ELECTRONOTES

WEBNOTE 23 2/22/2015

ENWN-23

TRIG IDENTITIES – USEFUL FOR PHASE SHIFTERS

The 90° PDNs

Phase shifters are to be used with caution. First of all because the ear is, roughly-speaking, “phase deaf” according to Ohm’s Acoustic Law, a phase shifting filter in audio may well be inaudible. That is not to say that we don’t need and use phase shifters. Most of the uses are for cases when a phase is shifted and then recombined (or otherwise reused). A popular use of phase shifters is inside so-called 90 degree phase differencing networks (90°PDNs).

These 90°PDNs are used in frequency shifter design. While we have probably not done a good comprehensive summary of this art, the following two sources give the necessary ideas. The idea is to have two cascades of phase shifters (all-pass filters) such that one “chases” the other while maintaining an approximate 90° phase difference at the parallel outputs. This is an “approximation problem” of which the Weaver method is as good as any and was described in the Musical Engineer’s Handbook (1975). This showed how to calculate the poles (and mirror zeros) of the two networks:

http://electronotes.netfirms.com/MEHCh6aPart.PDF

About the only significant improvement appeared in 1987 in EN#168 (also called Special Issue G). This had a good listing of reference materials, and illustrated how poles could be combined into second-order sections so as to DRASTICALLY reduce component spread – **way too much of an improvement to ignore**:

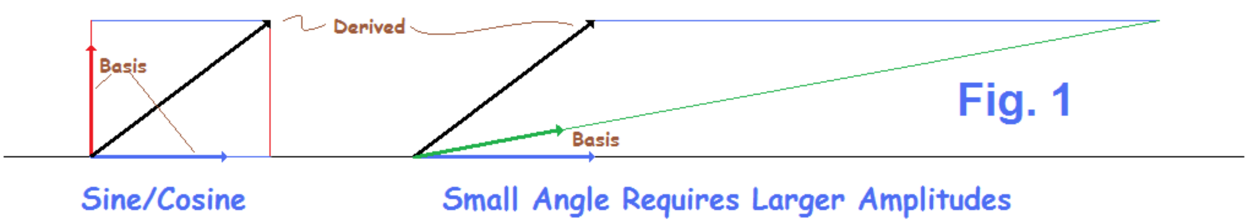
http://electronotes.netfirms.com/EN168-90degreePDN.PDF

That’s about all we did. Perhaps we will eventually get to a full “Revisit” of this topic.

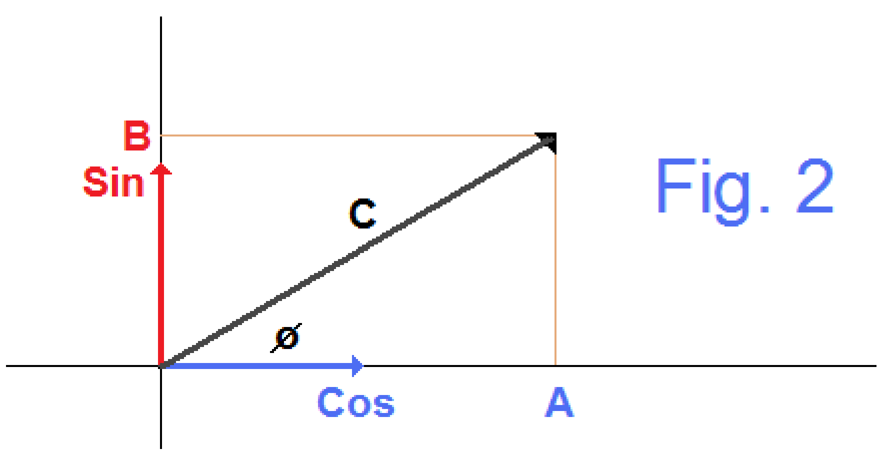
Other Than 90°

A new customer asked about these designs with particular regard for how an angle other than 90° might be achieved. It is probably clear that we could redesign the networks for other phase differences, and also clear that this would be a lot of effort. In consequence, we would look to have something that might be easier, and perhaps more general at the same time. That is, can we form the desired phase from some combination of the 90° phases. Of course we can. But is this trivial? Not really – at least not a linear process. The reason is that we have two non-linear equations involved: that amplitude in terms of the Pythagorean theorem and the phase in terms of an inverse tangent.

But first, we need to be clear that we can generate any arbitrary phase and amplitude by having any two different phases, not just two phases differing by 90°. This is necessary because we have said above that even with the attempt to achieve 90° of phase difference, we only get this approximately (typically with an error called ε of perhaps 2° or 3°). Fig. 1 shows a desired phase represented by a black vector as it can be generated as a weighted sum of two basis vectors 90° apart (red and blue) on the left. On the right, the same black vector is generated as a weighted sum of two basis vectors (green and blue) at a much smaller angle. Because we allow arbitrary weights, the basis vectors can be any non-zero length. The one thing to note is that the amplitudes are greater for the smaller angle.



We shall look here at the case of a 90° basis as shown in Fig. 2 where we assume a unit cosine (blue) and a unit sine (red) weighed by amplitudes A and B to form a sinewave of amplitude C at a phase angle ø . So, now for the trig equations:



By the Pythagorean theorem, we have:

$$C = \sqrt{A^2 + B^2} \tag{1}$$

and by the trig definition of tangent:

$$\varphi = \ tan^{-1}\left(\frac{B}{A}\right) \tag{2a}$$

or:

$$B = A\ tan\ \varphi \tag{2b}$$

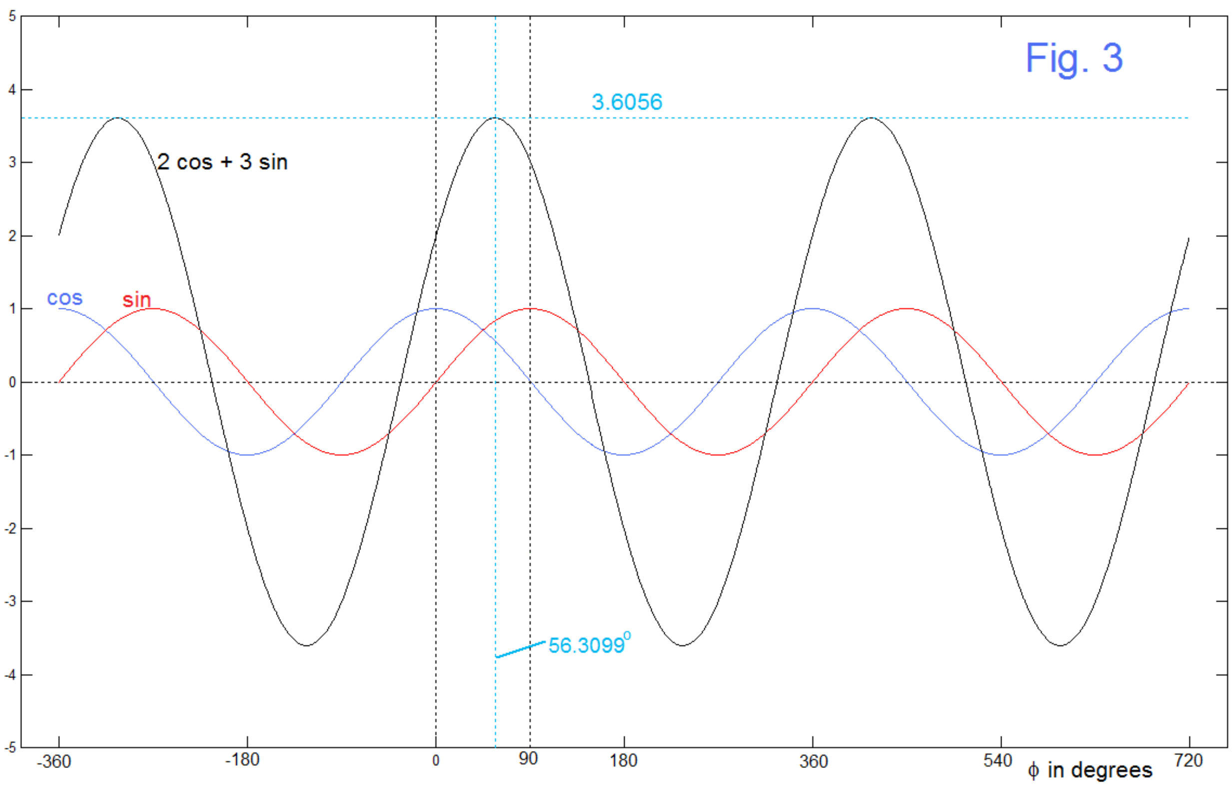
When we require a unity magnitude sinewave as output, C=1 requires:

$$A^2 + B^2 = 1 \tag{3}$$

Solving (2b) and (3) together gives:

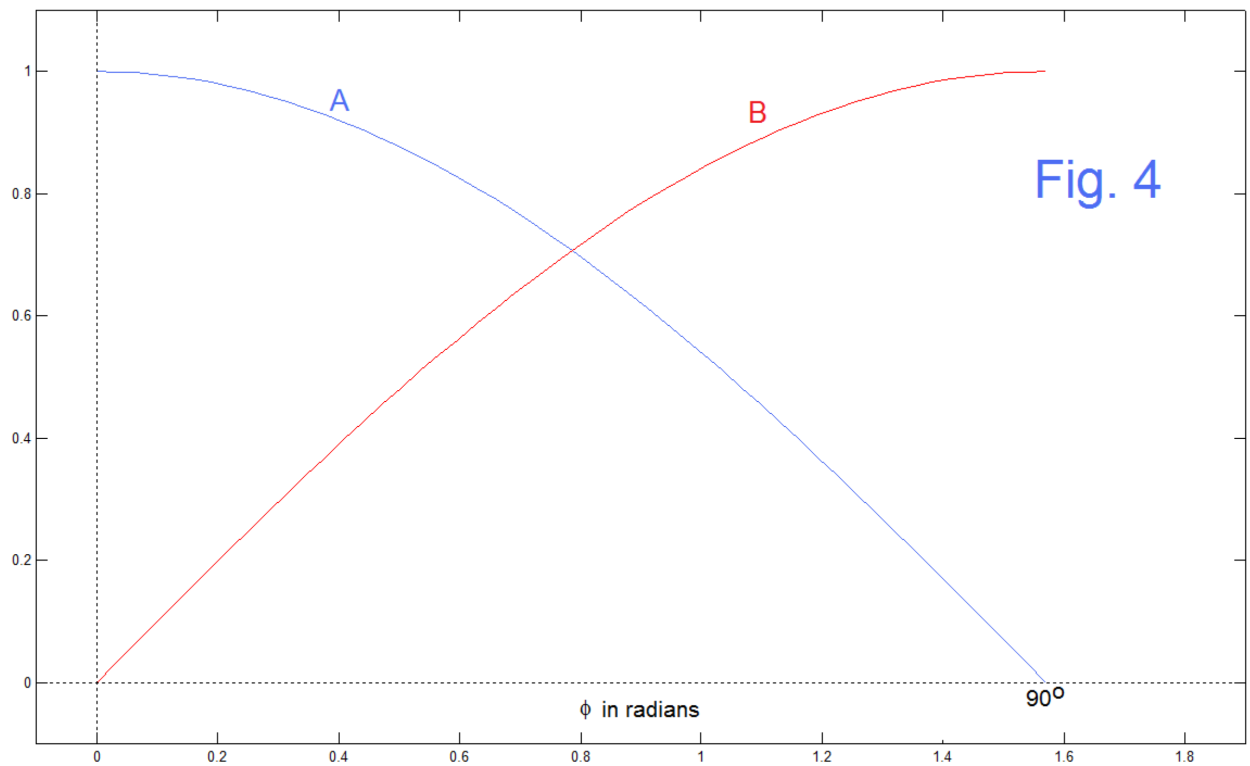
$$A = \frac{1}{\sqrt{1 + tan^2 \emptyset}} \tag{4a}$$

$$B = \ \frac{tan\ \emptyset}{\sqrt{1 + tan^2 \emptyset}} \tag{4b}$$



That's it.

At this point, we need to do some plotting related to these equations. First of all we will consider the summation of Fig. 2 where we now set A=2 and B=3 just as an example. Equations (1) and (2a) give C=3.6053 and ø=56.3099°. Fig. 3 shows a plot of the sine and the cosine along with the weighted sum, with the dashed light blue lines marking the verification of the calculated results. Note that we could easily set the resulting sinewave to unity amplitude by dividing by 3.6053. Fig. 3 is just for perspective – nothing new.



Our next test is to see what the curves for A and for B look like as a function of ø. That is, we plot equations (4a) and (4b), and these curves (which involve tangent functions) are shown in Fig. 4. Note the symmetry, and the fact that to get 45° A and B are both $\sqrt{2}/2$ which is what we need to get C=1 there. In fact, it is obvious that A and B from equations (4a) and (4b) obey the Pythagorean theorem with C=1. BUT – Fig. 4 immediately answers the question as to whether a simple "cross-fade" (linear ramps) will yield a proportionate phase. It won't.

The curves of Fig. 4 are not, however totally unfamiliar. They at least look a lot like what we might use to do a triangle-to-sine waveshaping. For example, the overdriven OTA waveshaper (EN#82 – Oct. 1977 - see brief excerpt on page 6 below) has a similar-looking non-linearity, although it is a hyperbolic tangent. This might deserve a careful look.

The final look here is to consider what would happen with an ordinary cross-fade. That is we have a cross-fading variable g that runs from 0 to +1, and this would be our weight B for the sine. At the same time, we would use A = 1-g for the cosine weight. Clearly we would have a pure cosine at g=0 and a pure sine at g=1. At the midpoint of the cross-fade where g reaches 1/2, 1-g is also 1/2, and the magnitude as is $\sqrt{2}/2$ or 0.7071, which represents a loss of amplitude (to 71%) relative to the starting and ending points. On the other hand, the phase at the exact middle would be exactly 45°, but this is a special result. Equations (1) and (2a) tell us the full story, as plotted in Fig. 5. The phase, Fig. 5 top is not all that non-linear with g. The magnitude, C, in the lower portion does have that dip.

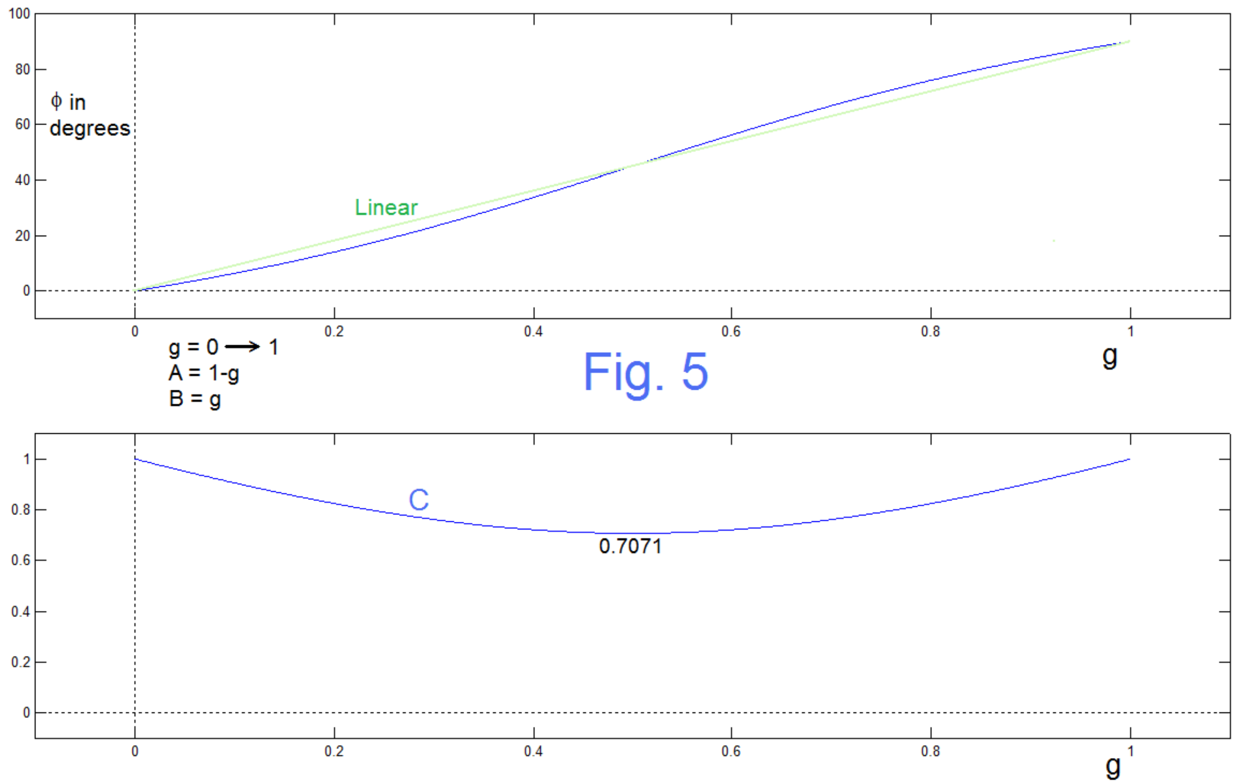
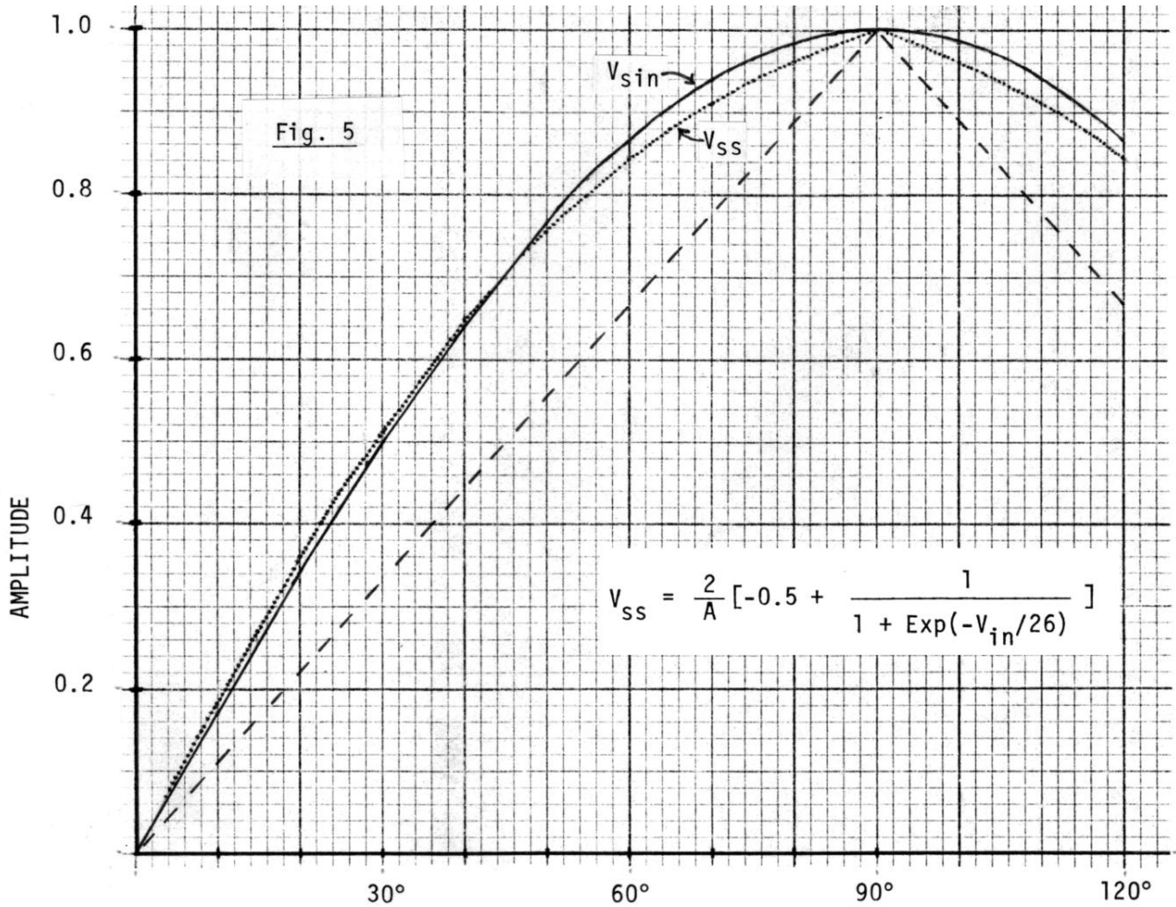


Fig. 5

Probably what we would seek to learn is whether or not this simple cross-fade would be satisfactory. It is after all just what we would achieve with a pot as a voltage-divider - a manual control in this case. Beyond that, the actual desired control curves might be achieved, at least to an approximation. And, keep in mind that if the input sine and cosine come from a 90°PDN, the phases already have some error to start with.



- - - - - - = Driving Triangle Wave

———— = Ideal Sine Wave

············ = Single Stage Uverdriven Differential Amplifier Approximation, 79 mV In.

EN#82 (8)

Figure Above: Excerpt from EN#82

Final Notes: (1) Here we have hopefully given enough information to put together an investigation of how to achieve a phase differing network of phases other than just 90°. (2) Keep in mind that the all-pass networks used do have ("exactly") unity gain (but not exactly 90°), once we combine the 90° phases, some amplitude variations will be expected. Contrast this with the discrete time approach to a 90° shift (Hilbert transformer) where we have exactly 90° in the shifted output, but not the exact unity gain we found in the analog all-pass. Typical engineering tradeoffs.