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A FREQUENCY-SENSITIVE FEEDBACK PATH EXAMPLE

Recently [1] we reviewed the often-confusing ideas of feedback gain as it affects overall gain, and often the very stability of a circuit. This was a classical approach. In the end it relied on checking the results with actual op-amp circuits on the bench. The ideas are simple but it is easy to get confused. In fact, one of the figures (Fig. 12 there – and also Fig. 12 here - before it was crossed out with the orange lines!) was drawn before I “smelled a rat” and did that check. (This is what we call “full disclosure”.) Then it was thereafter obvious. Let me suppose that the difficulty even EEs have with feedback suggests that non-EEs are at least as likely to misunderstand, and further, they are far less likely to enunciate their claims in the clearest conventional engineering terms.

An additional complication recently is the attempt to apply the notion of “Positive Feedback” to models of the Earth’s climate system. It is abundantly clear that too often it is <u>falsely</u> supposed that there must not be a temperature feedback that is positive, or at least not NET positive. In fact, positive feedbacks less that +1 usually won’t “blow up”. Even worse perhaps, the Earth’s system is chaotic and non-linear and full of unknown unknowns. To suggest that it is even “kind of like” a circuit is a large stretch. It is barely an analogy worth populating with actual numbers. That said, anything presented in the guise of EE notions should be done correctly. Some do a pretty good job of it. Too often it’s more like: “one more cow fart and the world goes up in flame – established physics!”.

Here I want to discuss a circuit sketch that is somewhat peripheral to a climate model, discussion in the sense that it was just offered as an example [2]. The author of the blog post on “Watts Up With That” was Joe Born and his contribution I mainly criticize on the basis that he confused me at first! He’s not an engineer. Says he is a retired lawyer – so not so bad. His presentation is quite involved, and my attention went to the diagrams, of which two are reproduced in Fig. 1 here. Readers here will note that the first (a) is a perfectly correct feedback flow graph. The bottom (b) is a – well – what is it?

Well it looks like an op-amp circuit. We do note that it seems to be a finite gain op-amp (the A) and that positive feedback is used (it says this too of course). Is it a Schmitt trigger? No component values! Why the capacitor? Why the differential input – the (-) input just confuses things.

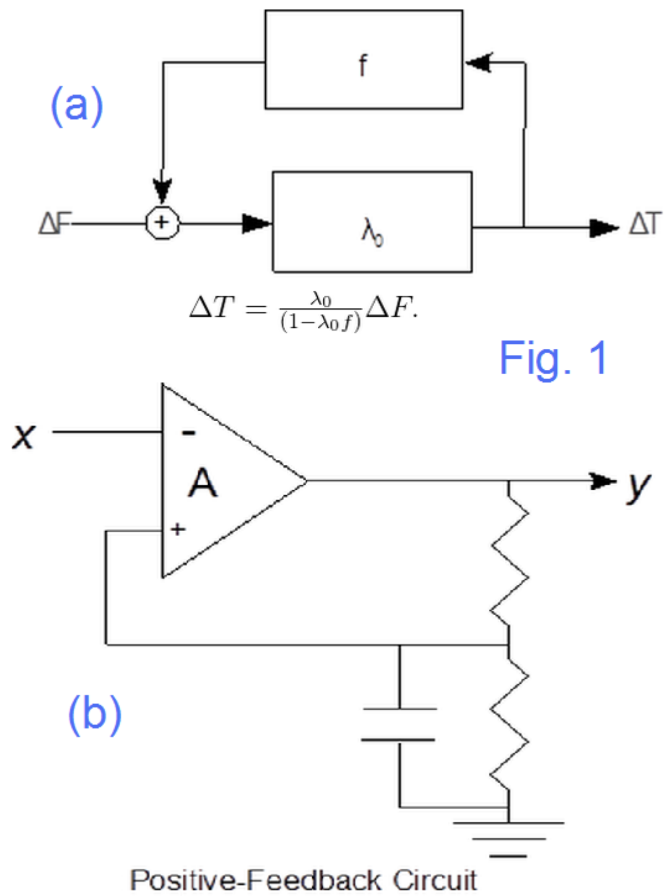


Fig. 1

Okay, it's a finite gain (-A) amplifier which happens to be inverting, and in the loop there is an attenuating low-pass filter. The inversion is of no consequence, and we can get rid of it. It may be stable, or not. Until we say what A is and say what the resistors are, we can't know. So it is very definitely not a usual finite gain (A) op-amp, where A might be a very large but not well-defined number (millions perhaps), and more realistically, A would be the frequency-dependent G/s. Instead, for example, A might be 1, in which case unless the upper resistor is R=0, the gain y/x can be much larger than 1, but it's stable. Got it.

TRANSFER FUNCTION

The revised version of Fig. 1b is Fig. 2 where we have replaced the differential amplifier with a non-inverting summer followed by a finite gain of A (thus getting rid of the awkward op-amp look-alike), and we have assigned component values as R_1, R_2, and C. We want to find the transfer function of this circuit, and this we can do in two different ways. The second way will be a direct analysis. The first way will employ the shortcut of the feedback equation and a Thévenin equivalent for the resistor divider.

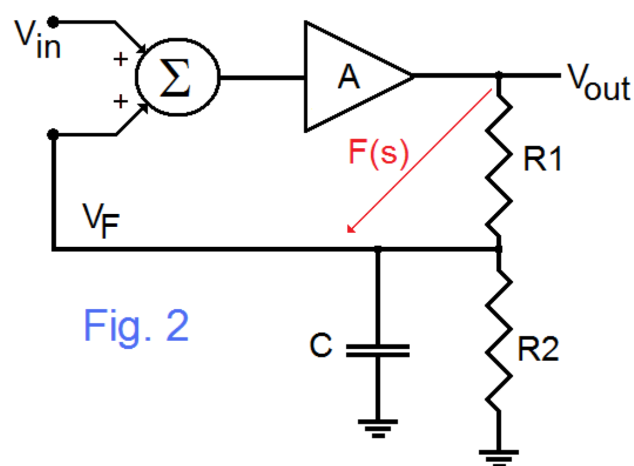


Fig. 2

Method 1 - Shortcuts

The Thévenin voltage of the voltage divider formed from R_1 and R_2 is $V_T = V_{out}R_2/(R_1+R_2)$, the Thévenin resistance is the parallel combination of R_1 and R_2, thus $R_T = R_1R_2/(R_1+R_2)$ and if we are familiar with the usual first-order R-C low-pass thus:

$$F(s) = [\ R_2/(R_1+R_2)]\ /\ [\ 1 + sCR_1R_2/(R_1+R_2)\ ] \qquad (1)$$

and using the feedback equation as in Fig. 1a we easily write:

$$T(s) = V_{out}(s)/V_{in}(s) = A\ /\ [1 - AF(s)\ ]$$

$$= [A(R_1 + R_2 + sCR_1R_2)]\ /\ [R_1 + R_2(1\text{-}A) + sCR_1R_2\ ] \qquad (2)$$

Method 2 - Systematic

For the second method, we start by finding the parallel impedance of R_2 and C as:

$$Z_{R2||C} = R_2(1/sC)/(R_2+1/sC) = R_2/(1+sCR_2) \qquad (3)$$

which checks in the limits: at DC (s=0) it becomes R_2, when C goes to 0 it also becomes R_2, and of R_2 becoming infinite (it's gone) it becomes just the capacitor 1/sC. This $Z_{R2||C}$ is the lower leg of a voltage divider with R_1 in the top, so V_F becomes:

$$V_F = V_{out}\ R_2/(1+sCR_2)\ /\ [R_1 + R_2/(1+sCR_2)]$$

$$= V_{out}\ R_2\ /\ [R_1 + sCR_1R_2 + R_2)\ ] \qquad (4)$$

Further it is clear that:

$$V_{out} = A\ [\ V_{in} + V_F] = A\ [\ V_{in} + R_2V_{out}\ /\ (R_1 + sCR_1R_2 + R_2)\ ] \qquad (5)$$

Solving equation (5) for V_{out}/V_{in} gives us equation (2) back.

The transfer function, T(s), equation (2), is useful for finding the frequency response as well as time responses such as impulse response and step response. Note that the zero of equation (2), T(s), is at $s_Z = -\ (R_1+R_2)/R_1R_2C$ independent of A, and is the same as the pole of F(s). The pole of T(s) does depend on A as:

$$s_P = -\ [R_1 + R_2(1\text{-}A)]\ /\ CR_1R_2 \qquad (6)$$

so T(s) being stable requires that s_P is less than zero or $R_1 > (A\text{-}1)R_2$. This can be manipulated to:

$$R_2\ /\ (R_1+R_2) < 1/A \qquad (7)$$

which says that the attenuation following A must knock the signal gain below 1 around the loop.

AROUND THE LOOP !

So which is the feed-forward and which is the feedback – looking at the loop as a whole. Clearly, with regard to stability, it does not matter – the two are just in series. The distinguishing feature is that the feed-forward is the path from the input to the output as agreed upon. It wouldn't matter which one (or both, or neither) has a frequency-sensitive element to it.

Consider a typical example where we use a basic notion of positive feedback to design a filter. This is shown in Fig. 3 where the very popular "Deliyannis" bandpass [3] is shown. It is really just a common "multiple-feedback infinite-gain" bandpass with positive feedback from the resistive voltage divider. As such, we view it as a relative to Fig. 2. While we suspect that we might analyze this by making use of the feedback equation, we will almost certainly be more comfortable with a direct analysis based on what we already know about op-amps. That is, we are still just going to need to use the fact that the differential input is zero. Further we are alert for the possibility that this NEGATIVE feedback might fail.

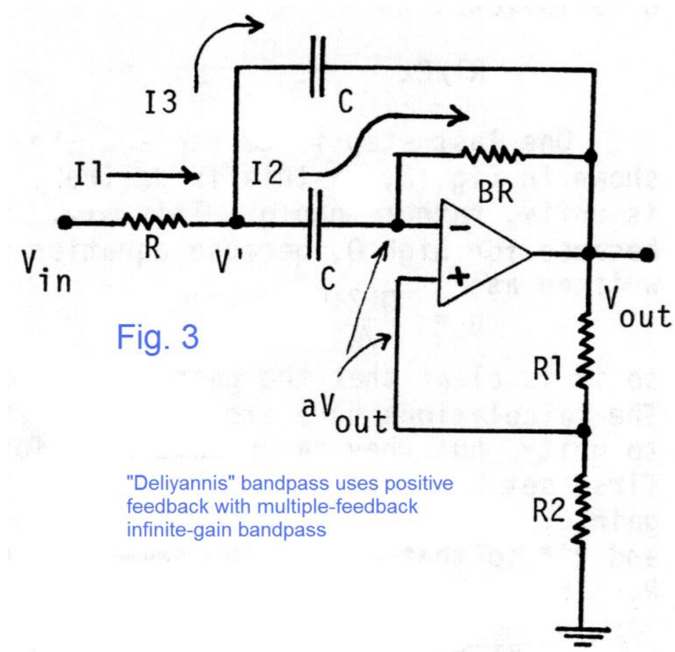


Fig. 3

"Deliyannis" bandpass uses positive feedback with multiple-feedback infinite-gain bandpass

In this view, Fig. 2 is just a case where we have a fixed gain (A) in the feedforward loop and Fig. 3 is the case where there is a fixed gain (attenuation actually, a) in the feedback. How would we tell if Fig. 3 is stable or not? Well, generally the notion of finding the poles is far more attractive then looking at loop gain. Readers of these notes are probably very familiar with Bob Moog's famous 4-pole low-pass as yet another example where positive feedback is used to sharpen a filter's response. In Bob's circuit, there was a loss of $1/\sqrt{2}$ in each of four stages (feed-forward) with a feedback gain of 4 for oscillation. In the case of Fig. 3, the maximum feedback gain is:

$$a = 2/(B+2) \qquad (8)$$

as derived in [3], which is the counterpart of equation (7).

POLES/ZEROS VS FEEDBACK EQUATION

It is clear that we can obtain transfer functions in various ways including a direct analysis, an analysis based on familiar sub-structures (a VERY common case), or perhaps using a feedback equation where appropriate. Of course it is often the case that multiple methods are used as checks on each other.

In the case of Fig. 2, in Method 1 since we knew the transfer functions of the feed-forward and feedback paths, plugging into the feedback formula was practical. Further, while the feedback criterion can uncover conditions of stability, a proper s-plane analysis is more general and adaptable to methods of obtaining additional information. Particularly in cases where paths have a frequency dependence, we are interested in such things as the frequency response, the impulse response, and the step response.

FREQUENCY RESPONSE AND STEP RESPONSE FROM POLES

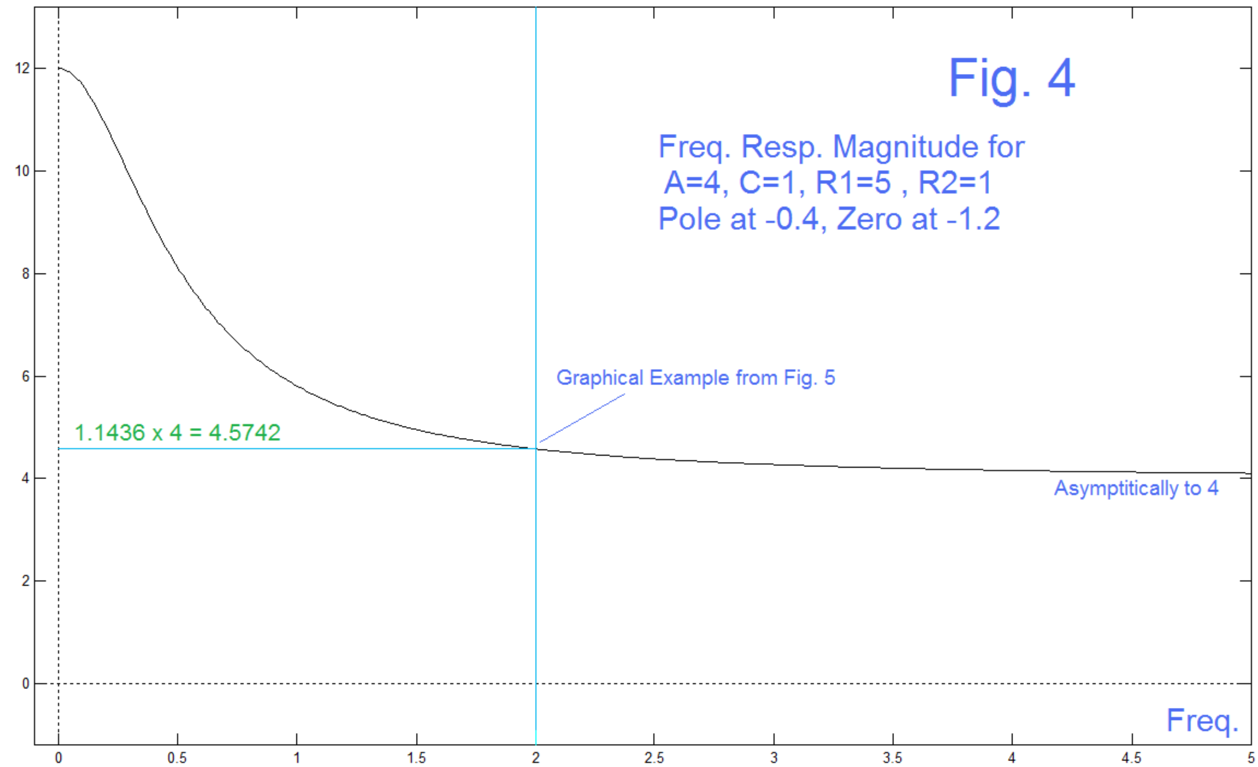
We have at least several good methods of calculating the frequency response from the transfer function. Foremost among these is plugging $j\omega$ in for s and taking the magnitude [square root of the product $T(j\omega)T(-j\omega)$]. This gives us a closed form from which we can calculate for all ω. We can also calculate from distances: from a point of interest on the $j\omega$-axis to each pole and each zero (see Fig. 5 below). There is an arbitrary constant multiplier possible here, but the response is proportional to the product of the distances to the zeros divided by the product of the distances to the poles. Another "method" is to just use a canned program such as Matlab's ***freqs***. Perhaps all these methods should start with an <u>estimation of the expected response</u>. Given that the transfer function was from equation (2):

$$T(s) = [A(R_1 + R_2 + sCR_1R_2)] \ / \ [R_1 + R_2(1-A) + sCR_1R_2] \qquad (9)$$

we can find the limit for DC (ω=0 or s=0) as $A(R_1+R_2) / [R_1 + R_2(1-A)]$ while the limit of very high frequency ($\omega\rightarrow\infty$, $s\rightarrow\infty$) is just A. Suppose for example that we look at A=4, R_1=5 and R_2=1, with C=0. This is stable by equation (7). The DC gain is 12 and the high frequency limit goes to A=4. This, given that the network is only first order, has to be just low-pass of some sort. Fig. 4 shows the full calculation (using ***freqs***).

So the frequency response starts at 12 and drops to 4 at the high frequency end. Not exactly a popular low-pass, but that's what we have here. Fig. 5 merely makes the point about the geometric interpretation of the frequency response magnitude. In general we plot the frequency response from a large set of calculated response values. For example, a particular choice of frequency ω=2, we have the geometry of Fig. 5. The response is proportional to the ratio of the distances dashed blue divided by dashed red, which is 1.1436. If we make this calculation for enough values of ω, we can sketch the shape of the curve. It is convenient to also consider the case where ω=0 (the origin in the s-plane), where the distance to the zero is 1.2 and to the pole it is 0.4 for a ratio of 3. As noted, we need to find the overall multiplier by a separate calculation. Since we found the limit at ω=0 to give a response of 12, we recognize an overall multiplier of 4 (that is, obviously, A=4) and the response at ω=2 is the ratio of the lengths there (1.1436) times 4, which is 4.5742 which is exactly what we have in Fig. 4.

Another thing that we can calculate that is related to stability is the step response. That is, a step appears at the input, and we assume that the network has been at rest for a very long time prior. Accordingly, the capacitor is discharged and the voltage across it is zero. Because of the discharged capacitor, there is not yet any positive feedback and the step, as multiplied by A=4, appears at the output (Fig. 6). But now the low-pass in



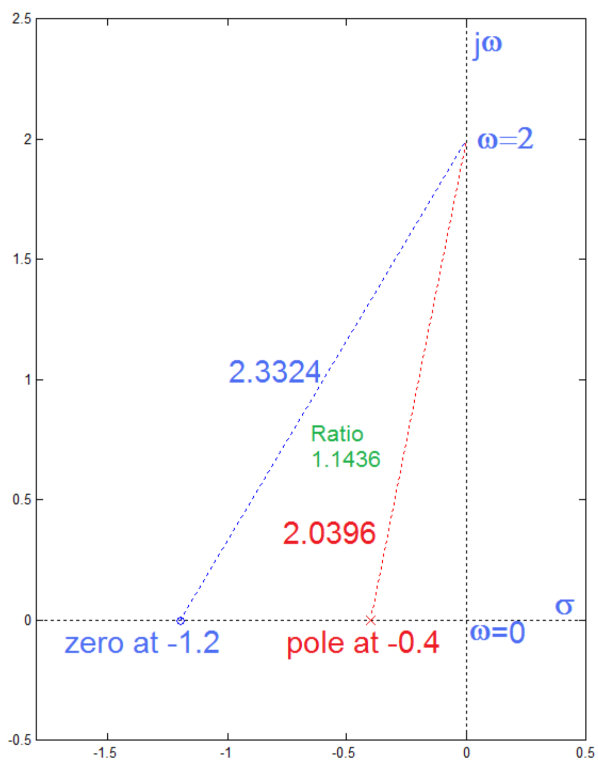
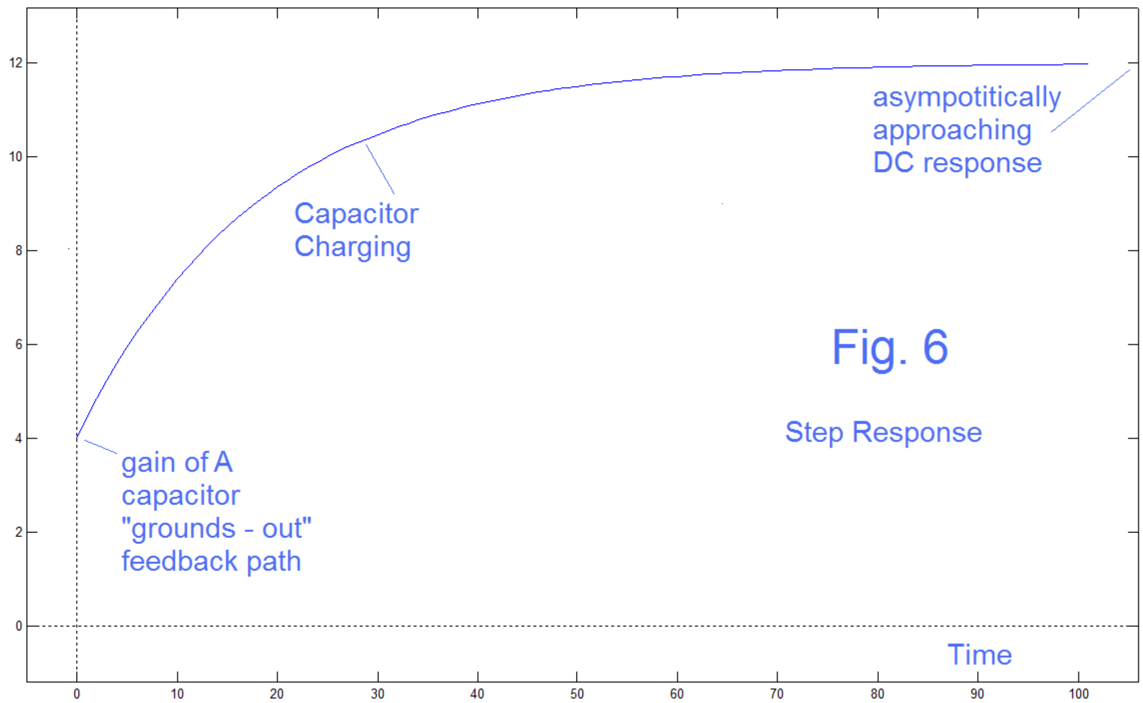


Fig. 5

s-Plane View



the feedback starts to charge its capacitor, and the output sees this positive contribution added to the step that remains at the input. Fig. 6 was actually generated by Matlab's ***step*** function which is a numerical simulation. Note that the step response begins to tip over and asymptotically heads toward a value of 12. It is no coincidence of course that this is the DC gain of the system (left side of Fig. 4), since a step goes on forever and eventually looks like the long-term application of a DC input. Notice how the numerical values at the endpoints are reversed for the step response and the frequency response. The horizontal axis for the step is time while for the magnitude response it is frequency. Either is probably satisfactory for looking at stability (blowing up to infinity or not) although the step response is probably the one that most closely relates to our notion of stability.

The fact that the example is stable comes as no surprise because we saw that we started with a stable pole (left half-plane). Further, we noted that the condition that the gain around the loop, $A[R_2/(R_1+R_2)]$ was less than one was met. While there is a capacitor involved, this makes the system additionally stable as well as simulating a delay. Accordingly it is convenient to write $F(s) = F = [R_2/(R_1+R_2)]$ as the feedback. This puts us pretty much at the position of a classic feedback flow-graph (Fig. 7).

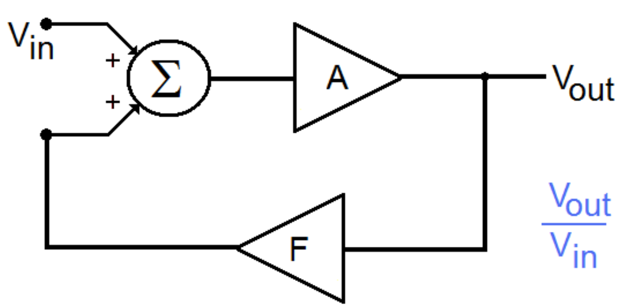
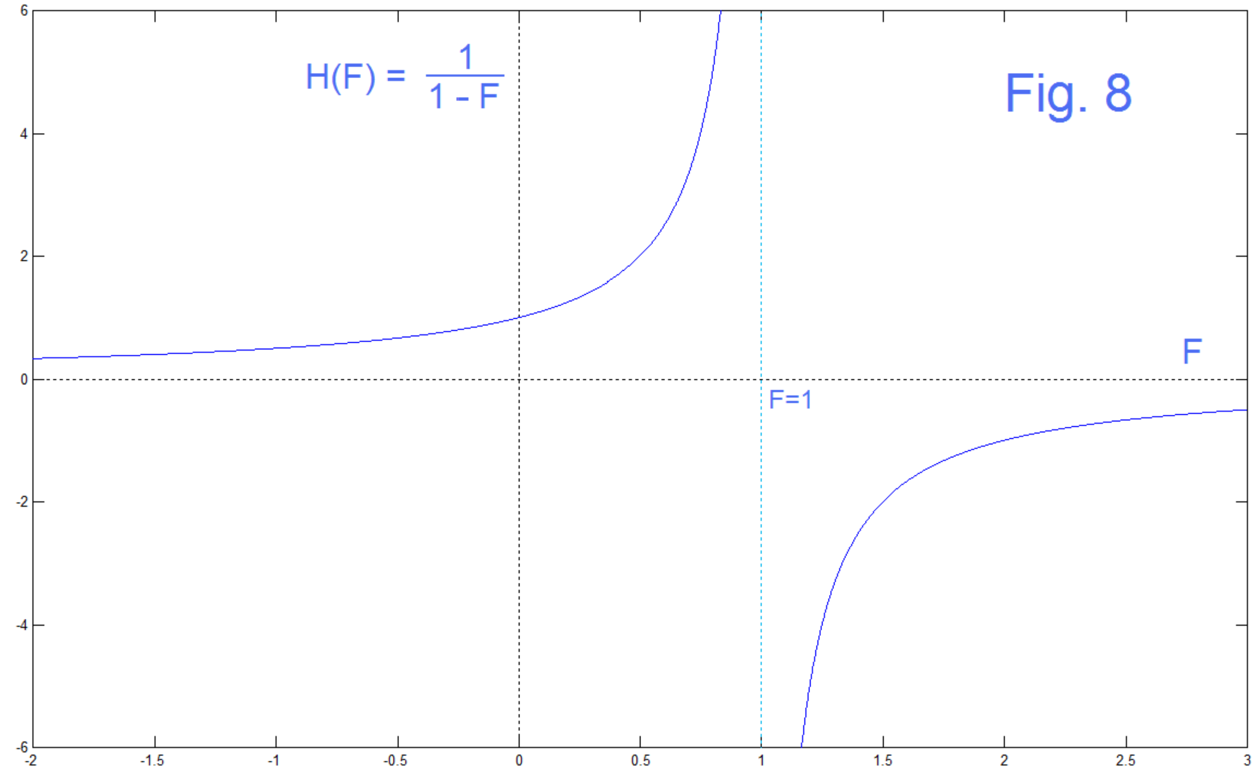


Fig. 7

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 - AF}$$



FEEDBACK EQUATION – FOR A = 1

In the one sense, looking at F as the voltage-divider ratio, F can only run from 0 to 1, (and stop just short of 1 for a stable result). With this situation (Fig. 2) we have no means of looking at F = -0.5 for example, or F=+1.3. A restriction such as that imposed by the voltage divider here is something we need to look for to come out of a study of the particular circuit or system. The feedback equation H(F) = 1/(1-F) is just math (Fig. 8).

The famous feedback curve of Fig. 8, or a similar curve when A is not required to be 1, is not a frequency response, an impulse response, or a step response. Indeed the horizontal axis is not frequency or time, but a parameter F. It has no poles (or zero) in the sense of a frequency plane (s-plane or z-plane) although it does have a "singularity" at F=1 where it goes to ∞, and indeed, comes back from –∞ in one of those head scratching math puzzles. We suspect that there is no "reality" associated with F=1. Even if we had a circuit that accepts F=1, as it searched for ∞ in one polarity or the other it could get no further than the power supply range (often called "rails"). But is there a physical reality (an apparent inversion of the amplifier polarity) for F > 1? NOPE!

So let's look at some examples. Fig. 9 shows feedbacks (positive) of F = 3/4, 1/2, 1/4, and 0. According to H = 1/(1-F), the resulting gains should be 4, 2, 4/3, and 0. These are the ends of the step response (at least approached asymptotically). These step response all start at 1, and ramp to the DC value. This is offered in verification to Fig. 8 for F=0 to F=1. The fact that Joe Born added the capacitor C just delays the response. This is (pedantically) very useful. We see something happening – not just that it has happened.

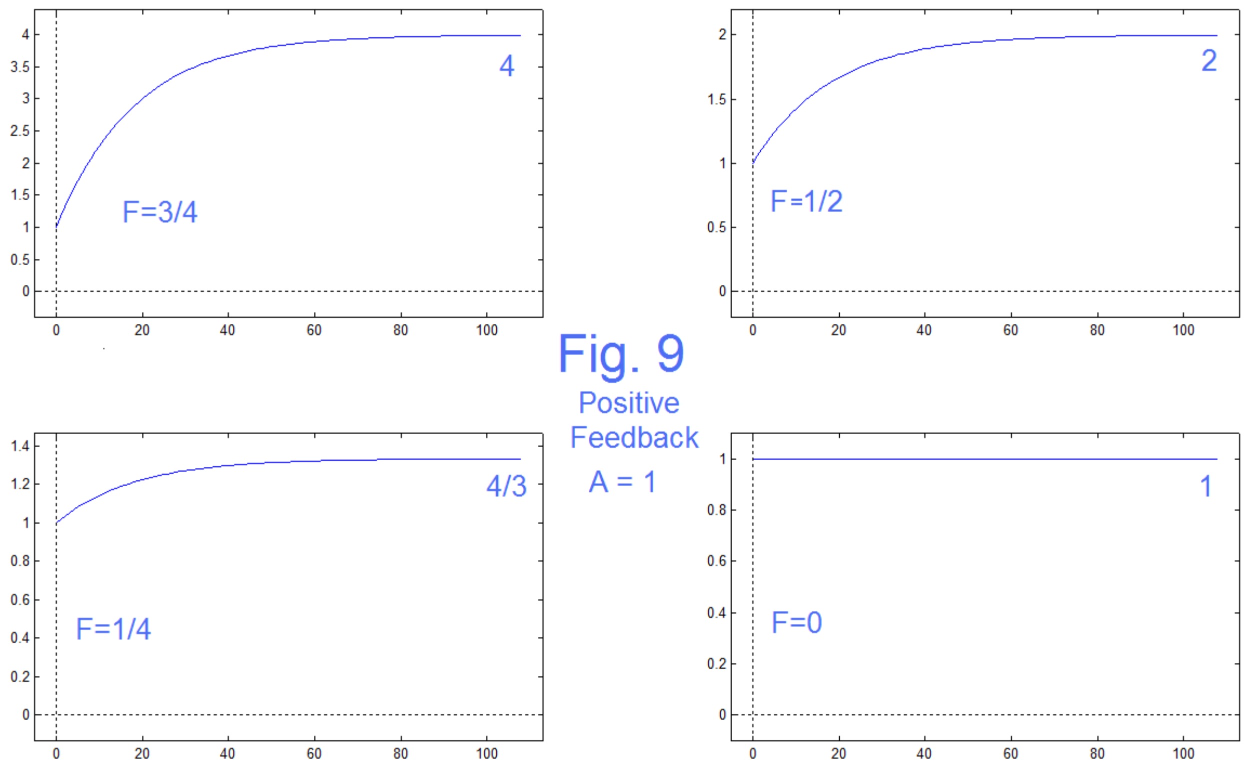


Fig. 9
Positive Feedback
A = 1

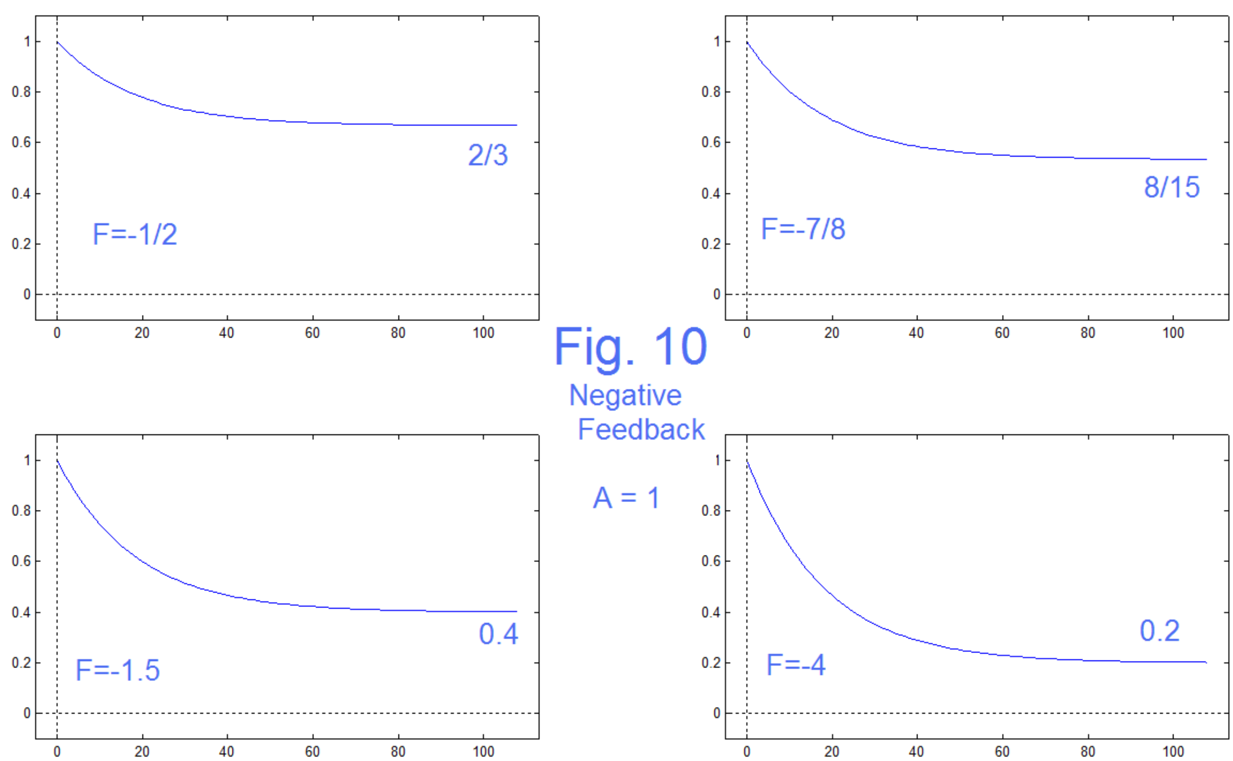
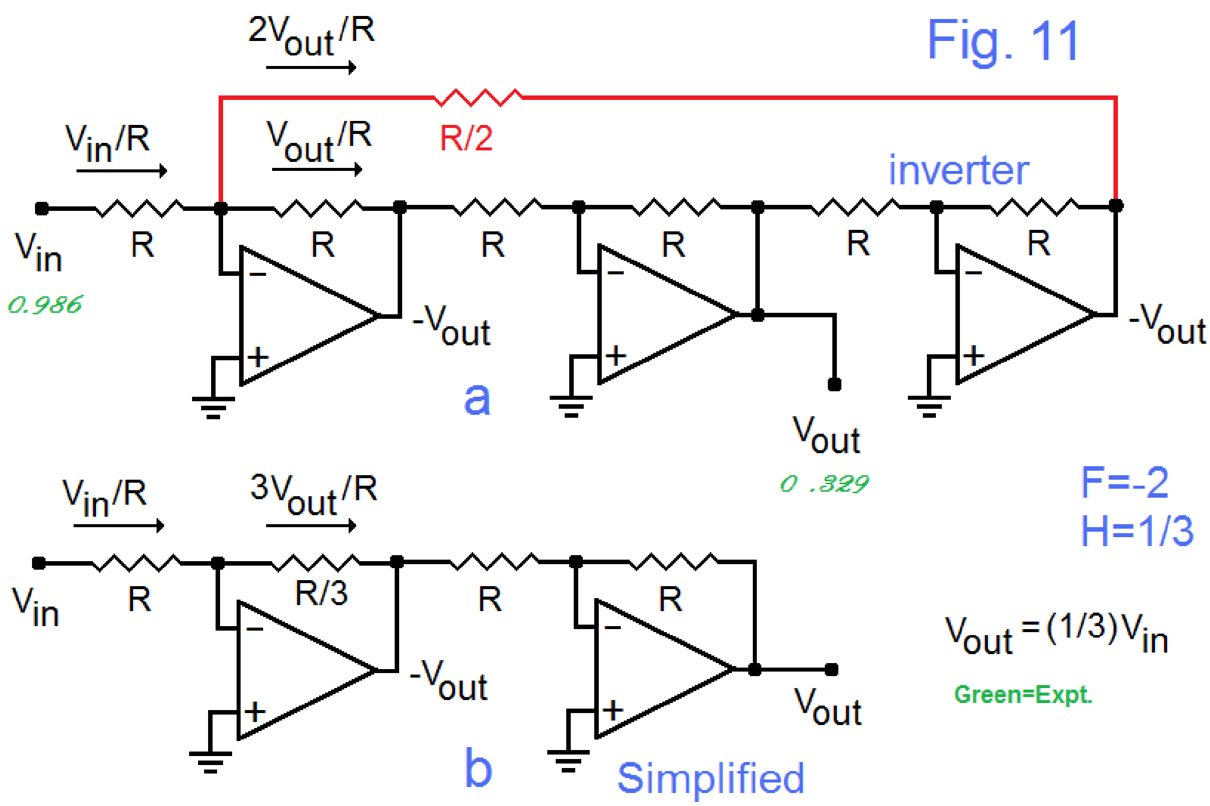
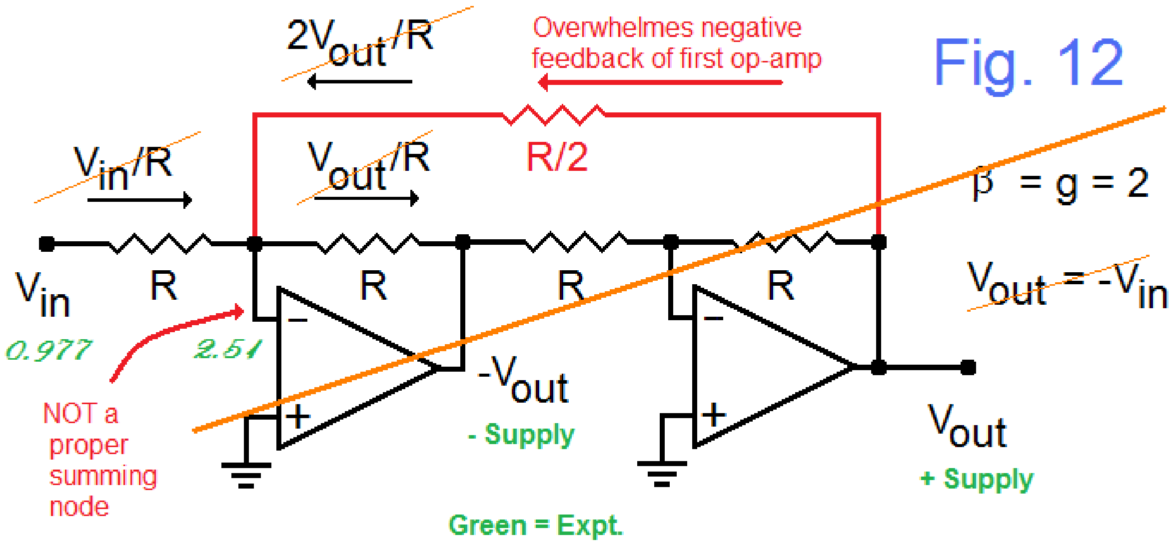


Fig. 10
Negative Feedback
A = 1

Next we look at cases where the feedback F is negative. This we can not do by adjusting the resistor ratio in Fig. 2. [Well, we can and did cheat in the computer program by using a negative resistance for R_1 – yes for R_1 – think about it.] This gives the result as shown in Fig. 10 showing F = -1/2, -7/8, -1.5, and -4, with corresponding gains H=1/(1-F) of 2/3, 8/15, 2/5, and 1/5, as seen as the step responses in Fig. 10. Note well that the negative feedback can go further back to -1 and as far negative as we want. The singularity and "inversion" occurs at F=+1 (Fig. 8). Lacking negative resistors we breadboard up a working circuit using an additional op-amp inverter as seen in Fig. 11 [1]. This was for F = -2, H = 1/3. Note that the additional inverter can be eliminated (b) and the result is just our familiar inverting amplifier (with feedback resistor R || R/2 = R/3) followed by a single inverter.



As we said at the top of this note, and wondered about just above: Why can't we use F > 1 and invert – the equation says we can. Well, in terms of describing an actual circuit, we see that it apparently fails because the summation requires the use of a proper electrical summer and the summation requires negative feedback. Fig. 12 here (also Fig. 12 in the reference [1]) shows simply that the positive feedback beyond F=1 exceeds the necessary negative feedback, and the summing node is destroyed, and the right-side op-amp has a non-zero differential input (2.51 volts in the bench test). [The left op-amp might have a proper virtual ground, or not, to the same output result.] Certainly it IS possible to come up with a number of op-amp circuits that have a gain of -1. For example, a simple nverter. The problem is that they don't involve the positive feedback, from the output, as postulated.



FEEDBACK EQUATION WITH A > 1

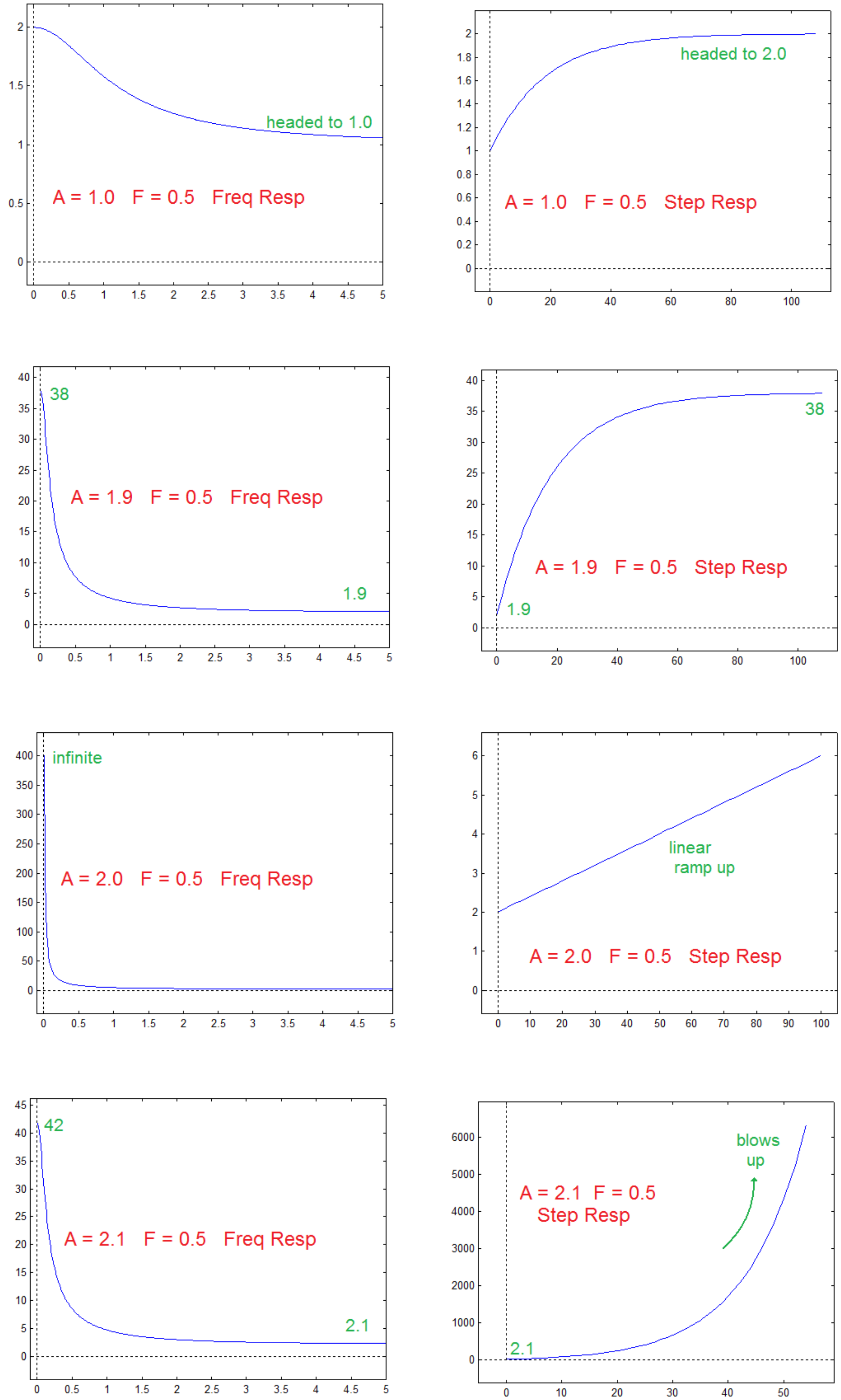
Above we had a forward gain of A=4 and then cut it back to A=1 while at the same time restricting F to be less than 1 because it was an ordinary voltage-divider. This did have the consequence that we could only use the circuit of Fig. 2 up to the singularity. If we allow for a positive gain of A >1 (say A around 2) then the voltage divider with approximately equal resistor is exploring the region about that singularity. That is, we use (Fig. 7):

$$H = A / (1 - AF) \qquad (10)$$

Fig. 13 shows plots of the frequency response and the step response for F = 1/2 ($R_1 = R_2$) and for values of A of 1, 1.9, 2, and 2.1. The last three values thus illustrate the condition just before the singularity, exactly at the singularity, and just above the singularity (A = 1.9, 2, and 2.1 respectively). Here we have merely amplified first so that a normal voltage-divider that follows will have a larger output. Roughly we double A so that F being around half allows us to look at loop gains AF on both sides of +1. An honest trick.

The top panel of Fig. 13 shows a baseline case of A=1 and F=1/2, and we note the expected (modest) frequency and step responses. In the second panel we have A=1.9 and F remaining at 1/2. So H = A/(1-AF) = 38, and this is what we find for the response limits. Here the loop gain is 1.9x0.5 = 0.95 so it is approaching +1, and in practice, we may be uncomfortably close (given component tolerances, noise, etc.). The third panel is for A=2, so the loop gain reaches 1 and H becomes infinite. This is a pole exactly at s=0 and is thus an integrator. The frequency response blows up at ω=0, so the first point is not plotted. The step response is just the expected linear ramp – the integrated step. Not useful except as an example. Beyond this the bottom panel shows A=2.1, so AF=1.05 is a loop gain exceeding +1. While the frequency response does not seem to

Fig. 13



Pole -1
Zero -2

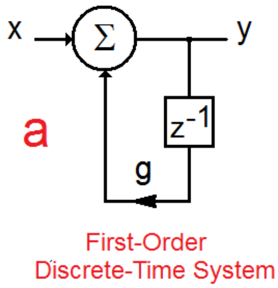
Pole -0.1
Zero -2

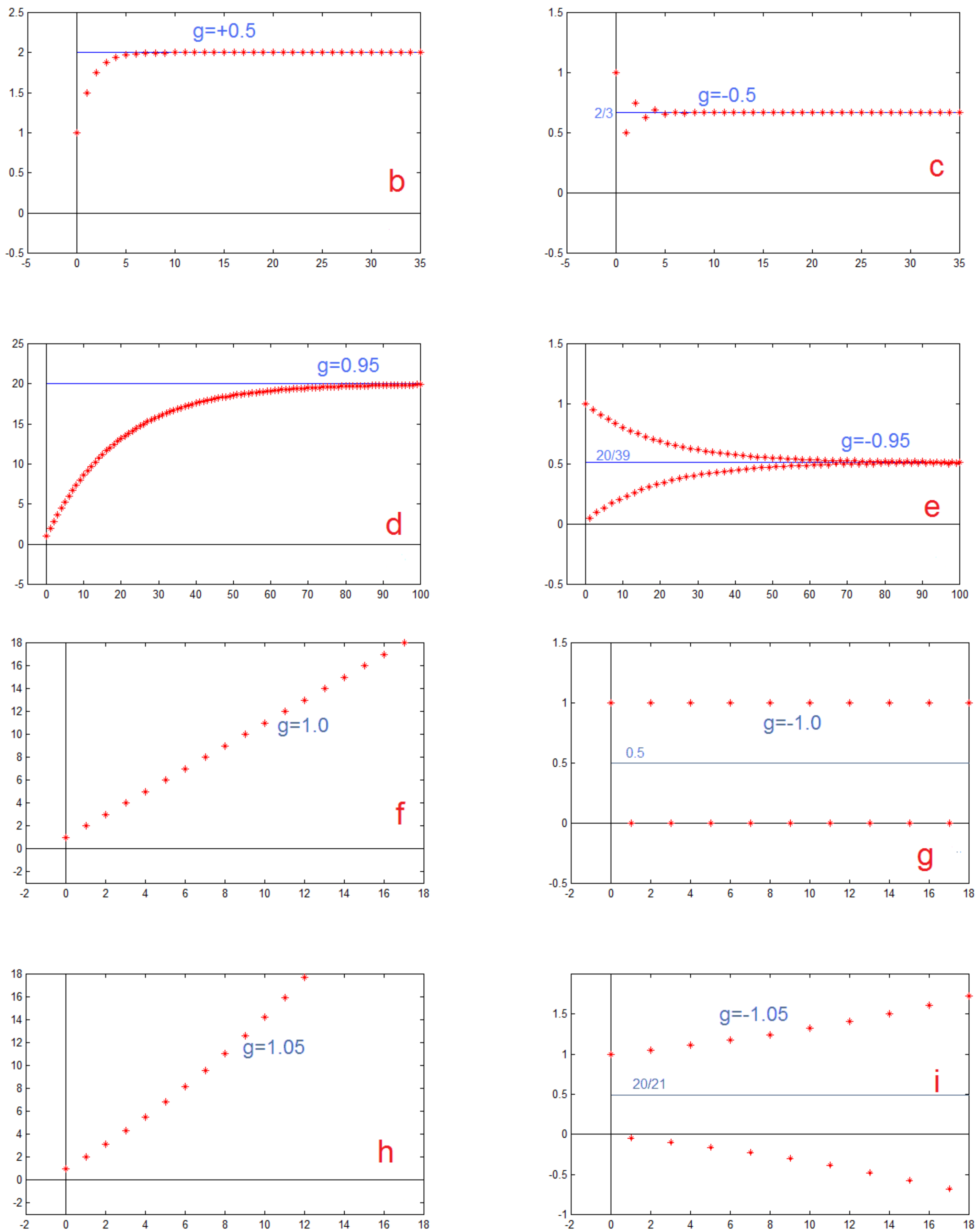
Pole 0
Zero -2

Pole 0.2
Zero -2

Fig. 14

Discrete Time





blow up, it corresponds to a right half-plane pole at +0.2. So while the feedback equation suggests the response of the system would be to multiply by -42, this is <u>not</u> what we find. Rather the step response blows up exponentially.

THE DISCRETE TIME CASE

At this point we have covered much of the same material as previously within a particular context and been more emphatic about what lies beyond the supposed limits. We have not gotten around to the discrete-time case, although this was actually the starting point of the previous review [1]. The delay makes an essential difference. The added perspective at the moment is that in this note adding a capacitor made it possible for us to see things happening much as they appear naturally in the discrete time presentation. This does not, I fear, make everything crystal clear.

Fig.14a shows the feedback with the delay. Figures 14b, 14c, 14d, and 14e show cases where we are within the stable range. Note well that for the discrete time case, the stable range does end on the negative side at -1 as well as on the positive side at +1. This is best understood in terms of digital filter theory in the z-domain. Exceeding this range is shown Fig. 14f, 14g, 14h, and 14i. More discussion of these issues can be found in the original reference [1].

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