

ELECTRONOTES

WEBNOTE 24

3/10/2015

ENWN-24

NOTCH FILTER WITH REAL POLES

We have spent some time recently on notch filters [1-3] and here we want to look at a case where we have a relatively weak notch associated with real poles, and zeros not exactly on the $j\omega$ -axis (slightly into the left half of the s -plane). At the same time, the poles are not necessarily on the same radius as the zeros. Thus it is perhaps less like a notch one would design with a specific application in mind, and more like the analysis of an existing physical notch.

We can start from a number of places. Generally we might be starting from a measured frequency response, and from it (perhaps by trial and error) get a possible pole/zero plot. Here we will start from a pole/zero plot AND the corresponding frequency response (Fig. 1 and Fig. 2). We chose the zeros to be at a radius of $1/11$ at an angle of 87° , and the two poles to be real and both at $-1/7.6$ (Fig. 1). The pole/zero plot is not enough to give us the exact transfer function, since an arbitrary constant multiplier is always possible. We will take this multiplier to be 1. Thus:

$$z_1 = (1/11)[- \cos(87^\circ) + j \sin(87^\circ)]$$

$$z_2 = (1/11)[- \cos(87^\circ) - j \sin(87^\circ)]$$

$$p_{1,2} = -(1/7.6) = -0.131579$$

so:

$$T(s) = [(s-z_1)(s-z_2)] / [(s-p_1)(s-p_2)] = (s^2 + 0.0095156s + 0.0082645) / (s^2 + 0.263158s + 0.017313)$$

This allows us to calculate the frequency response (like with Matlab's *freqs*) the magnitude of which is seen in Fig. 2.

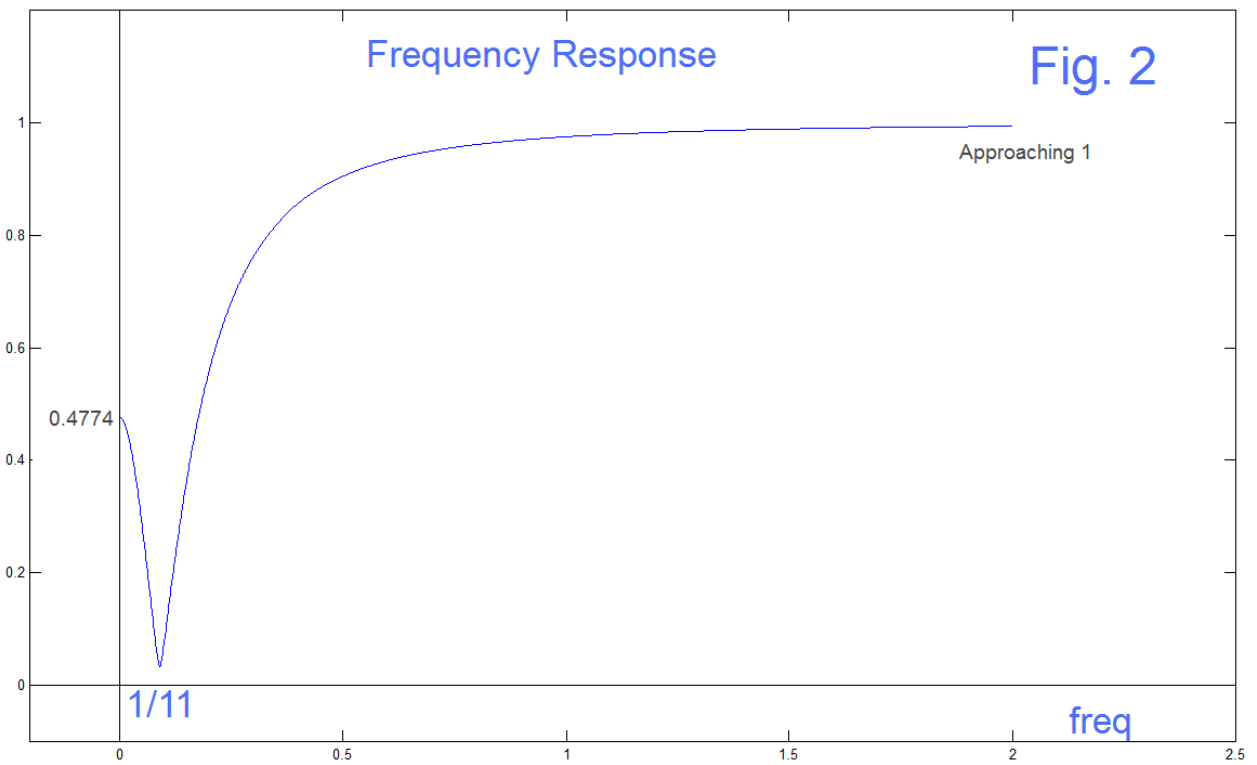
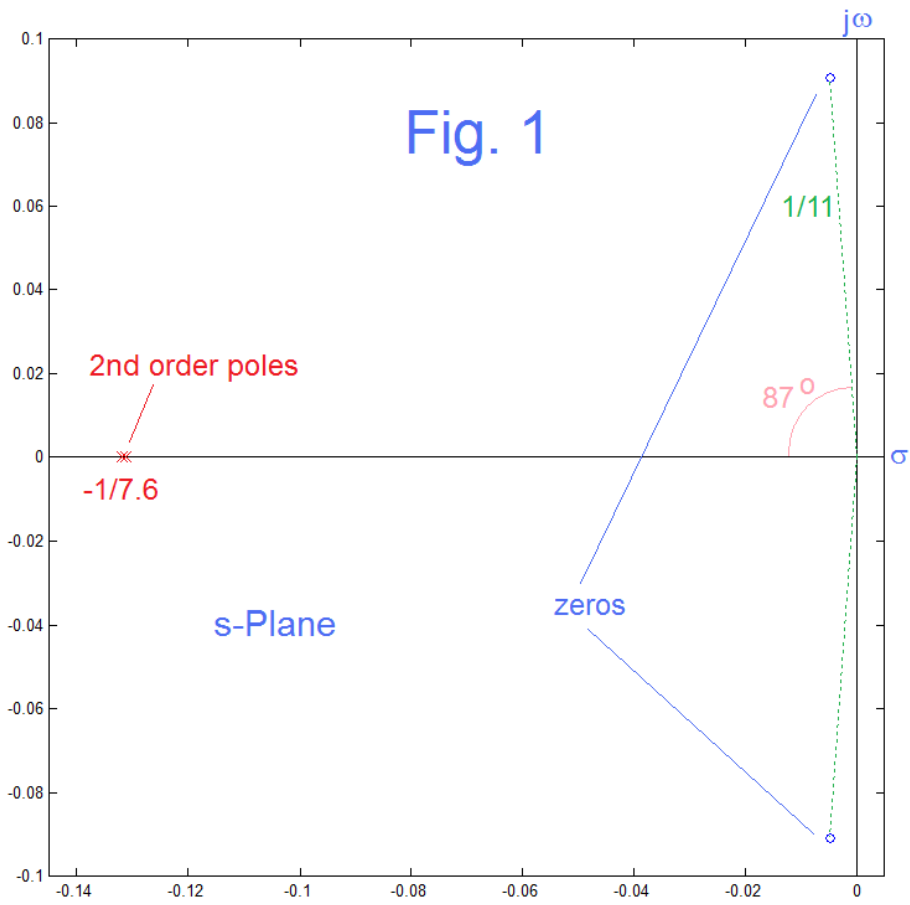
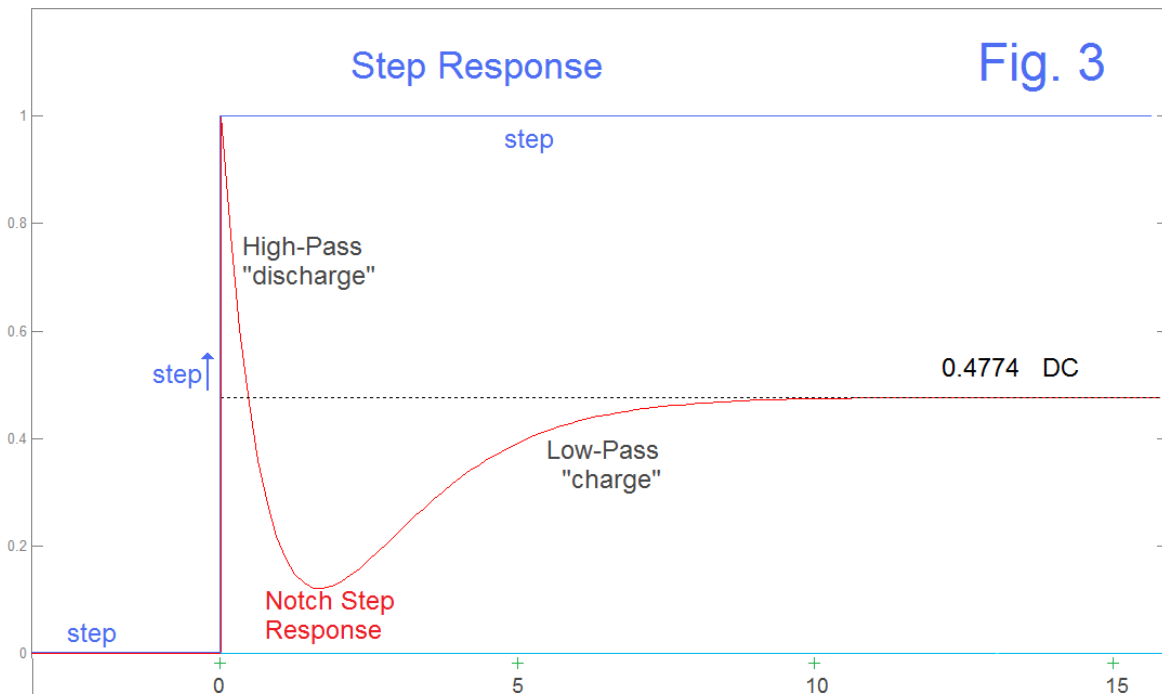


Fig. 2 shows the expected frequency response magnitude, showing a decent but not fully-rejecting notch at a frequency of $1/11$. We also note that the low-frequency “shelf” begins at 0.4774 and not at 1.00, which the high frequency “shelf” approaches. This is because the two poles are back at $1/7.6$ while the zeros are at a frequency $1/11$. Normally when we actually design a notch filter we are interested on having comparable shelves. Further, as noted, because the two poles are real (and we must have the same number of poles as we have zeros) the notch is not very sharp.

When here is interest in a step response of a notch filter, we have numerous ways to calculate this [1-3]. Most simply it is seen as the sum of a low-pass step response (the canonic RC low-pass response to a switched-on voltage step), an exponential charge, and the corresponding exponential discharge of a high-pass. [In the same flavor, the frequency response was the sum of a low-pass and a high-pass, which is a much deeper notch than one might expect because we have phase cancellation.] The calculation for the poles/zeros of Fig. 1 leads to the correct-looking result of Fig. 3, which is of course purely causal. This we got using Matlab’s *step*.



The step driving the notch is blue and is zero until time 0 at which time it becomes 1. The step response of the notch (red) is rigorously zero for negative time, and jumps to 1 at time 0, at which time it immediately starts to discharge, being dominated by the high-pass step response. At the same time, at time 0 the contribution of the low-pass step response has just gotten started. By about time 3, the low-pass is starting to dominate. Eventually the notch step response becomes almost entirely that of the low-pass, which in this example, approaches 0.4774 (the DC response) asymptotically. The swing is relatively modest because of the fact that the poles are real and back relatively further than we often see.

Here and in previous related examples [1-3] we have made the point that the response of the general nature shown in Fig. 3 is correct (because it IS correct) and because other possibilities are non-causal, which are forbidden in a physical system. Just below we will consider what certain non-causal (incorrect) results might look like. This is a familiar game. Usually we arrive at a result and rapidly come to the conclusion that we have done something wrong. [Many instructors give considerable exam credit when a student just recognizes that he/she has to be wrong!].

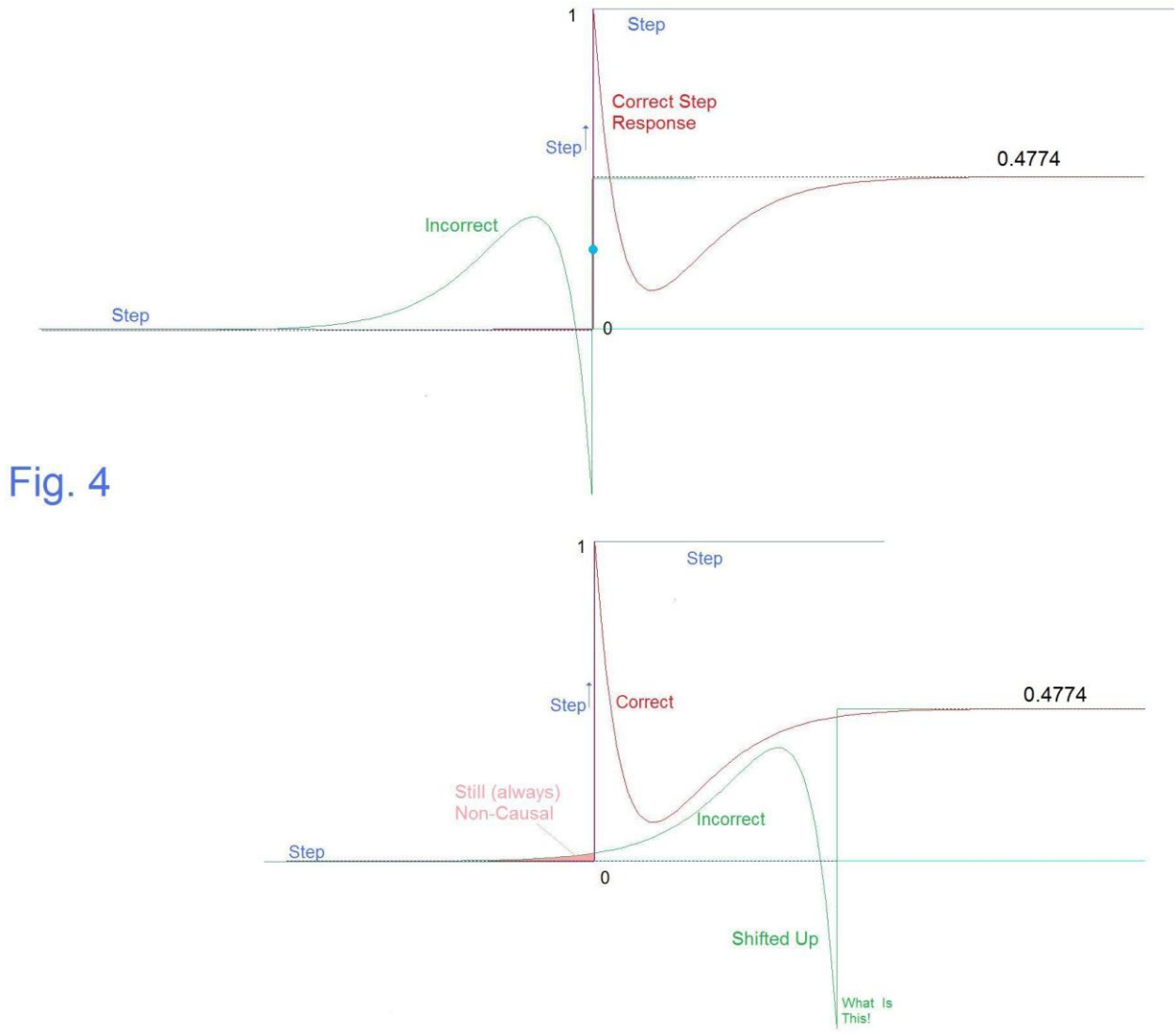


Fig. 4

Accordingly, we ask if there are sign flips, scales, rotations, or shifts of the correct result that would give a result that is perhaps more subtly wrong. One such case is Fig. 4. Here we show two examples, top and bottom, where the correct curve is shown in red while the proposed pretender is in green. First however note that any attempt to reverse time is going to fail as strictly anti-causal. Similarly, any inversion of the waveform would be acceptably as it would be causal but just a sign error. Time shifts, moving the red curve to the right would be fine (although an explanation of the delay would be

necessary). Rotating the whole thing by 180° is visualized as having the plot printed out and rotating the paper around the point (0, 0). It would be the same as reversing both time and amplitude. There would be the horrible negative-time component asymptotically at -0.4774. If however we shifted the rotated result up by 0.4774, this would go asymptotically to 0 at negative infinity, and would “look better” (Fig. 4, top). This is the same as rotating the upper plot of Fig. 4 about the blue dot that is on the upstep exactly half way to the DC response value (0.4774/2).

It would look better still if the rotated and shifted up curve were shifted to the right a bit (Fig. 4, bottom). Here we might have shifted it even further than the actual diagram, so that the actual plot would be fully to the axis on the left. Instead, we left the little pink non-causal region as a reminder. In fact, it is impossible to shift it enough to make it causal. You can't be a little-bit non-causal. Further, the ugly jag (“What is this!” in Fig. 4, bottom) leading to an absolutely flat plateau would remain, as would be a requirement to justify any delay tried. Alarm bells!

Accordingly, results other than Fig. 3 are incorrect.

REFERENCES

[1] B. Hutchins, “Notch Filtering – Some Notes,” Electronotes Application Note No. 413 July 30, 2014

<http://electronotes.netfirms.com/AN413.pdf>

[2] B. Hutchins, “Step Response From Laplace Transform,” Electronotes Application Note No. 414, Aug 15, 2014

<http://electronotes.netfirms.com/AN414.pdf>

[3] B. Hutchins, “The Difference Between All-Pass and Notch,” Electronotes Webnote ENWN-19 , 9/15/2014

<http://electronotes.netfirms.com/ENWN19.pdf>