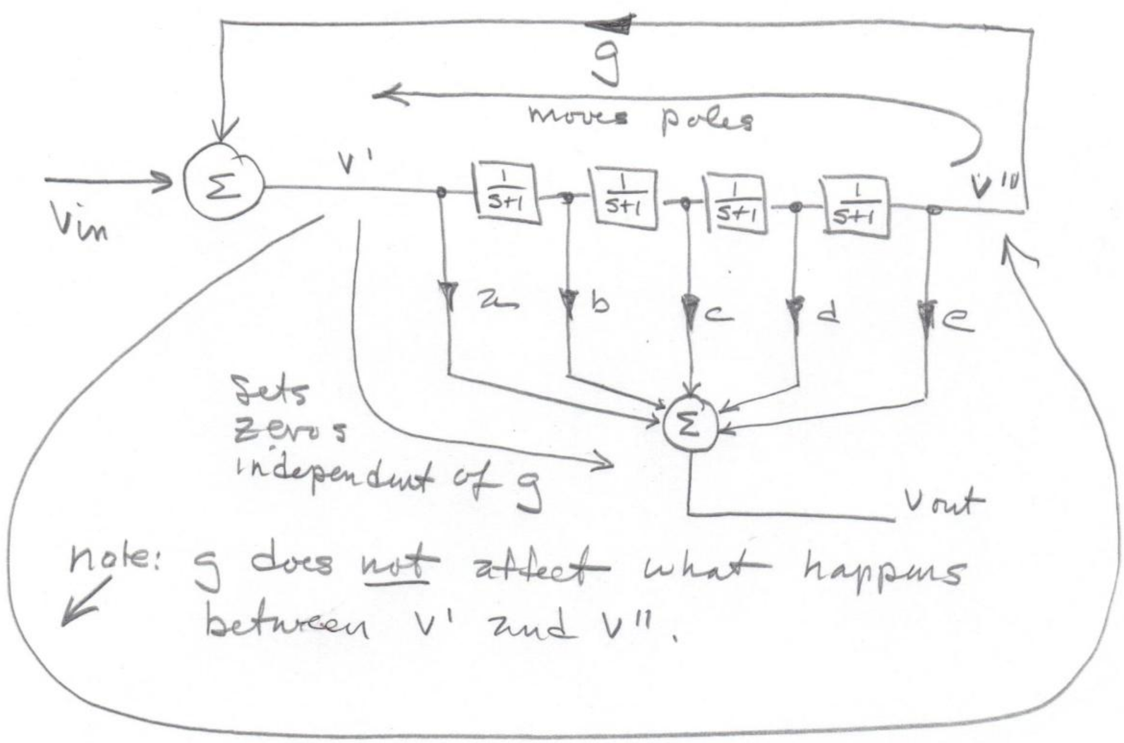

Fig. 1 Four-Pole with Weighted Output

Now, the idea is that we would like to set the summation weights a, b, c, d, and e for a response other than low-pass, and we would also like to be able to change g to "sharpen" the response, as we do with low-pass. This is not possible in the general case, but there are some important special cases.

Since we can not change the zeros, we need to choose responses where the zeros do not <u>need</u> to move. This really means the zeros need to be at s=0, or s=∞. The low-pass has four zeros at s=∞. The high-pass has four zeros at s=0. The bandpass has two zeros at s=∞, and two at s=0. All three of these work.

The problem comes up with finite zeros not at s=0. Consider a notch, as suggested in reference [2], where we have second-order zeros at s=± j. (That is, there are four zeros total, two at +j and two at –j.) This will of course produce a notch at a frequency of 1, which may be our main goal. On the other hand, we might want the response away from the notch to be relatively flat. In such a case, if we move the poles to sharpen the notch, they should leave s=-1 and approach (in pairs) the zeros at j and –j along a circle centered at s=0 of radius 1. This is not what happens with the four pole where the poles move singly on the corner of a square. So you get a kind of a notch, but certainly not one that remains balanced on either side.

The case of the all-pass is much more problematic. It is not clear to me if anyone has claimed or demonstrated a useful all-pass. Here is the problem. An all-pass must have zeros that are mirror images (reflected in the s-plane about the jω–axis). As the poles moved with g, the zeros would have to move too – and they can't move.

REFERENCES (References [1] , [2], and [5] are linked on News Page)

[1] B. Hutchins, "Multi-Mode Filter Based on First-Order Low-Pass," Electronotes Application Note No. 71, January 18, 1978

[2] B. Hutchins, "Additional Design Ideas for Voltage-Controlled Filters," *Electronotes*, Vol. 10, No. 85, January 1978

[3] This was a supplement by B. Hutchins and Walt Luke. Perhaps I will find a copy some day. Little chance of finding a TI-59.

[4] B. Hutchins, "The Migration of Poles as a Function of Feedback in a Class of Voltage-Controlled Filters," *Electronotes*, vol. 10, No. 95, Nov. 1978.

[5] R. Bjorkman, "A Brief Note on Polygon Filters," *Electronotes*, Vol. 11, No. 97, January 1979

REFERENCES FOR WebNote 8/20/2009

http://electronotes.netfirms.com/AN71.pdf

http://electronotes.netfirms.com/EN85part.pdf

http://electronotes.netfirms.com/EN97part.pdf