ELECTRONOTES 71
Newsletter of the Musical Engineering Group
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GROUP ANNOUNCEMENTS:

CONTENTS OF EN#71

This newsletter is concerned mainly with filters. We start off with a study of the bandpass filter and analyze it from a mathematical and musical point of view. In particular, we develope some applications where the filter produces control signals. The filter has been used for years to produce complex patterns for "scope art" and it should be realized that these same patterns are useful as controls for musical parameters. Secondly, we are presenting the first two of the ENS-76 voltage-controlled filters. These came out a little earlier than we planed and since we found some very useful improvements, we decided to get them out right away. In particular, we have developed filters achieving high-Q at high frequency using a compensation technique that was available in the literature.

NEW MEMBERS AND CHANGES:

Edward A. Dudley	447 Blythwood Rd., Toronto, Ont. Canada M4N 1A8
Pierre FaFard	701 Duchesway, St-Justin, P. Quebec, Canada J0K 2V0
Bill Van Hassel	35 New Street, New Hope, PA 18938
David Kramlich	56 Bay State Rd #3, Boston, MA 02215
Robert Lange	105 Bingham, Rumson, NJ 07760
G. Oczkowski	6-703 Corydon Ave., Winnipeg, Manatoba, Canada R3M 0W4
Leonard Sasso	2983 Alexander Rd., Laguna Beach, CA 92651

NEWS ITEMS:

Copies of the proceedings of the Music Computation Conference II (Nov. 7-9, 1975) are available from: Proc. of the Music Computation Conference II, Continuing Education in Music, Univ. of Ill. at Urbana-Champaign, 608 S. Mathews, Urbana, IL 61801. Orders for these should be sent before Dec. 6, 1976. Available papers are:

Part 1. Software Synthesis Techniques, Papers by Ferretti; Justice; Petersen; Saunders; Kaehler; Zuckerman & Steiglitz; and Cherubini. (75 pages - $3.00)

Part 2. Composition with Computers, Papers by Beckwith; Chadabe; Gressel; Howe; Rothenberg; and Tipei. (83 pages - $3.00)

Part 3. Hardware for Computer-Controlled Sound Synthesis, Papers by Beauchamp, Pohlmann and Chapman; Gross; Kriz; Roy (59 pages - $3.00)

Part 4. Information Processing Systems, Papers by Austin and Bryant; Charnasse; Dal Molin; Rosenboom; Peters (96 pages - $4.00)

Make checks payable to the Univ. of Ill.

The following is a list of Audio Engineering Society Preprints from the Spring 1976 and Fall 1976 conventions that are of interest to engineers working with electronic music. These preprints can be obtained at a cost of $2.00 each ($1.50 each for AES members) and can be ordered from: Audio Engineering Society, 60 E. 42nd St., Room 449, New York, NY 10017.

Spring 1976 (Los Angeles)

1094 (D-2)	Nyle A. Steiner, "An Electronic Valve Instrument (Trumpet) for Controlling an Electronic Music Synthesizer"
1101 (C-1)	James G. Simes, "An Almost Locked Oscillator for Electronic Music Synthesis"
1102 (C-5)	Philip West, "Use of Tape Recorders in Real-Time Electronic Music"
1104 (D-3)	Tracy Lind Petersen, "Analysis-Synthesis As a Tool for Creating New Families of Sound"
1110 (D-6)	Lee Ferguson, "A Polyphonic Music Synthesizer Utalizing Master Programmed Electronic Synthesis Modules for Each Key"
1121 (D-1)	Thomas Wood, "A High-Speed Digital-to-Analog Conversion System for Digital Music Synthesis"
1123 (C-3)	Brent Gabrielsen, "A Patchable Electronic Music Percussion Synthesizer"
1129 (C-2)	Bob Moore "A 'Hybrid-Synthesizer' "
1133 (D-5)	Patrick Gleeson, "Things Any Boy Can Do With a 16-Track, a DBX, and an Eμ Polyphonic Synthesizer"

Fall 1976 (New York City)

1142 (L-2)	J. Stanley Kriz, "An Audio Analog-Digital-Analog Conversion System"
1146 (E-1)	James A. Moorer, "The Use of the Phase Vocoder in Computer Music Applications"
1161 (E-2)	James Michmerhuizen & Michael Gilbert, "A Digital Rhythm and Timing Generator for Electronic Music Applications"
1162 (B-4)	C. J. Evans & J. Dawson, "A Feedforward Controlled Delay-Line Limiter"
1165 (D-5)	Walter Jung, "Application Considerations for IC Data Converters Useful in Audio Signal Processing"
1166 (E-5)	Gene P. Weckler, "Making Music with Charge-Transfer Devices"
1166 (F-4)	Gene P. Weckler, "Signal Processing with Charge-Transfer Devices"
1172 (E-3)	Thomas Oberheim, "A Programmer for Voltage-Controlled Synthesizers"
1185 (H-6)	Eero Leinonen, Matti Otala, & John Curl, "Method for Measuring Transient Intermodulation Distortion (TIM)"
1190 (L-1)	Francis F. Lee & David Lipschutz, "Floating Point Encoding for Transcription of High Fidelity Audio Signals
1191 (L-6)	Peter W. Mitchell & Richard E. DeFreitas, "A New Digital Time-Delay and Reverberation System, Part II: Psychoacoustics vs. Practical Electronics"

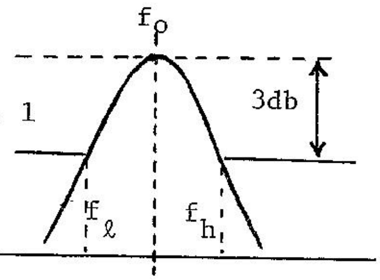
A position with title Audio Engineer/Technician and Training Associate is open at the University of Miami. Interested persons should send credentials to: Ted J. Crager, School of Music, Univ. of Miami, Coral Gables, FL 33124. We have sent them copys of the forms we have on individuals who contacted Electronotes seeking employment in electronic music.

THE BANDPASS FILTER RESPONSE AND ELECTRONIC MUSIC APPLICATIONS:

-by Bernie Hutchins, ELECTRONOTES

A typical bandpass filter response is shown in Fig. 1. The principal parameters which characterize the bandpass response are the center frequency (the maximum response) and the "Q" of the filter which is a measure of the sharpness of the response. The Q of the filter is generally taken to be the center frequency divided by the 3db bandwidth $f_h - f_\ell$.

Fig. 1



The voltage-controlled bandpass filter has an obvious function in electronic music in that it can filter a complex waveform and thus control the harmonic content that appears at the output. Other uses are of course possible. With a high Q and white noise at the input, the filter passes a narrow band of frequencies which serve to define a feeling for musical pitch. The sharper the response, the clearer the feeling for musical pitch at the output. The sharp response bandpass filter can also be made to "ring" by exciting it with an impulse and letting the energy thus inserted decay as it oscillates. These applications are suggested by figures 2a, 2b, and 2c.

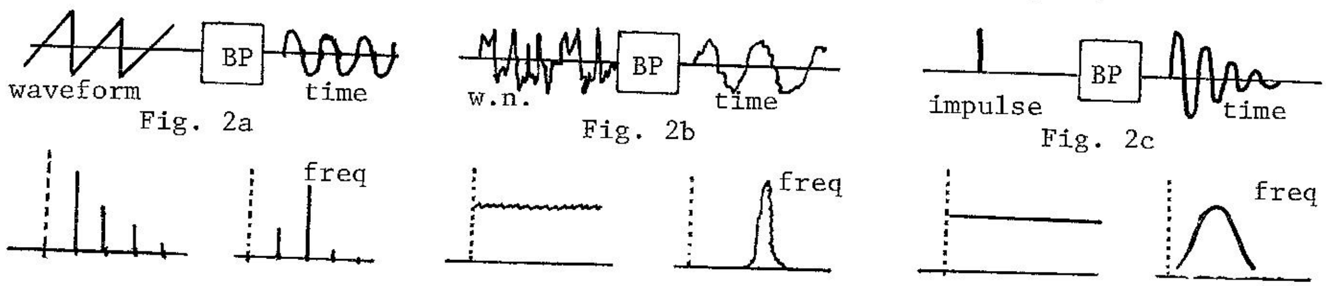


Fig. 2a Fig. 2b Fig. 2c

FREQUENCY RESPONSE

Although inductors are seldom used in audio filters these days, it is often useful to use them "on paper" to demonstrate certain principles which may not be so clear with active RC filters. A well known LC bandpass filter is shown in Fig. 3, and is probably familiar to readers who have studied radio frequency circuits where inductors are most certainly practical. We can easily derive the transfer function for the series RLC circuit in Fig. 3 by just considering it to be a voltage divider, and using the complex (Laplace) frequency variable "s". By inspection we have:

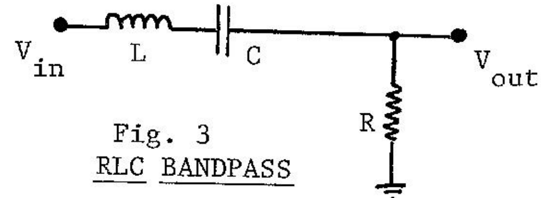


Fig. 3
RLC BANDPASS

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{R}{R + sL + (1/sC)} \qquad (1)$$

Clearly we are interested in the "resonant" case where $sL = -(1/sC)$ in which case T(s) becomes R/R = 1, its maximum value. This happens for a particular value of s which we shall denote s_o. Thus, $s_o^2 = -1/LC$, and we can let s_o take on the value $j\omega_o$ and we then arrive at $\omega_o = 1/\sqrt{LC}$. Next we note that nothing prevents us from multiplying the top and bottom of equation (1) by s/L and this gives:

$$T(s) = \frac{s(R/L)}{s^2 + s(R/L) + 1/LC} \qquad (2)$$

This second form of equation (1) shows more clearly the fact that the denominator of T(s) is really second order. Furthermore, we can identify the constant term in the denominator (1/LC) with the square of the center frequency (ω_o^2) and this will prove a useful reference point. We will then want to consider the coefficient of the s term in the denominator, and will find that it is this term which determines the sharpness (Q) of the bandpass response.

The approach we shall use to show the relationship of the coefficient of s and the Q of the filter will be to first assume the correct answer and then show that it all works out right. Thus, the general form of the bandpass response will be assumed to be:

$$T(s) = \frac{As}{s^2 + (\omega_o/Q)s + \omega_o^2} \qquad A = \text{constant} \qquad (3)$$

Note that for the moment we have not shown that the Q in equation (3) is the same as the Q described in the first paragraph of this report. We shall just note that since we propose that Q be dimensionless (since it is the ratio of frequencies), the constant ω_o is needed to keep the denominator second order in the dimension of frequency. Thus what we have done in equation (3) is reasonable and will be justified by the final results.

Since we will be concerned here with the values of the frequency response that are down 3db from the peak, it will be necessary to determine the actual frequency response, not just the transfer function. To do this we evaluate T(s) for $s = j\omega$ and take the magnitude $|T(j\omega)| = [T(j\omega)\cdot T(-j\omega)]^{1/2}$. This is applied to equation (3) and we shall set A=1 for convenience.

$$|T(j\omega)| = \left[\frac{j\omega}{(j\omega)^2 + \frac{j\omega_o\omega}{Q} + \omega_o^2} \cdot \frac{-j\omega}{(-j\omega)^2 - \frac{j\omega_o\omega}{Q} + \omega_o^2}\right]^{1/2} \qquad (4a)$$

$$= \left[\frac{\omega^2}{(\omega_o^2 - \omega^2 + \frac{j\omega_o\omega}{Q})\cdot(\omega_o^2 - \omega^2 - \frac{j\omega_o\omega}{Q})}\right]^{1/2} \qquad (4b)$$

$$= \left[\frac{\omega^2}{(\omega_o^2 - \omega^2)^2 + \frac{\omega_o^2\omega^2}{Q^2}}\right]^{1/2} \qquad (4c)$$

$$= \left[\frac{Q^2/\omega_o^2}{1 + Q^2\,\frac{(\omega_o^2 - \omega^2)^2}{\omega_o^2\omega^2}}\right]^{1/2} \qquad (4d)$$

$$= \left[\frac{Q^2/\omega_o^2}{1 + Q^2\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right]^2}\right]^{1/2} \qquad (4e)$$

The final result (4e) may seem a little awkward at first, but it is in a form that will be very useful to us. For example, we can see directly from this that when $\omega=\omega_o$, the frequency response is a maximum since only a one is in the denominator. Secondly, when we go to look at the 3db points (half power points), we will just want to see how a denominator of 2 will occur, and this is a matter of setting the second term in the denominator equal to one.

By consulting Fig. 1, we see that the frequency f_o is a unique point in that it is the only value of frequency that has no other frequency value giving the same response. All others have two frequencies for a given value of response, and one of these is below f_o and the other is above. We will want to use equation (4e) to find a relationship between such points. It will be convenient to use radial frequencies corresponding to the notation of equation (4e). We could use <u>any</u> value of response in what follows, but it will be most direct if we choose the frequencies where the response is down 3db from the peak value. The lower frequency is ω_ℓ, the center is ω_o, and the upper one is ω_h. It is then clear from equation (4e) that the 3db points are related by:

$$\frac{\omega_\ell}{\omega_o} - \frac{\omega_o}{\omega_\ell} = \frac{\omega_h}{\omega_o} - \frac{\omega_o}{\omega_h} \qquad (5)$$

This is easily solved to give:

$$\omega_\ell \omega_h = \omega_o^2 \qquad (6)$$

Equation (6) is quite interesting. It says that the center frequency is the geometric mean of the upper and lower 3db frequencies (and in fact, of any two frequencies having the same response value). This is a very convenient way of finding the actual center frequency of some low Q bandpass filters which have a broad top as it may be difficult to locate the maximum response of such filters. Also note that the center frequency is not the average of these values, although this is an excellent approximation for high Q, and is often used for high Q cases. However, for low Q, be sure to determine the center frequency as the square root of the product.

Next we want to determine the 3db bandwidth $\omega_h - \omega_\ell$, which will be denoted by B. Using equation (6) we get:

$$B = \left[\omega_h - \omega_\ell\right] = \left[\omega_h - \frac{\omega_o^2}{\omega_h}\right] = \left[\frac{\omega_o^2}{\omega_\ell} - \omega_\ell\right] \qquad (7)$$

We then select one of the 3db frequencies (ω_h) and arrive at the expression:

$$B = \omega_o \left[\frac{\omega_h}{\omega_o} - \frac{\omega_o}{\omega_h}\right] \qquad (8)$$

From equation (8) we can arrive at the expression:

$$\left[\frac{\omega_h}{\omega_o} - \frac{\omega_o}{\omega_h}\right]^2 = \frac{B^2}{\omega_o^2} \qquad (9)$$

We can now complete the task by observing that for 3db down, the frequency response $|T(j\omega)|$ should be down by $\sqrt{2}$ and thus the denominator in the brackets in equation (4e) should be 2, and thus:

$$Q^2 \left[\frac{\omega_3}{\omega_o} - \frac{\omega_o}{\omega_3}\right]^2 = 1 \qquad (10)$$

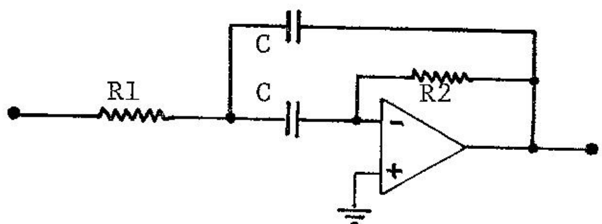
where ω_3 is one of the 3db frequencies, for which we can use ω_h. Substituting into equation (10) from equation (9) gives $Q^2B^2/\omega_o^2 = 1$ or:

$$Q = \omega_o/B = \omega_o/(\omega_h - \omega_\ell) = f_o/(f_h - f_\ell) \qquad (11)$$

Thus equation (11) verifies equation (3) and the information on Fig. 1.

The importance of the above calculations is that we can now pull the most important of the bandpass parameters (f_o and Q) directly from the transfer function of the network by cross-matching against equation (3). For example, the transfer function of the RLC series circuit can be used and it can be shown that $Q = (1/R)\sqrt{L/C}$ As a second example, consider the well known bandpass RC active filter of Fig. 4.

Fig. 4
ACTIVE BANDPASS



It is not our purpose here to show how transfer functions are derived, so we will just give the transfer function of the network of Fig. 4 as:

$$T(s) = \frac{-s/R_1C}{s^2 + 2s/R_2C + 1/R_1R_2C^2} \qquad (12)$$

From this we can easily get the center frequency and Q by comparing with equation (3).

$$\omega_o = 1/C\sqrt{R_1R_2} \qquad Q = (\sqrt{R_2/R_1})/2 \qquad (13,14)$$

The above filter is fine as a fixed filter where great precision is not needed. There are however other active filter configurations that give better bandpass response at higher values of Q. These can be found in books on active filters. We want to look here at two more examples, but instead of following up on the circuit of Fig. 4, we will be looking at the state-variable filter, and the "biquad" circuit as these are the types of networks we will be using in voltage-controlled filters. There are several forms of the state-variable filter which are in common use. The calculations for some of these are a little complicated because it is generally easiest to use the standard inverting integrator. In voltage-controlled filters using the CA3080 however, it is quite easy to make a non-inverting integrator, so we can then get the required negative feedback by summing into the simple inverting summer. The basic structure thus reduces to that shown in Fig. 5.

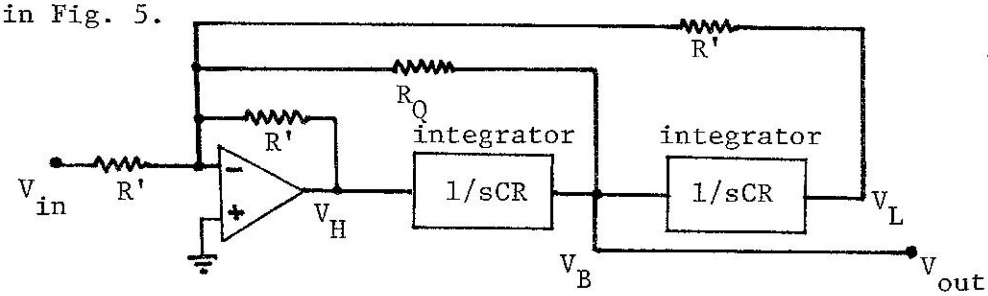


Fig. 5 STATE-VARIABLE FILTER

$$T(s) = \frac{-s/RC}{s^2 + \frac{R'/R_Q}{RC}s + \frac{1}{R^2C^2}}$$

From this, it is clear by comparing with equation (3) that $\omega_o = 1/RC$ and $Q = R_Q/R'$ which is a simple and easy to remember result. Note in particular that both ω_o and Q can be set simply and independently. This is a nice thing to have in a voltage-controlled filter for electronic music.

The structure of the "Biquad" is shown in Fig. 6. This filter has a bandpass and a low-pass output. Note that it is different from the state variable.

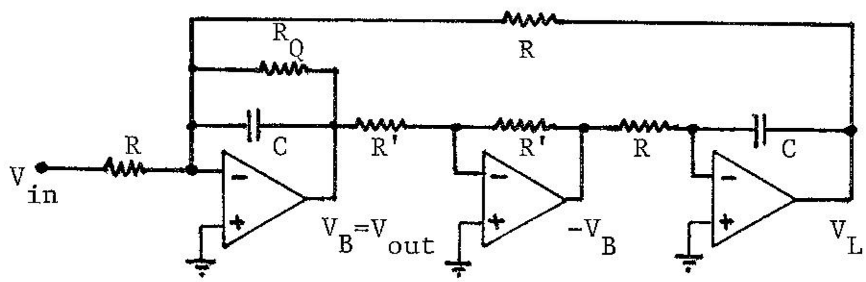


Fig. 6 BIQUAD ACTIVE FILTER

$$T(s) = \frac{-s/RC}{s^2 + (1/R_QC)s + 1/R^2C^2}$$

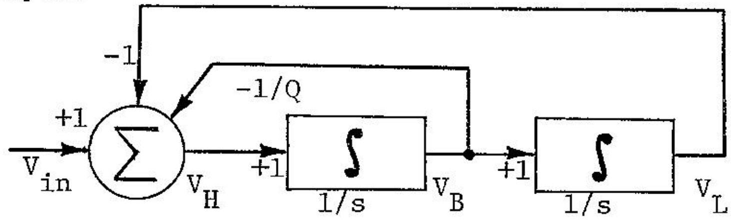
From the transfer function, we note by comparing with equation (3) that $\omega_o = 1/RC$ and $Q = R_Q/R$. Here we see that the Q is set by the ratio of one independent resistor (R_Q) and one resistor (R) which is a frequency determining resistor. In fact, it is clear that the Q and ω_o rise together as R decreases. The reader can easily see that this means that the filter has a constant bandwidth. That is, if the bandwidth is 50 Hz for a center frequency of 100 Hz, the Q is 2. Now, by decreasing R, we would eventually arrive at a point where the center frequency is 1000 Hz. If Q were constant, the bandwidth would be 500 Hz. But the Q has gone up by a factor of 10 and is now 20, and the bandwidth is still 50 Hz. Generally, the musical value of such (constant bandwidth) filters is not is great as constant Q filters. But, there is one thing to note. We will see soon that a constant bandwidth filter has a constant ring time, unlike the constant Q filter which rings longer at lower frequencies. We shall want to take a careful look at this later, and we should also note that with voltage-controlled Q, we may be able to get constant bandwidth with the state variable filter as well.

While on the subject of voltage-controlled filters, it should be noted that there are at least two ways to insert a signal into a state-variable VCF. The first way is basically as shown in Fig. 5. The second way provides a "Limit" input through a voltage-controlled Q section and has been used by Terry Mikulic in his filter in EN#34, page 17. Below in Fig. 7a and Fig. 7b we show the two methods where we have taken RC=1 to keep things simple.

Fig. 7a

STATE VARIABLE

STANDARD INPUT

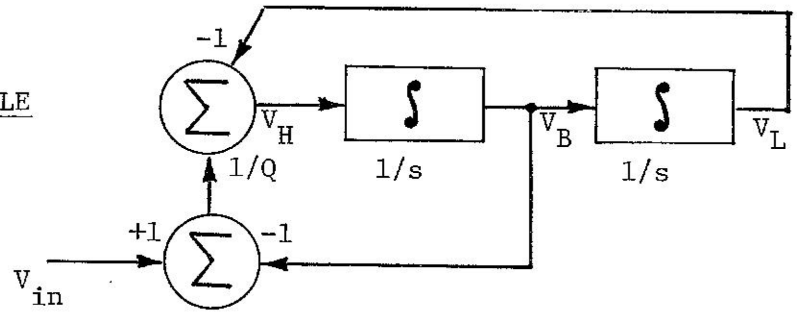


$$T(s) = V_B/V_{in} = \frac{s}{s^2 + (1/Q)s + 1}$$

Fig. 7b

STATE VARIABLE

LIMIT INPUT



$$T(s) = V_B/V_{in} = \frac{s/Q}{s^2 + (1/Q)s + 1}$$

The reader should note that the only difference between the two is while V_{in} is fed in through a +1 path in Fig. 7a, it is fed in through a +1/Q path in Fig. 7b. Thus, all that we have really accomplished is an attenuation of the input by a factor of 1/Q, and this carries through to the transfer functions T(s) as can be seen. We could convert 7a to 7b by just changing the +1 path to +1/Q and not use the second summer, but the second summer shown in Fig. 7b is what we find in practical voltage-controlled Q setups, and it is certain that the factor 1/Q is the same for both the input and the V_B feedback in the case of Fig. 7b. The advantage of the limit input should be clear from a study of the transfer functions. In the case of 7b, the peak of the bandpass (and of the other outputs when used) is a constant as Q changes. This keeps the filter from saturating when high Q is achieved, as the input voltage level is cut back when the Q goes up. For fixed Q, there is probably no reason to choose one structure over the other if one just judges by the output sounds produced. When voltage-controlled Q is used, it is a different story. When the regular input is used, the output amplitude level will increase with Q, and the passband of course sharpens with increased Q. With the limit input, only the sharpening of the passband occurs. Probably a filter with voltage-controlled Q should have both types of input available.

TIME RESPONSE

At this point in our analysis, we want to take a look at the bandpass from the point of view of its time response. Here is where we want to see exactly what happens when we "ring" the bandpass filter. The first thing we want to do is to take the standard bandpass transfer function and factor its denominator (using the quadratic formula):

$$T(s) = \frac{As}{s^2 + (\omega_o/Q)s + \omega_o^2} = \frac{As}{(s + \alpha + j\omega_d)(s + \alpha - j\omega_d)} \qquad (15)$$

where $\alpha = (\omega_o/2Q)$ and $\omega_d = \omega_o\sqrt{1 - 1/4Q^2}$ as can be easily verified by substituting back. We are interested in the case where the denominator becomes zero (and hence, T(s) blows up). These are clearly values s_1 and s_2 such that $s_1 = -\alpha + j\omega_d$ and $s_2 = -\alpha - j\omega_d$. These values are called the "poles" of the transfer function. Note that the two poles have a real part (α) and an imaginary part (ω_d). Thus we have to represent them as points in the complex "s-Plane" where s is the complex Laplace variable we have been finding useful. A plot of these pole positions is shown in Fig. 8. The s-Plane is laid out in terms of a real axis (σ) and an imaginary axis ($j\omega$). More information on the s-Plane can be found by reading the article on low-pass filters in EN#41, reprinted in Chapter 5d of the Musical Engineer's Handbook. For the moment, we will just note that we are interested in the value of a function (such as T(s)) of a complex variable (s) and since the variable is complex ($s = \sigma + j\omega$), we must represent its value as the elevation of a surface above a plane (the s-Plane). The poles thus represent infinitely high points above the plane. The portion of the surface around them is thus expected to be very high. Note also that the point s = 0 causes the numerator of T(s) to be zero, so this point is called a "zero" of T(s). In between these poles and zeros, we have a curved surface. Points of equal elevation on the surface form flat contours, and these loop around the poles and connect up. We thus can plot contours such as the 3db (half power) contour as shown in Fig. 9. For frequency response, we were interested on the value of the transfer function above the $j\omega$-axis going from 0 to ∞. We can see that for s = 0, T(s) = 0 as well. Moving upward along the $j\omega$-axis, we move to higher ground. When we reach $\omega = \omega_d$, we are near the highest point, but the zero at s=0 has in effect lowered the ground at ω_d and the actual peak is not reached until we reach ω_o. Beyond ω_o, it is downhill all the way to $\omega = j\infty$.

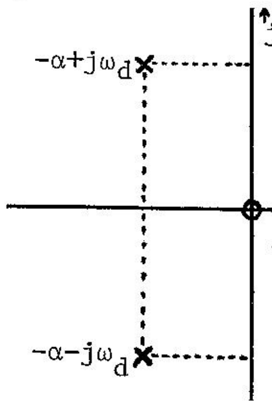


Fig. 8
s-Plane

x= pole of T(s)
o= zero of T(s)

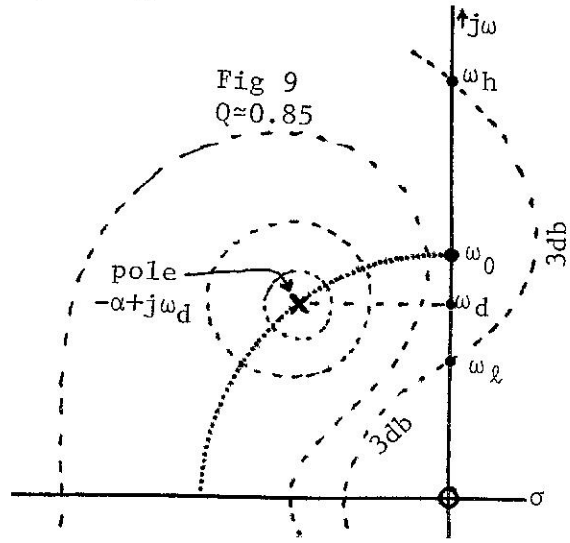
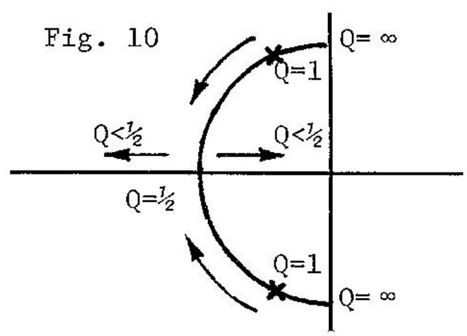


Fig 9
Q≃0.85

We have now had two looks at the way things are placed around the s-Plane, and should have some idea about it even if we don't know all the details. We want to take one more look at it, as seen in Fig. 10 where we see how the poles move as Q changes. We saw in Fig. 8 that the poles were placed according to the values of α and ω_d, and we obtained these two parameters as a function of Q only. Thus, we can plot a "locus" of points showing how the poles move as Q changes. Here we show mainly Q from infinity down to 1/2, as these are the only cases where the poles are complex (the Q = 1/2 case being the critically damped "Gaussian" filter case.) Before considering filter ringing, note that the peak of the frequency response remains at ω_o in all cases even though the pole frequency ω_d changes with Q. This is because the zero at s=0 has pulled down the surface along the the $j\omega$-axis across from ω_d as we have described above.

Fig. 10



In order to understand filter ringing, we have to study the impulse response of a filter, but first we have to see what is really meant by T(s). We have written $T(s) = V_{out}/V_{in}$ and we are also accustomed to measuring frequency response using sine waves, so we might think that V_{out} and V_{in} are functions of time. Well, they are functions of time, but we can't write a function of one variable equal to a function of another variable so what is meant by T(s) must be:

$$T(s) = V_{out}(s)/V_{in}(s) \tag{16}$$

where $V_{out}(s)$ and $V_{in}(s)$ are the Laplace transforms of their time waveforms. [The reader should take a moment to consider why it is that we can actually measure T(s) in terms of sinusoidal waveforms which vary in time.] The meaning of this is that T(s) is a function that we can use to get the Laplace transform of the output if we know the Laplace transform of the input. We want to ring the filter, and this takes a sharp spike waveform or impulse. Mathematically, the ideal spike is the delta function $\delta(t)$ and the Laplace transform of $\delta(t)$ is just 1. The reader will probably find this to be probable (believable) if he considers that the Laplace representation is a form of frequency representation (spectrum), and he probably already knows that a sharp spike sounds like a "click" which is wideband noise, or a flat, constant spectrum, which can be taken to be one. We can thus do the following:

$$T(s) = V_{out}(s)/V_{in}(s) \qquad \text{or:} \qquad V_{out}(s) = T(s)\cdot V_{in}(s) \tag{17a, b}$$

$$\text{for } V_{in}(t) = \text{impulse} = \delta(t),\ V_{in}(s) = 1 \tag{18a, b}$$

$$\text{Thus:} \quad V_{out}(s)_{\text{impulse}} = T(s)\cdot 1 = T(s) \tag{19}$$

Taking inverse Laplace transforms:

$$V_{out}(t)_{\text{impulse}} = \boxed{L^{-1}[T(s)] = h(t)} \tag{20}$$

where the symbol L indicates the Laplace transform (L^{-1} is thus the inverse transform). Equation (20) thus gives us the important result that the impulse response (in time) is just the inverse Laplace transform of the transfer function $T(s)$. We already have $T(s)$. Thus to find how the filter responds to an impulse, we just have to look up the Laplace transform pairs in the tables. One such pair is shown below:

$$\frac{s}{(s-a)(s-b)} \ \underset{L}{\overset{L^{-1}}{\rightleftarrows}} \ \frac{be^{bt} - ae^{at}}{b-a} \tag{21}$$

This is clearly of the same form as equation (15) for the bandpass filter. We could now plug into the inverse Laplace transform to get the time waveform of the ringing filter. However, the complete solution tends to obscure the important features rather than enhance them. Instead, we will just note that terms like

$$e^{(-\alpha - j\omega_d)t} = e^{-\alpha t}e^{-j\omega_d t} \tag{22}$$

will appear. This is a decaying exponential ($e^{-\alpha t}$) which controls the amplitude of a sinusoidal term ($e^{-j\omega_d t}$). Thus, we can justify to some extent our intuitive notion that the ringing filter produces a decaying exponential sinusoidal waveform. This waveform is shown in Fig. 11. We note further that the exponential decays with a time constant $1/\alpha = 2Q/\omega_o$. This is no surprise since we expect longer decay times as we increase the Q of the filter. What is more of a surprise is that the sinusoidal frequency is ω_d and not ω_o as we might have expected. We can see however from Figures 8, 9, and 10 that for high Q, there is very little difference between ω_d and ω_o.

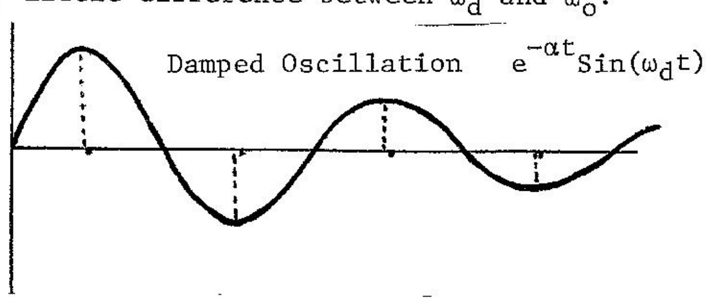


Fig. 11

An important formula on filter ringing is obtained by finding the amount of time it takes for the amplitude to die out to 1/e of its original value. This is just a matter of setting $e^{-\alpha t} = e^{-1}$ and hence $T_{ring} = 1/\alpha = 2Q/\omega_o$ which can also be written:

$$T_{ring} = \frac{Q}{\pi f_o} \tag{23}$$

There are several important things to notice about equation (22). First, it is a convenient way to measure Q in cases where the bandwidth is very small (thus for high Q) and where it would be difficult to separate the 3db points. Thus Q can be measured by ringing the filter and finding the time constant of the decay envelope.

Secondly, note that for fixed Q, the ring time gets shorter as frequency rises. This is an important musical result, although it is not possible to say ahead of time if it is particularly useful or if it is a drawback. On the one hand, we know that many acoustic musical instruments have faster decay at higher pitches (the piano for example). On the other hand, we may want to use the ringing filter as a self-enveloping system where we would otherwise use an envelope generator (fixed time constants) and a VCO. In this case, the shorter decay times at higher frequency that would result from the ringing filter would be a drawback. However, we can have both results if we wish by using voltage-controlled Q. This is a good case for suggesting that Q should double for each one volt change of control. This would allow us to set up either constant number of decay cycles (fixed Q) or constant decay time (using voltage-controlled Q). These setups are shown in Fig. 12a and Fig. 12b. Note that a constant number of cycles for 12a is implied by equation (23) since $T_{ring} \cdot f_o$ = number of cycles = πQ, which is a constant for a constant Q.

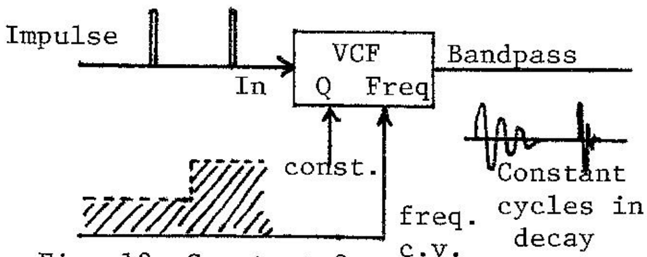


Fig. 12a Constant Q

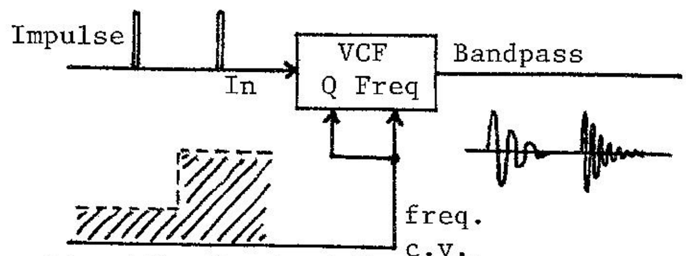


Fig. 12b Constant Decay Time

We now turn our attention to the question of the detuning from ω_o to ω_d where

$$\omega_d = \omega_o \sqrt{1 - 1/4Q^2} \tag{24}$$

This detuning for different values of Q is shown in Fig. 13.

Q	ω_d/ω_o	Ring Cycles (1/e)	Ring Cycles ($1/e^6$)
0.5	0	0.16	0.95
0.6	0.55	0.19	1.15
0.7	0.70	0.22	1.34
0.8	0.78	0.25	1.53
1.0	0.87	0.32	1.91
1.5	0.94	0.48	2.86
2.0	0.97	0.64	3.82
3.0	0.986	0.95	5.73
4.0	0.992	1.27	7.64
6.0	0.997	1.91	11.5
10.0	0.9987	3.18	19.1
50.0	0.9995	15.9	95.5
100.0	0.999987	31.8	191

FIG. 13

TABLE OF DETUNING VALUES DUE TO DAMPING OF FILTER

[Ring cycles down to $1/e^6$ is approximately to expected audible range.]

From the table above, we can make one preliminary conclusion. For a detuning value which we might expect to notice (say 1%), we would have a Q of about 4, and a total number of audible cycles of only about seven. This may not be enough for the ear to detect a proper feeling for pitch. A study of similar cases causes one to conclude that it may not be possible to detect this detuning by ear - hence its musical importance is very small.

Before we discuss some additional applications of bandpass filters, we should say a few additional words on what we have discovered so far. First note that equation (22) is really a damped sinusoidal of some form. Thus, we can look at the damped oscillation as shown in Fig. 11 as being of the form:

$$e^{-\alpha t}\mathrm{Sin}(\omega_d t) \tag{25}$$

The zero crossings of this waveform are clearly at times where $\mathrm{Sin}(\omega_d t) = 0$, which is just what we expect from a sinusoidal. However, the individual lobes between the zero crossings are not sinusoidal lobes. You could prove this by differentiating equation (25) and setting the derivative equal to zero. It will serve here however to just note that the maximum and minimum points of the waveform fall at points in time that occur before the midpoints between zero crossings. For example, the maximum point of the first lobe will occur when the rate of change of the upward sinusoidal is equal to the rate of change of the downward exponential. Clearly this is before the normal peaking point of the sine wave (which occurs at the midpoint between zero crossings), because at the normal maximum of $\mathrm{Sin}(\omega_d t)$, the combined waveform of equation (25) is moving downward because of the exponential damping factor. Now, beyond the maximum point of the waveform in equation (25), the exponential damping factor forces the sinusoidal down a little faster than normal, which tends to flatten the normally more rounded portion. Yet, the exponential damping works only on the amplitude of the sinusoidal (one in this case) and not on the waveform directly, so it is still necessary for the function $\mathrm{Sin}(\omega_d t)$ to go to zero in order to get the zeros of the composite waveform of equation (25).

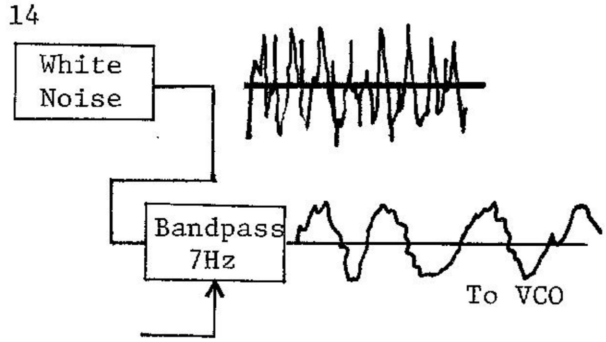
We have suggested above that the detuning from ω_o to ω_d [Equation (24)] is not likely to be detected by ear since in cases where there is enough pitch shift, there are too few cycles in the decay. The detuning may be more important at low frequencies (say 1 Hz.) Even though the waveform is not directly audible in such cases, these low frequency detunings may be detectable indirectly, as when they are used as control signals. It is just this sort of use of the filter output as a control signal that we want to discuss in the suggested applications below. Also, in cases where the filter output is used to modulate another process, there may be a great difference between a modulating frequency that is exactly harmonically related to another frequency, and one which is only close.

There is another fine point which we are not able to completely rationalize here. Perhaps some reader can provide us with the answer. First, we note that when we measure the frequency response of the bandpass, the peak is always at ω_o. Secondly, when we ring the filter, it oscillates (exponentially damped) at a somewhat lower frequency ω_d. All this is fine, but we now want to inquire about the response of the filter to a white noise input (flat spectrum). Since the input spectrum is flat in this case, we expect the output spectrum to have a peak at the maximum response point of the frequency response curve (which is ω_o). In another view, we can consider white noise to be a series of impulses of random amplitude and random time of occurance. In this view, the response of the filter is the superposition of the individual impulse responses, each of which is an exponentially damped sinusoid with zero crossings corresponding to ω_d. For practical purposes, thus probably makes no difference at all, but it would be interesting to know if these two views do lead to the same result.

APPLICATIONS

We are very much accustomed to listening to the output signals from filters rather than using them as a control for some other processing unit. The three primary applications in Figures 2a, 2b, and 2c are examples where we listen to the output. The signals from filters are of course usable as controls. An example of such a case is shown in Fig. 14 where white noise is bandpass filtered at a frequency of about 7 Hz with moderate to high Q (say about 100). The resulting signal is a useful vibrato signal which can be applied to a VCO as shown. Experiments with this show that this is quite different from vibrato produced with a steady state waveform. For one thing, the depth changes somewhat randomly with a time constant on the order of a second or two. Thus, it sort of fades in and out during an extended tone (and it is usually extended tones which employ vibrato). Another thing is that one can perceive the random nature of the process as some sort of subjective randomness in the output of the VCO. This is not to say that the resulting vibrato is more musical, or more natural, but just that it does have an interesting property. Some persons would perhaps describe it is a "spooky" vibrato as opposed to a "mechanical" vibrato from an oscillator control, and a "warm" vibrato which one would hear from a singer, for example.

Fig. 14



The value of frequency modulation (FM) and dynamic depth FM has been demonstrated both theoretically and through its use in musical compositions. A typical patch for FM is shown in Fig. 15a, for dynamic depth in 15b, and for modulation by the ringing bandpass filter in 15c.

Fig. 15a

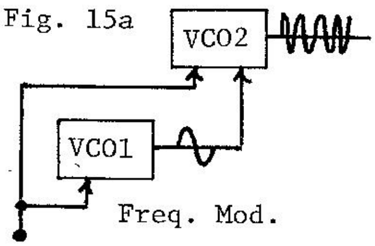


Fig. 15b Dynamic-Depth Freq. Modulation

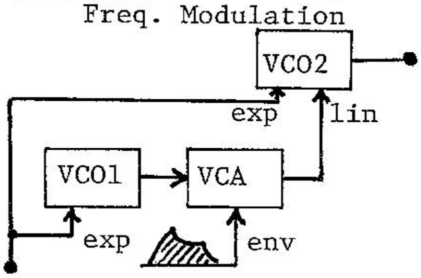
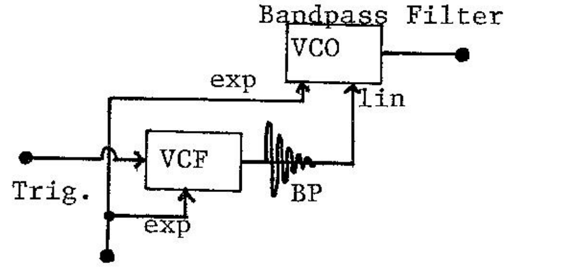
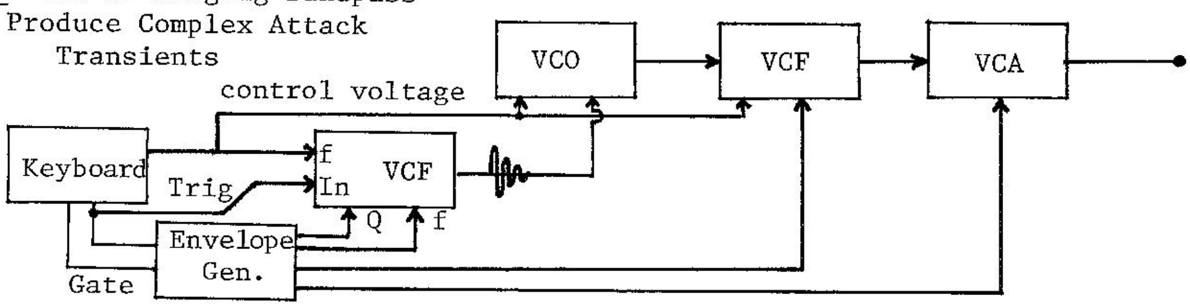


Fig. 15c FM by Ringing Bandpass Filter

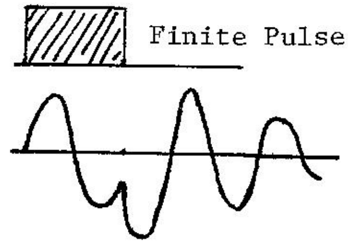


The patch of Fig. 15a is standard, that of Fig. 15b is being explored (very useful for percussion effects), and that of 15c is relatively unknown as far as we can tell. Linear frequency modulation by a ringing bandpass filter is however a very useful way of producing very interesting musical transients (usually attacks). We had an envelope generator which did this back in EN#9, but it was not much used. As shown in Fig. 15c, the bandpass is rung by the trigger from the keyboard. This damped expenential sinusoid goes into the linear FM input of a VCO. It is well established that many musical instruments produce very strange effects during the initial part of their tones. This attack phase may have many frequency components which do not exist during the steady state that follows. The bandpass filter rung by the trigger produces a useful waveform for adding features during attack. Immediately after the trigger arrives, the output of the bandpass is at a maximum (producing the deepest FM and thus the widest distribution of sidebands). As the bandpass response decays, the modulation depth decreases, and this means there is less and less energy in the sidebands, and more and more in the central frequencies. A more complete patch for this method is shown in Fig. 16.

Fig. 16 Use of Ringing Bandpass To Produce Complex Attack Transients



Of course, you can also use the filter at very low frequency (1 Hz for example) and at high Q to produce a control signal that lasts several minutes, and yet still does decay uniformly. In another variation, a finite width pulse can be used (meaning, one which has a duration longer than the time constants of the filter input stages). For example, the pulse of Fig. 17 will cause a complex ringing in that the filter will start to ring and decay on the positive going edge, and then will be restarted by the negative edge. This complex transient is useful, and can be easily domonstrated by ringing the filter with a pulse from a VCO and altering the pulse width. This can be used as a control signal for FM as in Fig. 16, or as a timbre producing element by directly listening to the output of the filter. In this latter case, the frequency of the filter would probably be in the upper audio range while the pulse ringing it would be in the lower audio range.



Finite Pulse

Complex Transient

Fig. 17

While we have been discussing here mostly bandpass filters, the same generally applies to high Q low-pass and high-pass filters which can also be rung. There may be some obvious differences however. For example, with the high-Q low-pass rung by a square wave, the DC term will come through (that is, the two levels of the square wave will be passed through). This is illustrated in Fig. 18. It is easy to think of patches where this sort of waveform could give interesting results since it is basically two levels with starting transients.

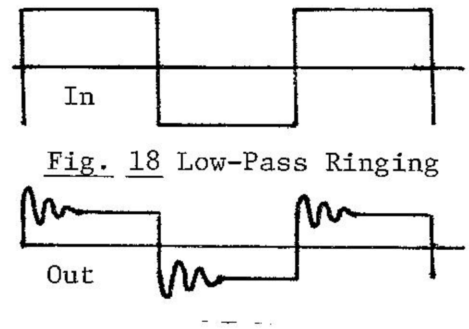


Fig. 18 Low-Pass Ringing

SUMMARY

Since the mathematics above may have obscured much of the useful information for some readers, we will provide a brief summary of the important results as far as musical applications are concerned.

1. The "Q" of a bandpass filter is obtained as discussed in the first paragraph.
2. Once a transfer function is obtained, the center frequency and Q can be easily obtained by comparing with equation (3).
3. A state-variable filter provides constant Q while a Biquad provides constant bandwidth. When ringing the bandpass, constant Q means a constant number of cycles during decay (shorter ring times at higher frequency), while constant bandwidth means a ring time independent of frequency. Constant bandwidth can be obtained with a state-variable by increasing Q with the frequency.
4. A "Limit" input for a state variable filter is often provided along with voltage-controlled Q. This input supplies less signal to the filter as the Q is increased.
5. A filter which is rung provides an exponentially damped sinusoidal waveform (Fig. 11)(Equation 25). The frequency of this ringing is slightly below the center frequency of the filter, but probably this detuning is not detectable by ear.
6. The relation between Q, center frequency, and ring time to $1/e = 37\%$ is given by equation (23). This equation should be carefully studied.
7. There are numerous applications of bandpass filters and other VCF's which use the output of the filter as a control signal rather than using the output signal as an audible sound. These include vibrato, linear FM, and a number of schemes for generating complex transients.
8. There is a need for additional analysis and/or experimental measurments on the output of bandpass filters excited by impulses and with white noise. In particular, it would be useful to know the exact mathematical properties of such outputs, and correlate this information with data on aural perception to get a better idea of musical implications. For example, Metzger [J. Acoustic Soc. Amer., Vol. 42, No. 4, 1967, pg 896] shows that the peak of the Fourier Transform of equation (25) is actually below ω_d.