ELECTRONOTES

WEBNOTE 19   9/15/2014

The Difference Between All-Pass and Notch

 Of course there is a large difference between an all-pass filter and a notch filter. Or is there?   Except for the one notch frequency (and a nearby frequency range) we are generally interested in passing the original signal with no attenuation. So it is often the difference in application rather than the gain at a particular frequency that is at issue. In addition to this, there is another "apples-vs-oranges" difference best illustrated in Fig. 1.


Fig. 1
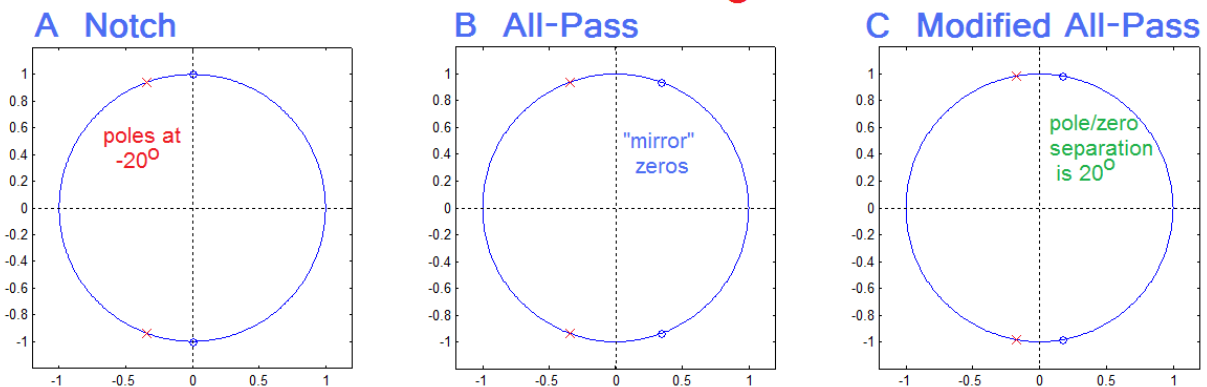
In Fig. 1, we have in A a familiar notch, having a pair of conjugate poles in the negative half-plane (at 20° to the jω-axis) with zeros on the same radius. The corresponding all-pass has zeros not on the jω-axis, but mirrored to poles as in B.   Note that in A the poles/zeros are separated by 20°while in B the separation is 40°. Here we will be more interested in an all-pass case where the pole/zero separation is again 20°, as in C. We want to compare the notch of A with the all-pass of C. Note that the damping in the case of C is half that of A.

Here we will be interested in cases with the geometry of Fig. 1A and Fig. 1C, but at an angle like 2.866 degrees (D=0.05) between the poles and zeros.   What this means can be appreciated from Fig. 1 if we consider the case where the poles and corresponding zeros are very close together and where we are relatively far from them, which would be the case for relatively low frequencies or relatively high frequencies, relative to 1. Viewed from such limits, the poles and zeros tend to shield each other and thus to cancel. In contrast, at a frequency close to 1, for the notch a zero is right in the path (the axis) while the poles is off to the left.   Thus the "supporting" effect of the poles is overwhelmed by the nulling of the unavoidable zero.   In the case of the all pass, the distances to the poles and zeros always cancel in corresponding pairs, but the phase angles add (double – the phase angle off the zeros being of opposite sign to the poles).

Fig. 2 shows the cases of the magnitudes of the frequency responses. Here for the notch we show for comparison a lower Q (larger damping D=1/2), but want to emphasize the less damped (D=1/20) case. This same D=1/20 is used for the all-pass. Note the relatively sharp notch. Also note that the responses are very similar (unity) for frequencies not close to 1. "Far away", the fact that the zero is waiting right on the jω-axis is not apparent in the notch magnitude response.   The frequency just wanders into the trap and the magnitude is forced to zero.
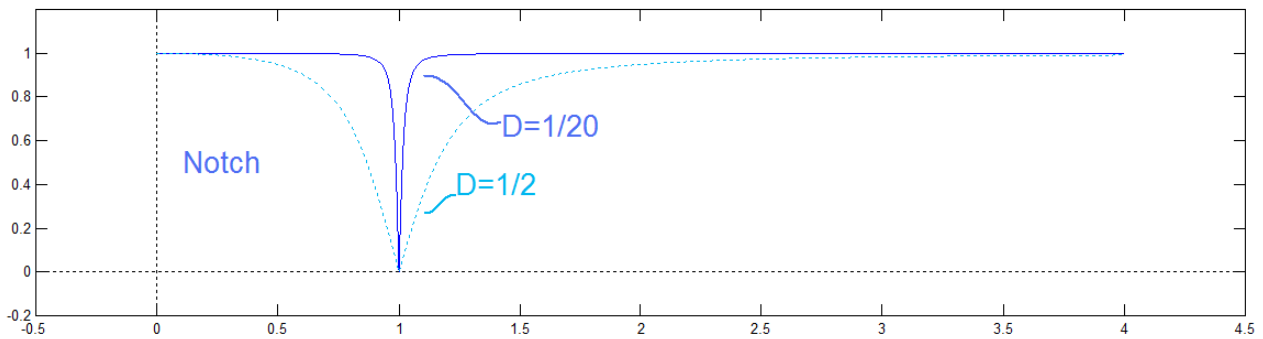

Magnitude Responses

Fig. 2
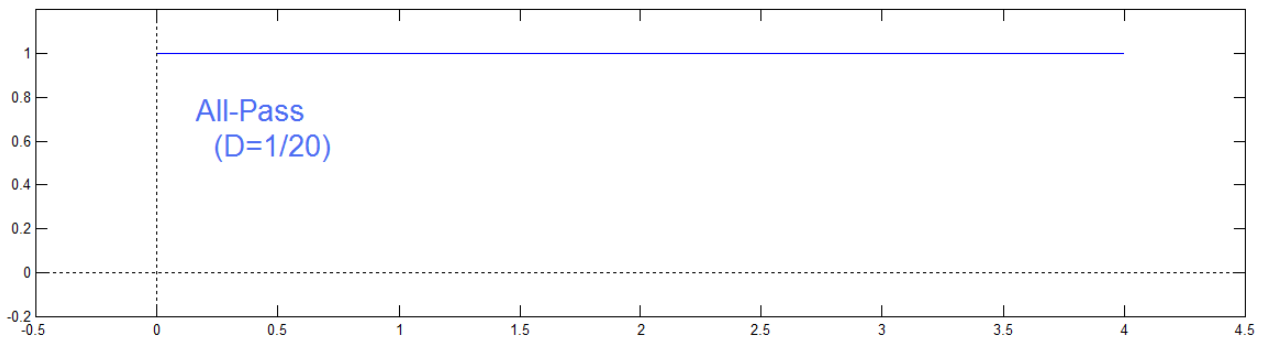
In addition to the magnitude response, we are interested in the phase responses. This is usually somewhat more difficult to interpret.   Here instead of taking the magnitude of a transfer function. We compute the angle, the inverses tangent of the imaginary part divided by the real part. This inverse tangent is subject to ambiguities of multiples of 2π. Fig. 3 shows the phase responses that correspond to the magnitude responses of Fig. 2, here showing the higher-Q (D=1/20 notch, D=1/40 all-pass) cases. It appears that the phase responses are much the same for the notch and for the all-pass, except the all-pass has twice the angle.   Can this be right?
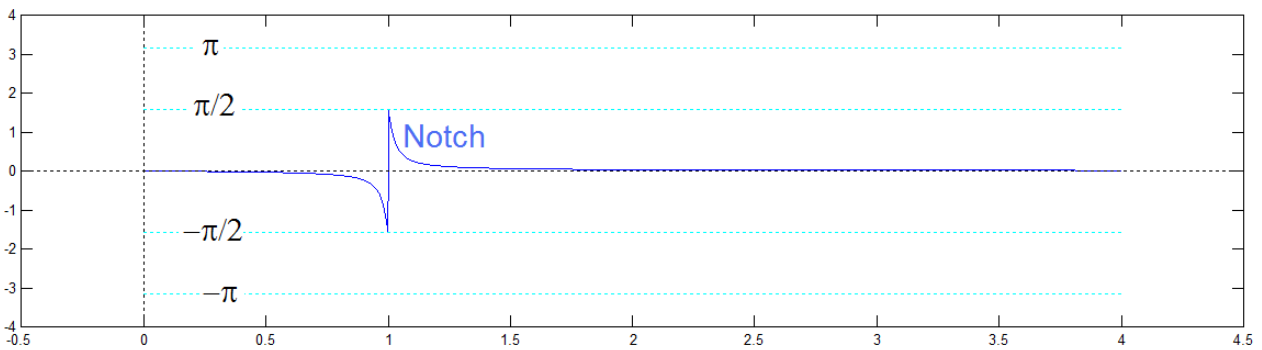
Phase Response   Fig. 3
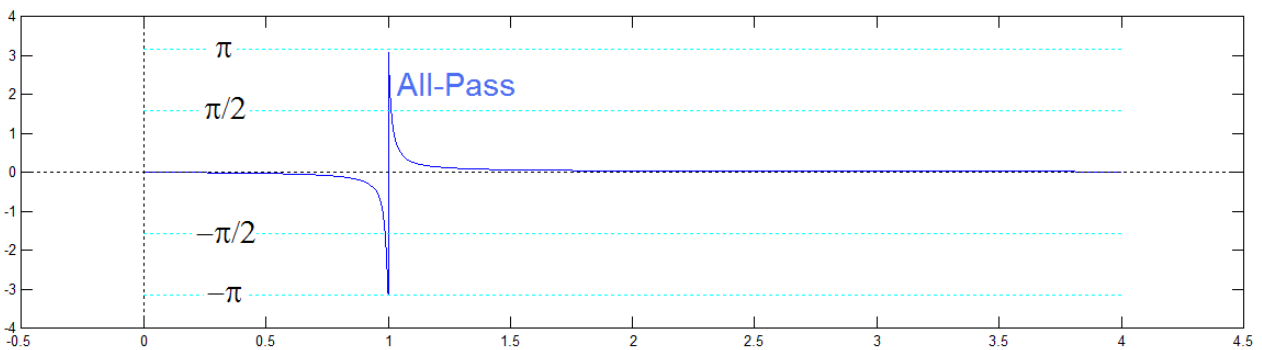
In Fig. 3 for the all-pass we see a jump of $+2\pi$ when the angle reaches $-\pi$ which is what we expect from the arctangent passing <u>through</u> $-\pi$. As we shall see in Fig, 4, this is what actually happens.   The jump of $+\pi$ at angle $-\pi/2$ in the case of the notch, is more mysterious.   It is not just a property of the calculated trig function.

One useful interpretation of phase is to consider each singularity separately, and imagine ourselves situated at that point in the s-plane watching the actual frequency increase from $\omega=0$ up the $j\omega$-axis.   The phase is the angle subtended by this segment. (Angles for zeros are opposite in sign from those of poles.) Thus we see that for something like a single real pole, the angle is an arctangent of the frequency, starting at zero, and approaching $-\pi/2$ (-90°) as the green dashed curve of Fig. 4.   The dashed black curve of Fig. 4 corresponds to a 2nd-order low-pass (two poles), and runs to $-\pi$.
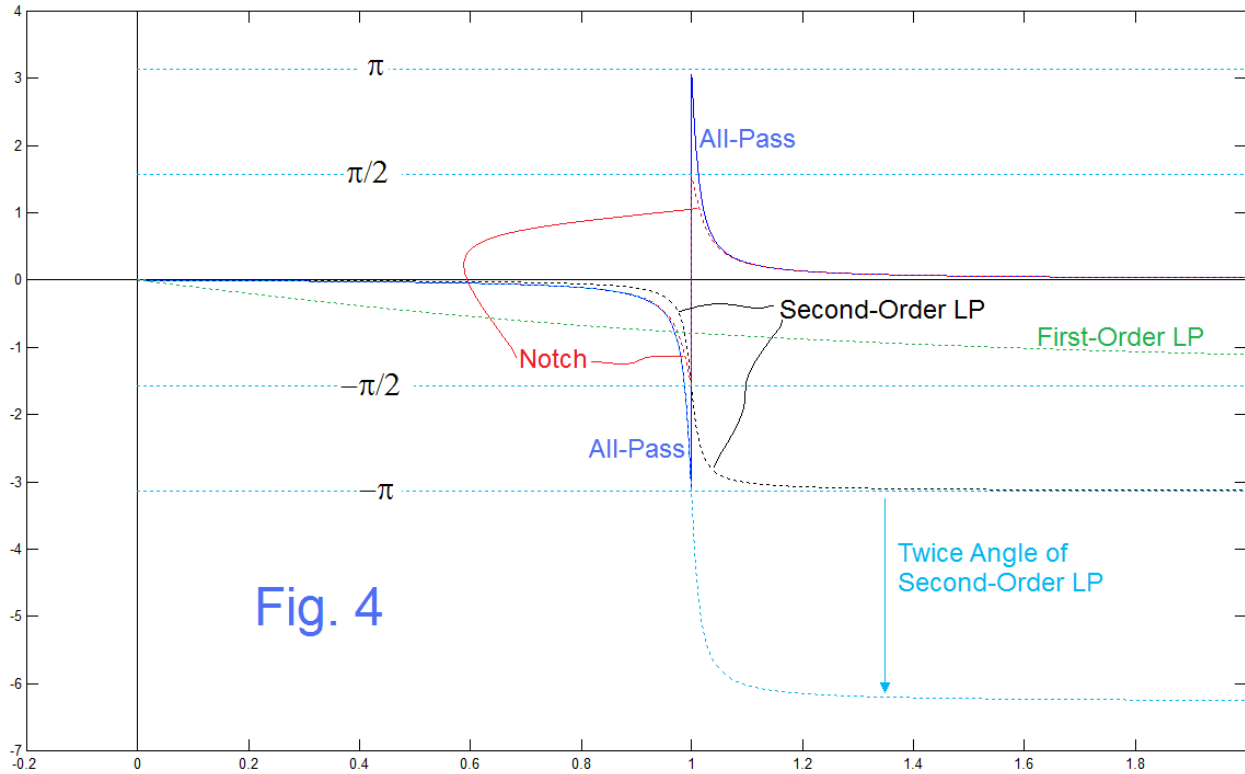
Fig. 4

The jump of π in the notch is explained by supposing ourselves to be sitting at the upper zero on the jω-axis as the frequency ω starts at zero, arrives at ω=1, and passes through. As the frequency is less than 1, the angle is always the same, directly below. As ω goes through 1, instantaneously the angle is directly above, a shift of π. It would be strange to suppose that we could have any instantaneous shift of π (or any discrete shift). (A mechanical system such as a mass and spring could not do this except with an infinite force.) But here, the shift occurs at an amplitude that is exactly 0 (the bottom of the notch) so it is possible.

There is much information in Fig. 4 and in Fig. 5 which is a follow-up. Fortunately, we can look at it one detail at a time. First we call attention to the calculation of the phase of the first-order LP (dashed green in Fig. 4) and the second-order LP (dashed-black in Fig. 4). The first-order curve is very gradual and is approaching –π/2, although it has a long way to go in Fig. 4. This is because we have a real negative-half-plane pole at -1 and when ω gets to 1, the angle from the pole looking at the frequency (ω=1) we have reached -π/2 (-45°) but then the rate of increase slows down. In the case of the second-order LP, we have two poles, so the total phase shift approaches π, as suggested as the right side of the dashed black curve. The shift is more sudden because of the two poles, and more importantly, they are very close to the jω-axis, so the subtended angle changes rapidly. Note that no finite zeros are in play here.

We see (again Fig. 4) that the phase of the all-pass (blue) advances faster than that of the notch (dashed red) (disallowing for the instantaneous π shift of the notch – Fig. 3) and of the dashed black 2nd-order LP (Fig. 4). That rapid phase shift is kind of the main claim-to-fame of the all-pass. In fact, the phase off the mirror zeros should exactly double that of the low-pass (the angles being in the opposite directions, but being from zeros, have the opposite signs). So what happens if we plot TWICE the LP phase shift (dashed blue of Fig. 4). This is the doubling of a calculated angle and reaches -2π, which is outside the range of the arctan. What we get is an exact fit to the all-pass phase (dashed light blue overplotting solid blue for the range from 0 to –π). The continuation of the dashed light blue from –π to -2π is the shifted version of the all-pass from π down to 0 above. This leaves the actual phase curve (dashed red in Fig. 4) of the notch unexplained.
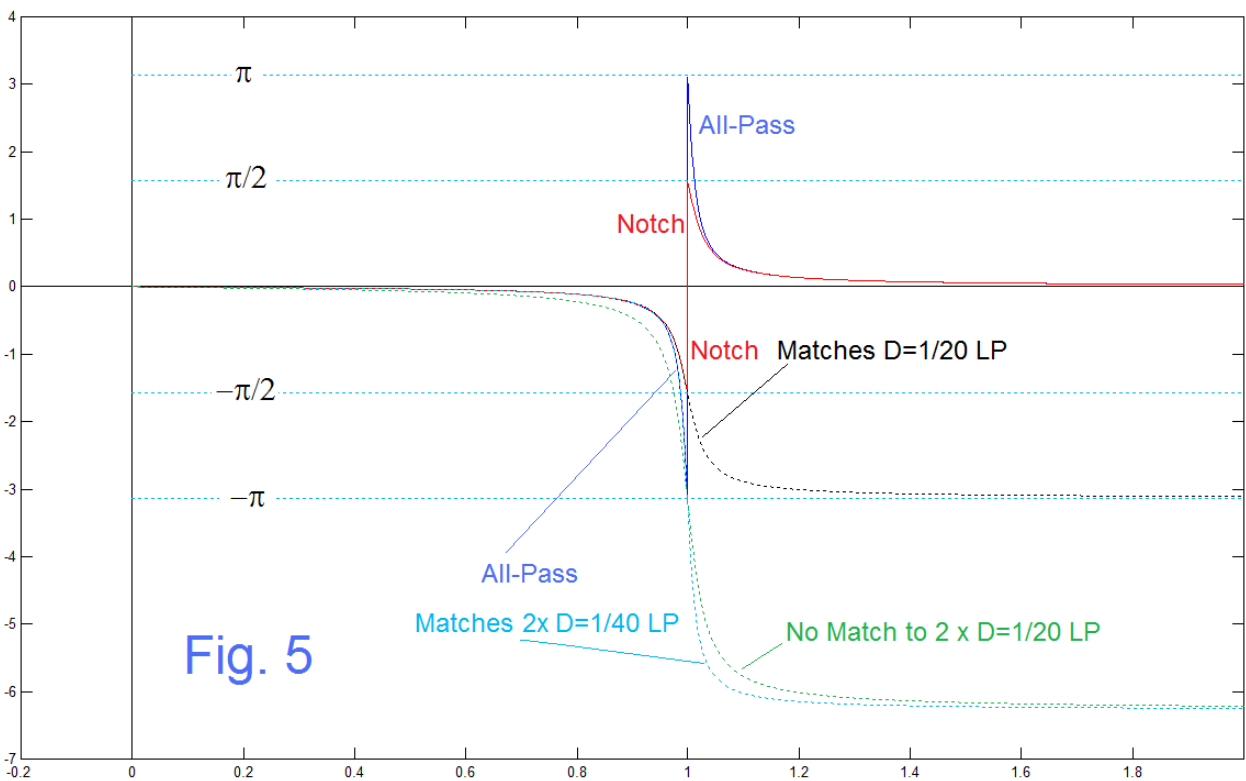

Fig. 5

Fig. 5 is a follow-up on Fig. 4 which shows additional details. The solid blue and dashed light-blue are the same as Fig. 4.   Here we recall that the poles of the notch and the all-pass were not the same. Specifically we used half the damping of the notch for the all-pass so that the separation of the poles/zeros was the same. That is, the notch had a damping of 1/20 while the all-pass damping was 1/40.   It is twice the angle of the D=1/40 LP that matches the all-pass.

We now look at the phase of the D=1/20 LP which is shown as the dashed black curve in Fig. 5. Here the notch phase is plotted as solid red, so we see that the D=1/20 LP exactly overplots the notch phase from 0 to –π/2. Similar to the case of the all-pass, we see the dashed black from –π/2 to –π corresponding to the red notch from π/2 down to 0.

We can now look at some time-domain responses (impulse and step responses) to supplement our understanding of the magnitude and phase responses.   These we do by simulation.   If we look back to Fig. 1 we recall that a notch and an all-pass had a very similar pole/zero plot, particularly as we kept the damping very low, and as we were agreeable to using half the damping of the notch for the all-pass.   This we suggest by comparing Fig. 1a and Fig. 1c for our postulated special relationships, the only significant difference being that the notch has its zeros exactly on the jω-axis while the all-pass has zeros ever so slightly in the right half-plane. Accordingly we would not be surprised if the two had similar time responses.
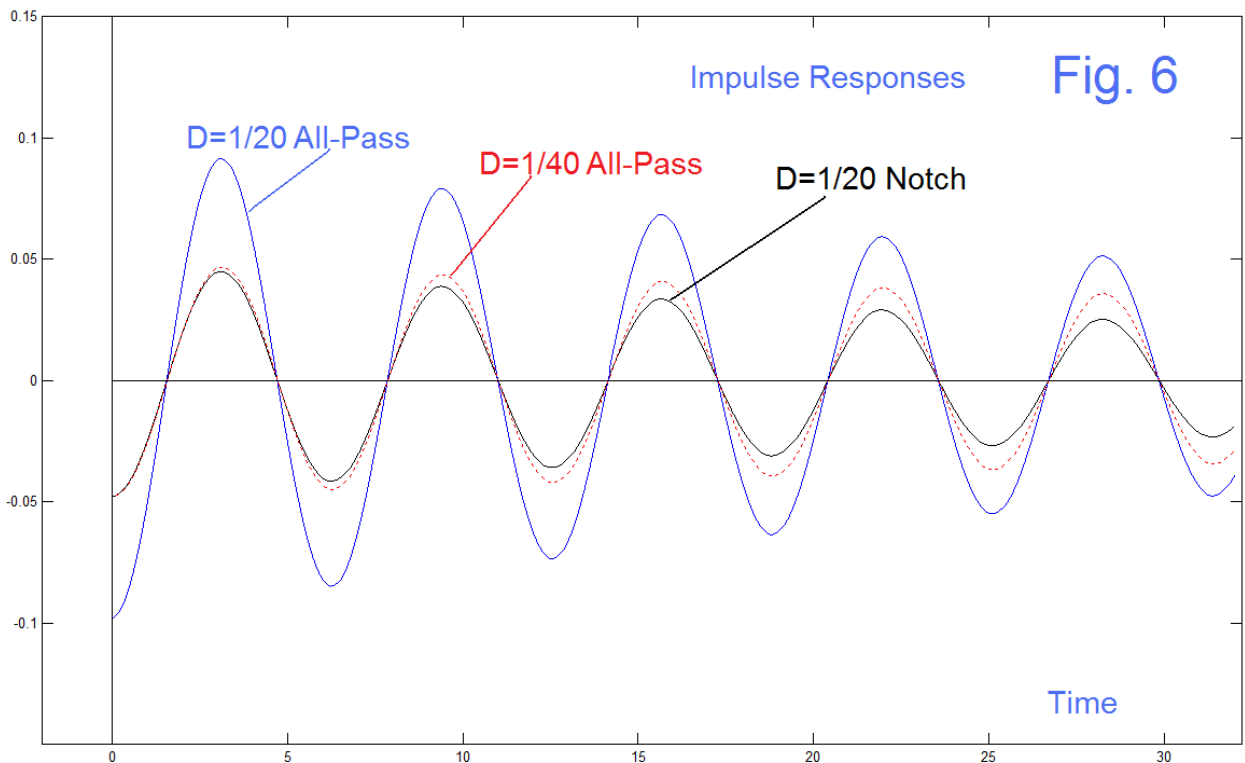

Fig. 6 shows three cases of interest.   There are two cases of an all-pass: for D=1/20 and for D=1/40. Then there is that case of the notch for D=1/20. Note first the similarly of the D=1/20 notch to the D=1/40 all-pass <u>on the left side</u>.   Further note that the D=1/20 all-pass and the D=1/20 notch decay at the same rate, while differing in amplitude by about 2:1.   As we might expect, the D=1/40 all-pass decays slower (rings longer) than the two D=1/20 cases. We might however, have expected the D=1/40 all-pass to have a larger response (initially) than the D=1/20 case. What we do see is apparently because the zeros better "shield" the poles in the D=1/40 case. See also the corresponding band-pass results in Fig. 8 below.

Fig. 7

Step Responses

D=1/20
All-Pass

D=1/40
All-Pass

D=1/20
Notch

Step
Up

Time

Band-Pass Impuse Responses

Fig. 8
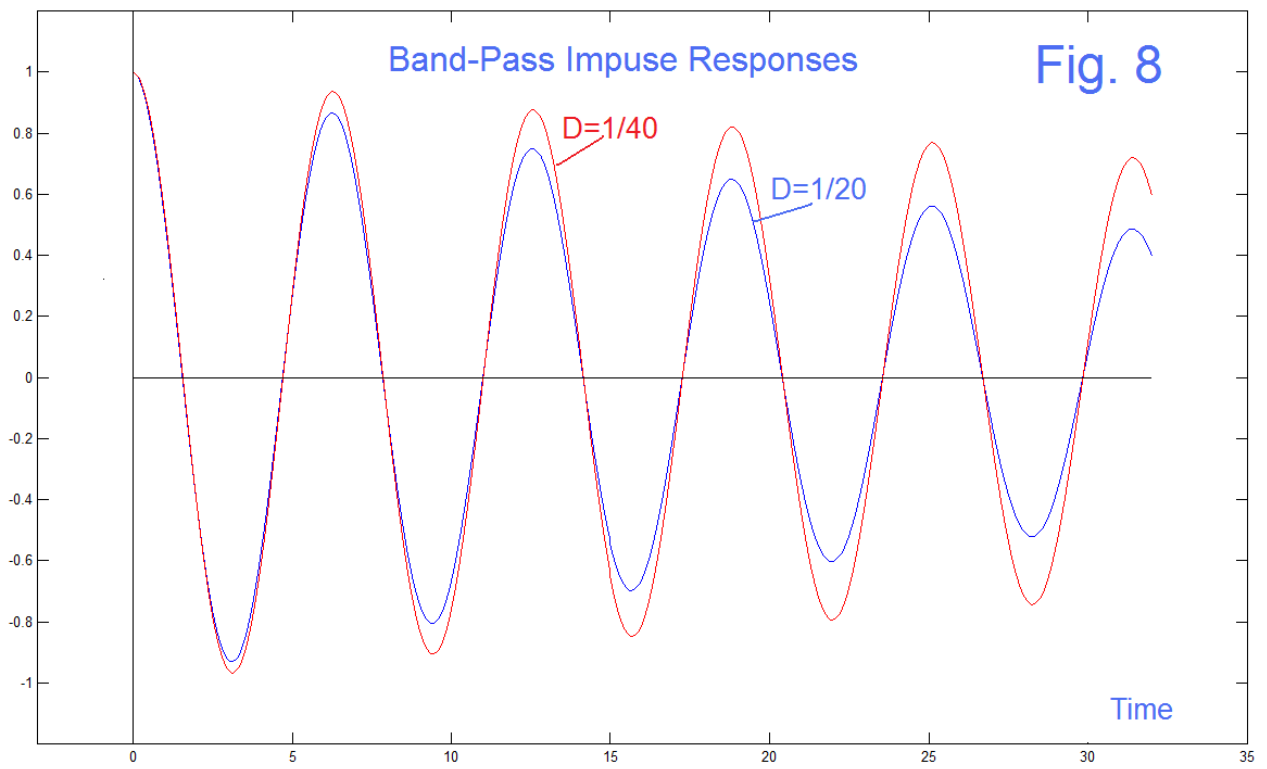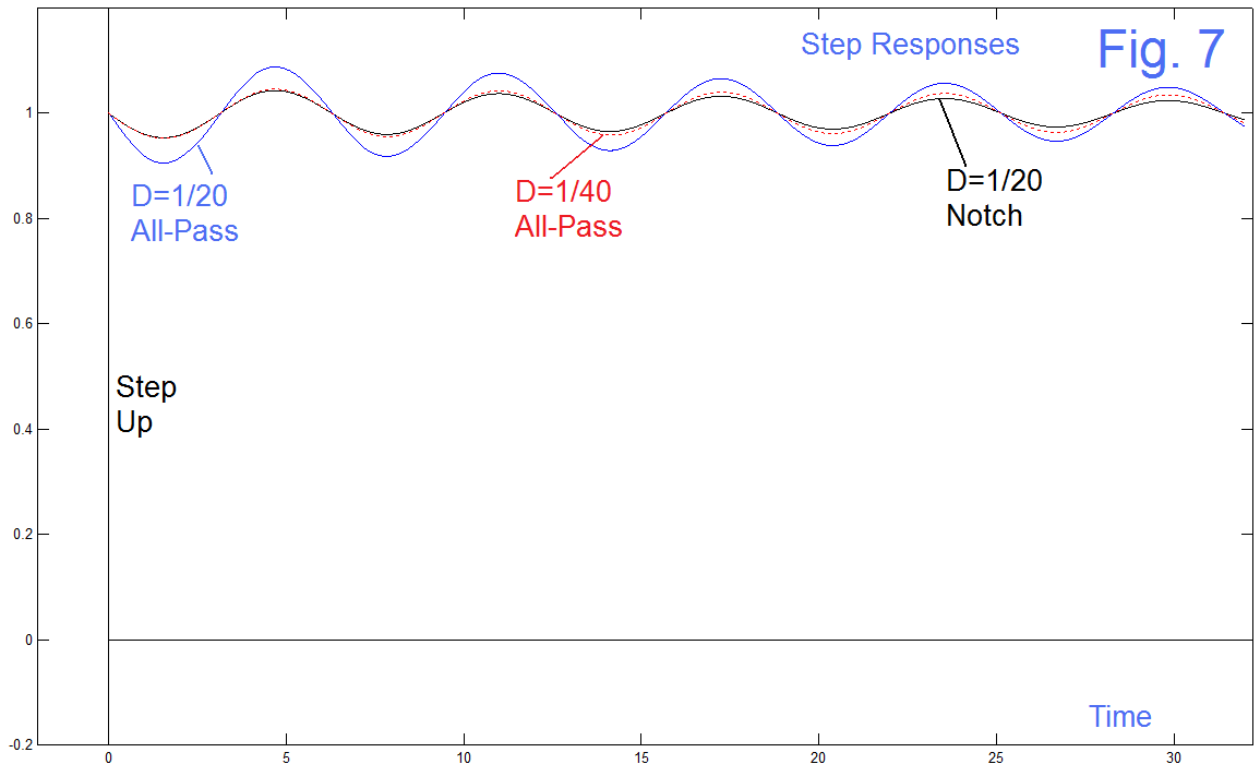
D=1/40

D=1/20

Time

ENWN-19 (7)

Fig. 7 corresponds to Fig. 6 except here we show the step response. The results are similar for similar reasons.   One difference is that all three have the step response beginning at 1 at time=0.

Fig. 8 shows impulse responses for band-pass responses – the expected exponentially damping sinusoidal waveforms. Here we see <u>both</u> band-pass responses starting at 1 at time=0 and decaying, with the less damped case (D=1/40, red) decaying more slowly.   No surprise, but here the blue curve is always smaller than the red one, except at t=0 where they are equal.   In contrast, in Fig. 6, the D=1/20 all-pass (blue) is greater than the D=1/40 all-pass (dashed red) for the portion plotted.   However, this was just a matter of the head start it got, and it is easy to see, by comparing relative amplitudes on the left and right sides, that the blue curve will dip below the red somewhere a little further to the right.

PROGRAM

The following code is included for full documentation. The figure numbers in the code correspond exactly to the figure numbers of the text.

```
% WN019.m
a=0:0.005*pi:2*pi;
x=sin(a);
y=cos(a);
figure(1)
subplot(131)
plot(x,y,'b')
hold on
plot([-1.2 1.2],[0 0],'k:')
plot([0 0],[-1.2 1.2],'k:')
pa=20*2*pi/360
Dpa1=sin(pa)
plot(-sin(pa),cos(pa),'xr')
plot(-sin(pa),-cos(pa),'xr')
plot(0,1,'ob')
plot(0,-1,'ob')
axis([-1.2 1.2 -1.2 1.2])
axis('square')
hold off

subplot(132)
plot(x,y,'b')
hold on
plot([-1.2 1.2],[0 0],'k:')
plot([0 0],[-1.2 1.2],'k:')
pa=20*2*pi/360
plot(-sin(pa),cos(pa),'xr')
plot(-sin(pa),-cos(pa),'xr')
plot(sin(pa),cos(pa),'ob')
plot(sin(pa),-cos(pa),'ob')
axis([-1.2 1.2 -1.2 1.2])
axis('square')
hold off
```

```
subplot(133)
pa=10*2*pi/360
Dpa2=sin(pa)
plot(x,y,'b')
hold on
plot([-1.2 1.2],[0 0],'k:')
plot([0 0],[-1.2 1.2],'k:')
plot(-sin(pa),cos(pa),'xr')
plot(-sin(pa),-cos(pa),'xr')
plot(sin(pa),cos(pa),'ob')
plot(sin(pa),-cos(pa),'ob')
axis([-1.2 1.2 -1.2 1.2])
axis('square')
hold off


D=1/20
NN=[1 0 1];
ND=[1 D 1];
NN1=[1 0 1];
ND1=[1 1/2 1];
AN=[1 -D/2 1];
AD=[1 D/2 1];
w=[0:.0005:4];
TN=freqs(NN,ND,w);
TN1=freqs(NN1,ND1,w);
TA=freqs(AN,AD,w);
%
TL1=freqs([1],[1 1],w);
TL2=freqs([0 0 1],AD,w);
%

figure(2)
subplot(211)
plot(w,abs(TN),'b')
hold on
plot([-1 5],[0 0],'k:')
plot([0 0],[-1 2],'k:')
plot(w,abs(TN1),':c')
axis([-.5 4.5 -0.2 1.2])
hold off
subplot(212)
plot(w,abs(TA),'b')
hold on
plot([-1 5],[0 0],'k:')
plot([0 0],[-1 2],'k:')
axis([-.5 4.5 -0.2 1.2])
title ('Magnitude')
hold off

figure(3)
subplot(211)
plot(w,angle(TN),'b')
hold on
%
plot([-1 10],[0 0],'k:')
plot([0 0],[-10 10],'k:')
plot([0 4],[pi pi],'c:')
plot([0 4],[pi/2 pi/2],'c:')
plot([0 4],[-pi -pi],'c:')
plot([0 4],[-pi/2 -pi/2],'c:')
```

```
%
axis([-.5 4.5 -4 4])
hold off
subplot(212)
plot(w,angle(TA),'b')
title('Phase')
hold on
%
plot([-1 10],[0 0],'k:')
plot([0 0],[-10 10],'k:')
plot([0 4],[pi pi],'c:')
plot([0 4],[pi/2 pi/2],'c:')
plot([0 4],[-pi -pi],'c:')
plot([0 4],[-pi/2 -pi/2],'c:')
axis([-.5 4.5 -4 4])
hold off


figure(4)
plot([-0.5 4.5],[0 0],'k')
hold on
plot([0 0],[-8 4],'k')
plot(w,angle(TA),'b')
plot(w,angle(TN),'r:')
plot(w,angle(TL1),'g:')
plot(w,angle(TL2),'m:')
plot(w,2*angle(TL2),'c:')
%
plot([0 4],[pi pi],'c:')
plot([0 4],[pi/2 pi/2],'c:')
plot([0 4],[-pi -pi],'c:')
plot([0 4],[-pi/2 -pi/2],'c:')
%
hold off
axis([-.2 2 -7 4])
figure(4)


figure(5)
TL22=freqs([0 0 1],[1 D 1],w);
plot([-0.5 4.5],[0 0],'k')
hold on
plot([0 0],[-8 4],'k')
plot(w,angle(TA),'b')
plot(w,angle(TN),'r')
%plot(w,angle(TL1),'g:')
plot(w,angle(TL22),'k:')
plot(w,2*angle(TL2),'c:')
plot(w,2*angle(TL22),'g:')
%
plot([0 4],[pi pi],'c:')
plot([0 4],[pi/2 pi/2],'c:')
plot([0 4],[-pi -pi],'c:')
plot([0 4],[-pi/2 -pi/2],'c:')
%
hold off
axis([-.2 2 -7 4])
figure(5)
```

```
% IMPULSE RESPONSES
VH=zeros(1,16000);
VB=zeros(1,16000);
VL=zeros(1,16000);
dt=0.002
D=1/20
imp=500
for n=2:16000
    VL(n)=VL(n-1)+VB(n-1)*dt;
    VB(n)=VB(n-1)+VH(n-1)*dt;
    VH(n)=-VL(n-1)-D*VB(n-1)+ imp;
    imp=0;
end
VN=VH+VL;
VA=VH-D*VB+VL;
VB05=VB;
VH=zeros(1,16000);
VB=zeros(1,16000);
VL=zeros(1,16000);
dt=0.002
D=D/2
imp=500
for n=2:16000
    VL(n)=VL(n-1)+VB(n-1)*dt;
    VB(n)=VB(n-1)+VH(n-1)*dt;
    VH(n)=-VL(n-1)-D*VB(n-1)+ imp;
    imp=0;
end
VAhalf=VH-D*VB+VL;
VB025=VB;
figure(6)
plot([0:15998]/500,VN(2:16000),'k')
hold on
plot([0:15998]/500,VA(2:16000),'b')
plot([0:15998]/500,VAhalf(2:16000),'r:')
plot([0,16000]/500,[0 0],'k')
plot([0 0],[-2 2],'k')
axis([-1000/500 16100/500 -0.15 0.15])
hold off
title('Black: Notch, Blue: AP, Red-dot: D->D/2 AP')


VH=zeros(1,16000);
VB=zeros(1,16000);
VL=zeros(1,16000);
dt=0.002
D=1/20
s=1
for n=2:16000
    VL(n)=VL(n-1)+VB(n-1)*dt;
    VB(n)=VB(n-1)+VH(n-1)*dt;
    VH(n)=-VL(n-1)-D*VB(n-1)+ s;

end
VN=VH+VL;
VA=VH-D*VB+VL;
figure(7)
plot([0:15998]/500,VN(2:16000),'k')
hold on
plot([0:15998]/500,VA(2:16000),'b')
plot([0,16000]/500,[0 0],'k')
plot([0 0],[-2 2],'k')
```

```
VH=zeros(1,16000);
VB=zeros(1,16000);
VL=zeros(1,16000);
dt=0.002
D=D/2
s=1
for n=2:16000
    VL(n)=VL(n-1)+VB(n-1)*dt;
    VB(n)=VB(n-1)+VH(n-1)*dt;
    VH(n)=-VL(n-1)-D*VB(n-1)+ s;
end
VN=VH+VL;
VAhalf=VH-D*VB+VL;
plot([0:15998]/500,VAhalf(2:16000),'r:')



axis([-1000/500 16100/500 -0.2 1.2])
title('Black: Notch, Blue: AP, Red-dot: D->D/2 AP')

hold off


figure(8)
plot([0:15998]/500,VB05(2:16000),'b')
hold on
plot([0:15998]/500,VB025(2:16000),'r')
plot([0,16000]/500,[0 0],'k')
plot([0 0],[-2 2],'k')
hold off
axis([-3 35 -1.2 1.2])
title('BP Imp Resp Blue D=0.05  Red D=0.025')
figure(8)
```