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REVISITING: COMPENSATION OF OP-AMP CIRCUITS FOR THE EFFECTS OF THE OP-AMP POLE

INTRODUCTION:

It has always been true that all ideal op-amps are equivalent, as well as non-existent. Since the beginning of widespread use of IC op-amps (about 1970) it has been true that real op-amps are looking more and more like ideal op-amps. Indeed, they can have tiny input bias currents, and very respectable speed in terms of both bandwidth and slew rate. And they are cheap. To a large extent, one type can be expected to serve in most all cases where an op-amp is needed. We have made a slight distinction of suggesting that a slower op-amp might be better advised in cases where speed is not needed, since the faster ones might become unstable [1].

In one area, op-amps remain non-ideal and will likely remain that way. They have their own pole (at a relatively low frequency)! (An ideal op-amp has no such pole.) The pole is put there intentionally. We will spend just a few moments to remind readers why this is:

Remember that the op-amp has two inputs, one inverting, and the other non-inverting. Because so many op-amp applications involve negative feedback, we generally think of "inverting" as being normal, and hence the rather strange term "non-inverting" (a double negative of sorts). The inputs are inverting or non-inverting with specific reference to the op-amp output. An op-amp used with feedback has an external path, or paths, from the output back to one or both inputs. These external paths are <u>under our control</u>, and they are designed and implemented <u>for a particular result</u>. So far, so good.

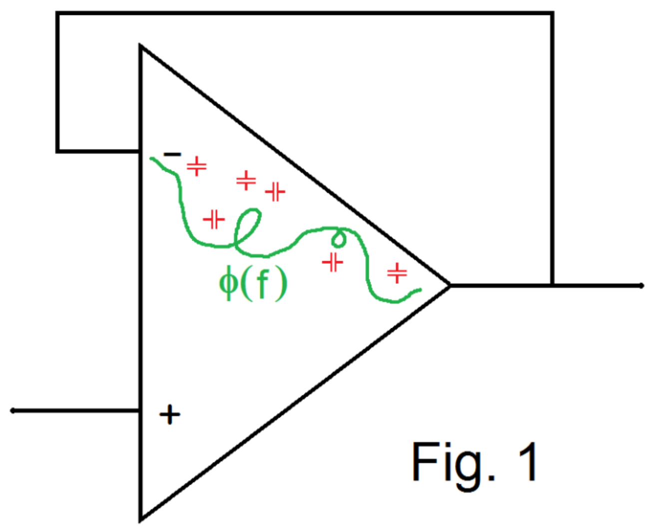
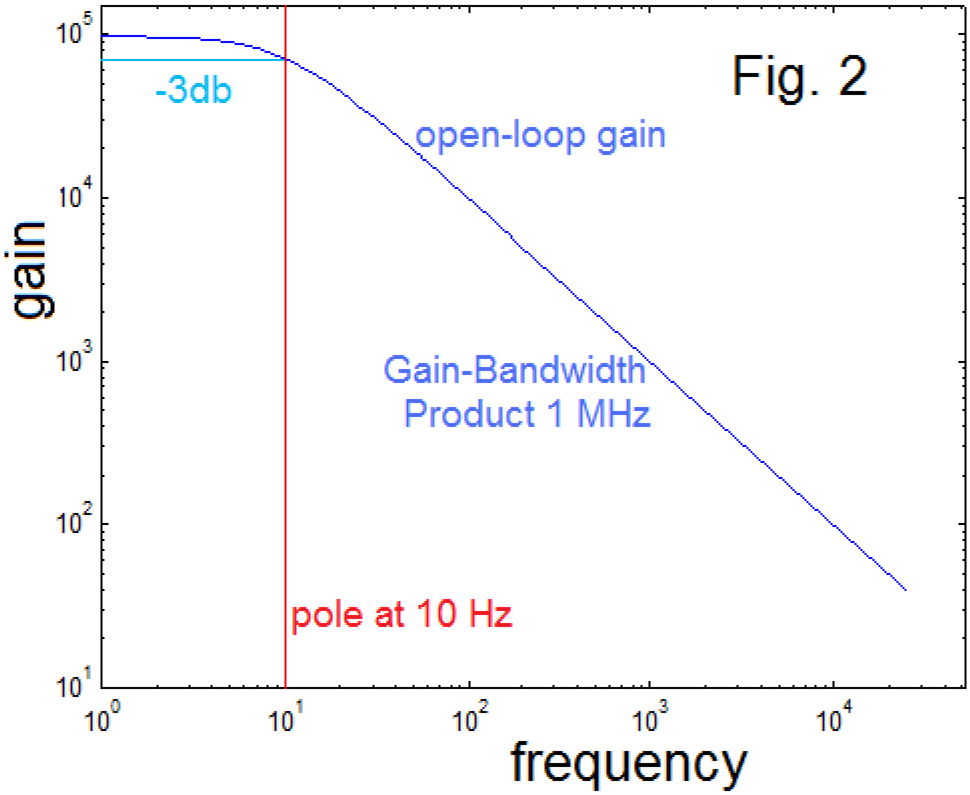


Fig. 1

But we also have the case that the op-amp "chip" is very small, by intention. As with anything that is electrical and small, we anticipate that there will be small stray capacitances, and these contribute to small phase shifts in general. Fig. 1 shows an op-amp configured as a follower (100% negative feedback) and we there indicate also stray capacitances (red) and an associated stray phase shift φ(f) in green. Well, the phase shift is a function of frequency. It increases with frequency. One might say that: so what – the excess phase is 0.01° at the highest frequency in my application – I can easily tolerate that. True enough, but we are not concerned with just the frequencies <u>intended</u> in the application. Small amounts of noise are plenty to introduce frequencies that can be very high; enough so that the value of φ(f) around the loop can be many degrees, perhaps even reaching 180°. Clearly if there are at least three capacitances (and we can always find one more smaller one) there is certainly a frequency where φ(f) reaches 180°. Again you may protest that even if there is an extra 180° across the chip, the frequency is too high to make it across without considerable loss. Indeed, that is what we want to assure.

In the actual case, there is far too much gain, so even if there is "considerable loss" of gain along the φ(f) path, all we need is a gain of 1 to have an oscillator if the phase reaches 180° at some frequency. Note that the unity-gain follower of Fig. 1 is the hardest to control since it has 100% feedback. [It <u>is</u> counterintuitive that the use at the lowest gain is the most susceptible to oscillation.] The solution which is almost universally now found is to intentionally degrade the frequency response so that when the phase shift reaches a frequency where φ(f) is 180°, the gain is already below 1. This is called "Unity-Gain Compensation" and means that a pole is built into the op-amp's open-loop gain curve, so <u>the op-amp is a low-pass filter</u>.



This internal compensation, obtained with a small capacitor fabricated on chip, is the price we pay and the cure we obtain. The part can now be used much more easily, and it is good enough (usually much more than good enough) in the vast majority of our applications. The remaining surprise is that this added pole is at the astoundingly <u>low</u> frequency of something like 10 Hz! Fig. 2 shows such an "open-loop gain curve". The plot is log-log, and is the 45° roll-off one expects from a single-pole roll-off. For this curve (straight line!), the product of the gain and the bandwidth (frequency), appropriately called the gain-bandwidth product or GBP, is a constant, 1 MHz in this case. For example, a gain of 10^4 is found at about a frequency of 10^2. The GBP of 1 MHz is typical of the "741" type of op-amp, standard where high speed is not needed. Other op-amps may have a GBP of 10 MHz or more.

Note that the curve in Fig. 2 is a first-order low-pass and thus obeys the transfer function:

$$T_{open-loop}(\mathrm{s}) = \frac{\mathrm{A}}{(1 + \frac{\mathrm{s}}{\mathrm{s_p}})} \tag{1}$$

where s_p is the pole position (-2π10 rad/sec) and A is the DC gain (about 10^5 here). The op-amp gain is not only non-ideal (ideally it would infinite gain) but varies with frequency, rolling off. If the GBP could be applied to DC (frequency 0) the gain at DC would indeed by infinite (GBP/0). Instead, it is finite of course, as we see for the curve bending level for low frequencies.   So the gain A is not the GBP.   Notice that the gain A, if held constant, would intersect the GBP line at exactly 10 Hz.

While the open-loop curve is nice to know about, it is not of much value in suggesting practical designs. Viable open-loop (no negative feedback) applications of op-amps are numerous, but do not include those for AC amplifiers. Rather open-loop applications are things like comparators where the output is either saturated at one supply rail or the other, or is racing as fast as possible between these choices.   In an amplifier application, an AC gain of say 10,000 at 100 Hz (GBP=1,000,000) might be what we want overall, but this would mean that the input AC signal would have to be 10,000 times smaller (perhaps ±1 mV) than the output range (determined by the supply levels). Unavoidably the DC gain would be 100,000 (Fig. 2) and the DC offset would be at least several mV. The output would be clipped. So the curve of Fig. 2 tells us something, but not how to do an actual amplifier design. And it may well worry us that this open-loop curve began falling off at 10 Hz. Not in your audio system – you say!   All designs of amplifiers and filters, etc., require negative feedback: controlled gains.

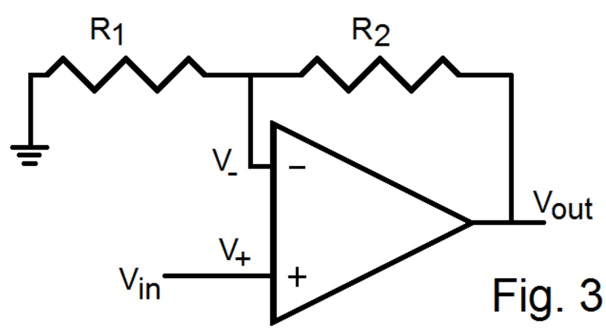


Fig. 3

Fig. 3 shows an application where two resistors are placed in a voltage divider for negative feedback.   This is the standard "non-inverting amplifier".   To a near-perfect approximation, the input bias current of the op-amp is zero, so the voltage V_- is the result of the voltage divider.

$$V_- = V_{out}\left[\frac{R_1}{R_1+R_2}\right] \qquad \text{\{ IDEAL \}} \qquad (2)$$

For the ideal op-amp, the gain is considered infinite, and thus $V_- = V_+ = V_{in}$, so:

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1} = K_{ni} \qquad \text{\{ IDEAL \}} \qquad (3)$$

In the case of the "real" op-amp, the major (perhaps only) significant changes is that we use the (G/s) model, the GBP curve, to relate V_{out} to V_+ and V_- as:

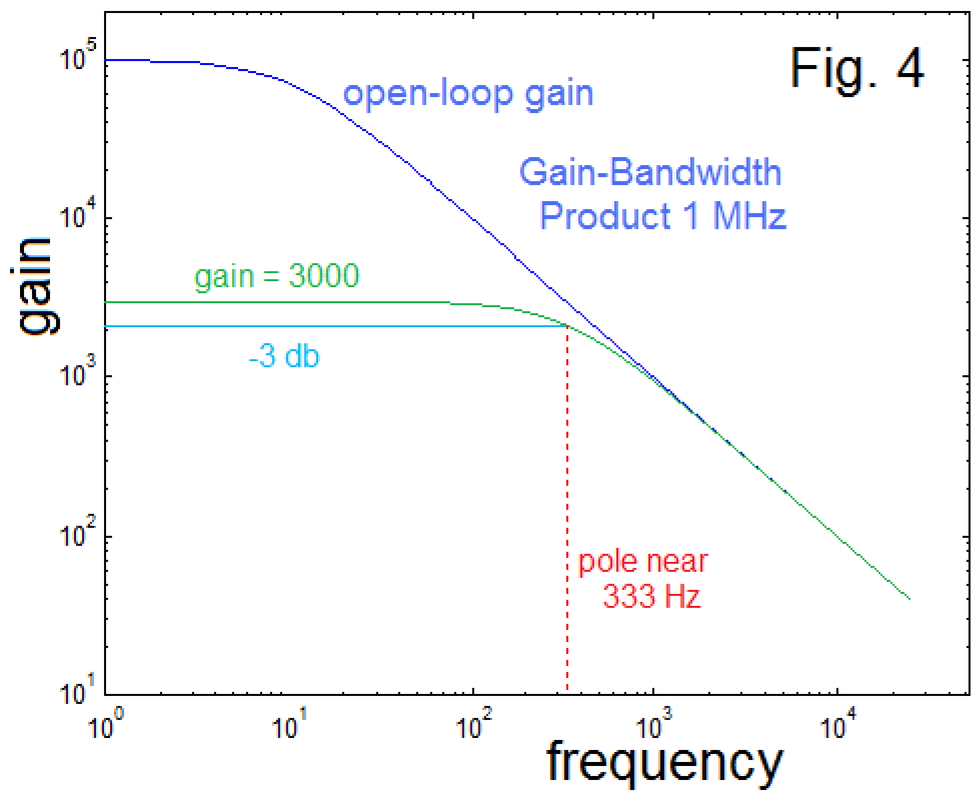
$$V_{out} = \left(\frac{G}{s}\right)(V_+ - V_-) \qquad \text{\{ REAL OP-AMP EQUATION \}} \qquad (4)$$

Here G is the GBP expressed in rad/sec, i.e., G = 2πGBP. This we can solve, using equations (2) and (4):

$$T_{ni}(s) = \frac{V_{out}}{V_{in}} = \frac{G/s}{1+\frac{(G/s)R_1}{R_1+R_2}} = \frac{G}{s+\frac{G}{K_{ni}}} \quad \text{\{ REAL NON-INVERTER \}} \qquad (5)$$

which is a first-order low-pass with a pole at:

$$s_p = -\frac{GR_1}{R_1+R_2} = -\frac{G}{K_{ni}} \quad \text{\{ REAL NON-INVERTER \}} \qquad (6)$$



COLOR KEY TO NOTES IN { } FOLLOWING EQUATIONS

Blue = Ideal Op-Amp = Cool

Red = Real Op-Amp = Alarm

Tan = Passive Compensation by Capacitor (Often Brown Color)

Grey = Active Compensation by Added Op Amp (Grey Paint Markings on the IC

Here we see that the pole of the op-amp remains (it is the only capacitor in the circuit) but it has moved, and may be quiet high in frequency.   For example, if $R_1 = R_2$, K_{ni}=2, and the pole is at G/2, which would be half a MHz for our example. Notice that the pole frequency is an excellent indication of the cutoff frequency of the amplifier (it being a low-pass filter). Accordingly we see how we may have an amplifier with plenty of bandwidth for our application even though the op-amp pole was at a very low frequency before the actual working gain was set by the negative feedback.

Fig. 4 shows the open-loop curve of Fig. 2, and we have added the frequency response (magnitude of equation (5)) as well, for a design gain of 3000 (quite a bit).   We see that the bandwidth is about 333 Hz (1,000,000/3000).   Notice that the working curve bumps into the open-loop curve above the pole.   This introduces the general problem within the area of an ordinary amplifier application.

HISTORY:

The basic ideas of open-loop gain and internal-compensation have been around since the early days of IC op-amps. A separate notion of "compensation" relates not to the op-amp's stabilization issue itself, but rather to various techniques of adjusting the performance of a circuit using the op-amps – allowing for the op-amp's internal compensation.   There are four ways to do this.

(1)   **GOOD ENOUGH?**   See if it really matters.   This is exampled by the graph of Fig. 4. Suppose we need an amplifier with a gain of 3000 that works up to 100 Hz. We have it. In the event that we do not get it, the alternative would be to obtain the needed gain in two stages – that sort of thing.

(2)   **OVERDESIGN.**   We can also use the classic "overdesign" of the engineer. The canonic example is that of building a bridge neglecting the weight of the bridge. The bridge sags. So you redo the design putting in a center hump, with the expectation that this will then fall back down to about where you wanted it.   Both experiment and calculations can be used to do this. Typically this is used in active filters.

(3)   **PASSIVE COMPENSATION.**   We try, usually by adding small shunt capacitors and/or small resistors in series with capacitors, to cancel and/or move an offending pole. Techniques like this certainly go back to the vacuum-tube era.

(4) **ACTIVE COMPENSATION**. Here we use an active element, almost certainly a second op-amp of the same type (often on the same chip) as the first in a feedback loop of the first. There are several ways to look at this. Basically we attempt to make individual building blocks such as summers and integrators into more ideal versions.

The historical effort here will be far short of providing a complete chronology of the various techniques. Indeed, I have been unsuccessful in pinning these issues down to one or even just a few sources. So I have here decided to discuss only our own publications and the items I personally recall.

Our main exposure was through the multiplier-controlled state-variable filter, soon to become the OTA based VCF. The publication of the OTA filter concept was a bit delayed, since while I knew of it for at least a year from company engineers, this was private information, and it was some time before I got it independently (along with an OTA-integrator-based VCO published by Sergio Franco). The idea of compensating the integrators with shunt capacitors across the input attenuators for the CA3080s followed shortly, although I don't recall just how this appeared. So this was mid-70s, and about that time, active compensation schemes also appeared, principally in IEEE journals.

The first reference I find to active compensation in our publications is in two application notes [2,3] from 1981 and 1982. In 1986 [4], we published a fairly extensive analysis of the then known passive and active methods of compensating the various circuit blocks. I recall working hard on that math! Not that the math was hard, but I was convinced that the case of the active compensation should show the cancellation of the op-amp caused pole by a zero via negative feedback. I couldn't find my algebra mistake. Well, it was a conceptual mistake with regard to what should happen, and indeed what even could happen. This led to the notion that we were not cancelling a pole, but rather replacing it with a zero and two poles, which together had a more favorable phase response. The "magic-circle" position of these pole positions was original (or at least independently) offered there – I don't know anywhere else it appears. Well, it did actually reappear in our own attempt at a text ***Analog Signal Processing*** which was written (and used in class) in 1987, and eventually published serially in ***Electronotes*** [5], which is online. The presentation in [5] is I believe, except for renumbering of figures and equations, exactly the same as [4].

PHASE – IT'S ALL ABOUT THE PHASE

As I stated, I knew about the purported use of active compensation before I had a good visual picture of how it worked (if it worked!). Likely this was because the result was offered in the form of equations, not of pictures, and because it concerned phase, which is usually a more remote concern than amplitude. Recall Ohm's Acoustic Law which seems to suggest that the human ear is phase deaf. Largely true, and a good reason why we often ignore it in audio applications. An obvious exception is where we sum signals of the same frequency. Now the result depends on the relative phases of the signals being added. An example of where signals are added is the state-variable filter structure. A few extra degrees of phase makes a difference. A filter which changes frequency (that's what a VCF does after all) can become unstable at high frequencies.

Let's right here look at an example, a very simple one, one at which we have not looked in our previous [4] study: a voltage-follower with a voltage-follower in the feedback. This is shown in Fig. 5. In Fig. 5A we have the real follower. Applying equation (4) we easily obtain

$$T_f(s) = \frac{V_{out}}{V_{in}} = \frac{G}{s+G} \qquad \text{\{ REAL FOLLOWER \}} \qquad (7)$$

Or we could just have used equation (5) with K_{ni}=1. So the real follower has a pole at s= -G. Of course, when G →∞ (ideal), $T_f(s)$ goes to 1. For the compensated case, Fig. 5B, we have added an identical follower replacing the wire from V_{out} to V_-. Equation (4) applies to both op-amps. In the feedback, the output of the back facing op-amp is $V_- = \frac{G}{s+G}\,V_{out}$. Plugging this in equation (4) gives:

$$T_{cf}\,(s) = \frac{G(s+G)}{s^2 + Gs + G^2} \qquad \text{\{ ACTIVE COMPENSATED FOLLOWER \}} \qquad (8)$$

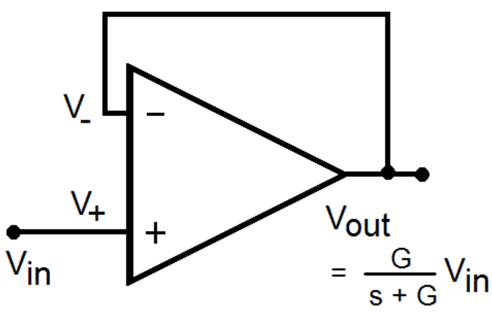


Fig. 5A
REAL FOLLOWER

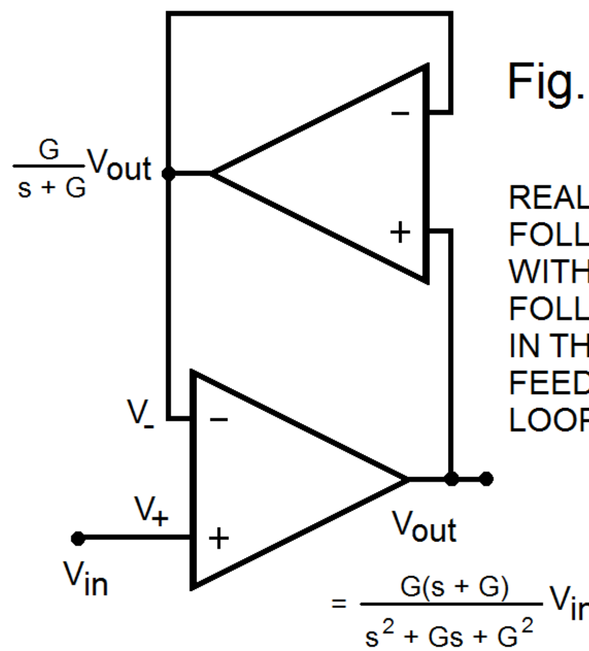


Fig. 5B

REAL FOLLOWER WITH REAL FOLLOWER IN THE FEEDBACK LOOP

Equation (8) does show a zero at s=G has appeared in the numerator, as we hoped, and might have expected by putting that in the feedback denominator. But we still have not only one pole, but two poles, because of having two op-amps (two capacitors), and we probably should have remembered this. The poles are at:

$$s_{p1,p2} = -\frac{G}{2} \pm \frac{jG\sqrt{3}}{2} \qquad \text{\{ ACTIVE COMPENSATED FOLLOWER \}} \qquad (9)$$

and we also have the zero:

$$s_z = -G \qquad \text{\{ ACTIVE COMPENSATED FOLLOWER \}} \qquad (10)$$

It is not obvious why this situation, equations (9) and (10), is better than just the single pole at –G. Here frequencies are angular frequencies ω.

We recognize that the ideal op-amp follower would have had a flat frequency response magnitude and zero phase shift for all frequencies. Using the real op-amps, we are curious as to how the response is changed by the pole at –G. Then we are interested to see what difference there is by having at zero at –G **and** the newly positioned pole **pair**. We can simply plot these responses. Fig. 6 shows these plots, and a zoom-in, a much smaller frequency range, is shown in Fig. 7. Likely the vast majority of what we need to appreciate about this approach is illustrated right here. The dashed red curves show the single-pole real op-amp follower, exactly what is expected. The blue curves show the result with the second follower in the feedback loop.

The magnitude response, Fig. 6A, shows a pronounced bump upward, and is perhaps alarming (although a quite lovely illustration of the expected feedback). The phase curve of Fig. 6B on the other hand shows a nice leveling – the response starts off with zero derivative (blue) instead of the linear relationship (dashed red). The degree of improvement is best seen in Fig. 7B, the zoom-in. In fact, keep in mind that the full frequency range here from 0 to 4 is in units of G. We show this full range to get well beyond the poles/zeros as we should with any investigation of analog response. Yet, we have no intention of going through a frequency of G, or anywhere close to G. As such, the responses of Fig. 7 which run only to G/10 are much more relevant. Looking at Fig. 7A, the alarming magnitude bump is not seen for this restricted range. It just shows a slight increase not much more than the slight decrease of the single pole in the range of ω=0 to ω=G/10 of Fig. 7A..

In fact, we can say that while both phase responses get to exactly 45° at ω=G, the dashed red curve is linear while the blue curve bends outward so as to approximate 0 initially. In fact, note that the dashed red curve is essentially φ=tan-1(ω/G) ≈ ω/G, or about 5.7° at ω=G/10.

Keep in mind here that the red curves apply to the followers, Fig. 5A and Fig. 5B (top), while the blue curves apply to the bottom op-amp of Fig. 5B. While the followers roll off to 0.7071 at ω=1, in the feedback (Fig. 5B) this reduction of gain causes the overall stage of Fig. 5B to have a gain of 1.414 at ω=1 (reciprocal relationship). The gain just below ω=1 is actually

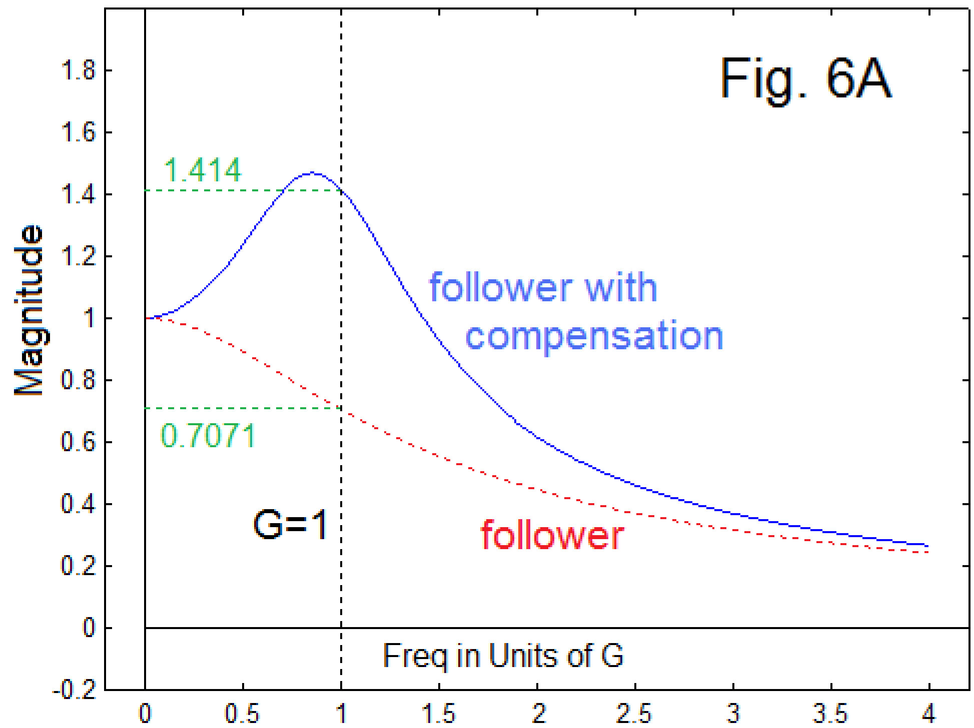


Fig. 6A
Magnitude
1.414
0.7071
follower with
compensation
G=1
follower
Freq in Units of G

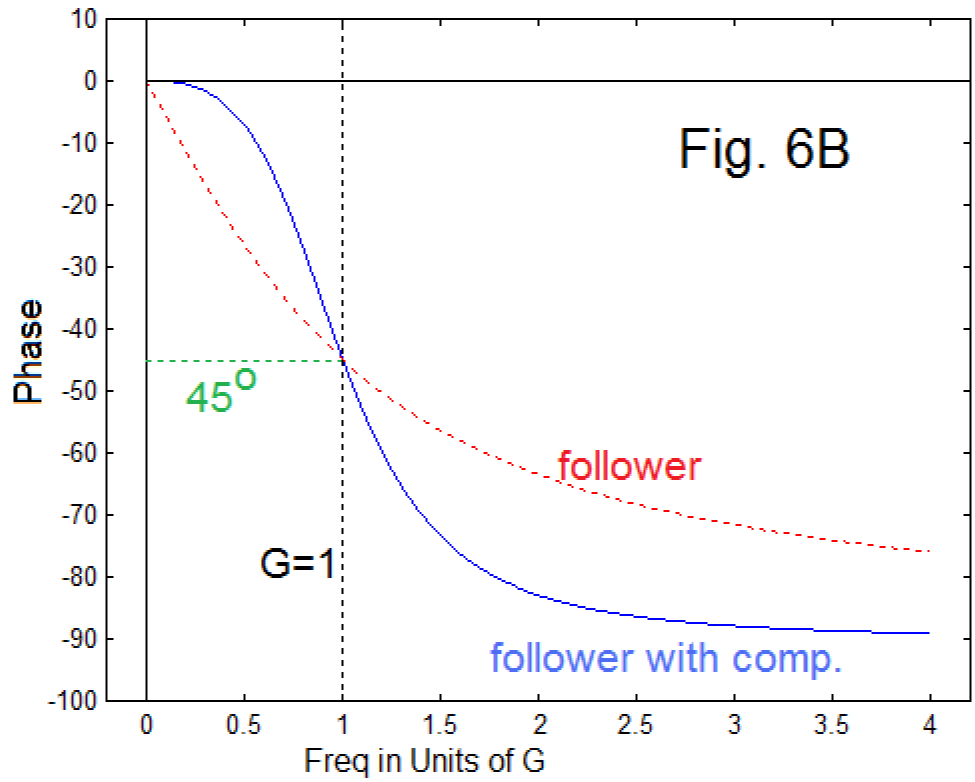


Fig. 6B
Phase
45°
follower
G=1
follower with comp.
Freq in Units of G

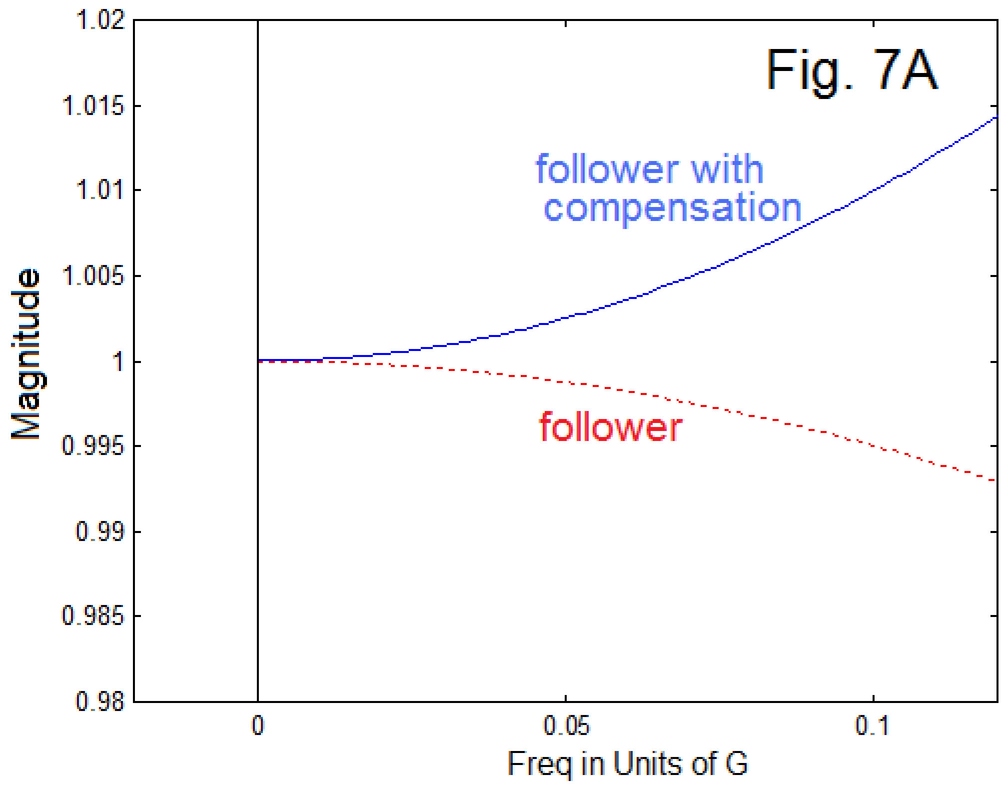


Fig. 7A
follower with
compensation
follower
Magnitude
1.02
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
0
0.05
0.1
Freq in Units of G

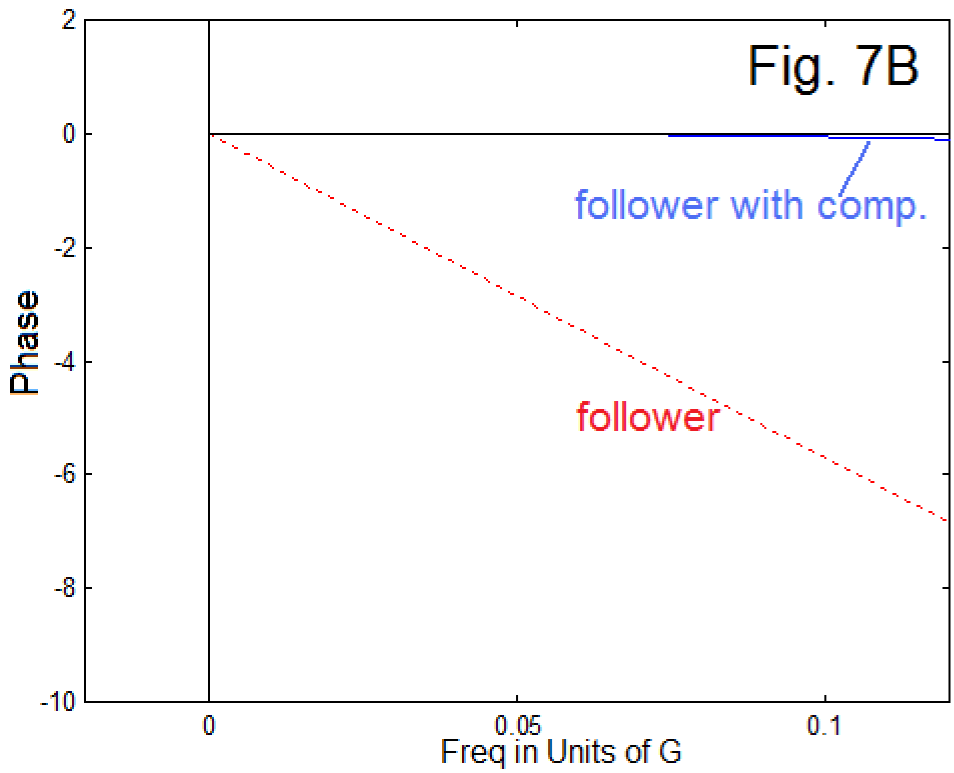


Fig. 7B
follower with comp.
follower
Phase
2
0
-2
-4
-6
-8
-10
0
0.05
0.1
Freq in Units of G

above the 1.4142 value. While the curves of Fig. 6 and Fig. 7 represent the situation of the follower, we note that they apply basically to the single extra pole, or the case of the zero and two extra poles. These op-amp caused amendments are added to any poles and zeros that may be the desired features of the network. For example, when we are designing an integrator, we really do want a pole at s=0. In such a case, the curves of Fig. 6 and Fig. 7 represent the deviations from nominal. For the follower (ideally flat and zero phase shift) the deviations represented the entire performance. These we will look at – but first, the "magic circle" mathematics.

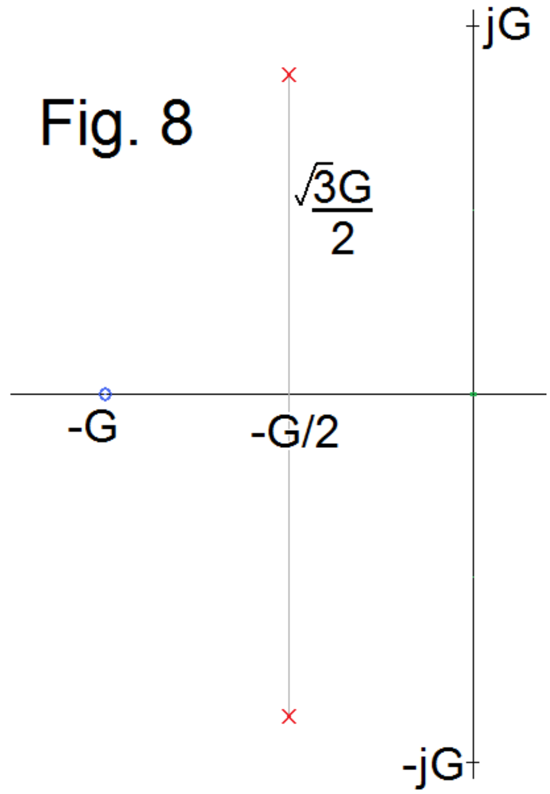
MAGIC CIRCLE

We found above in equation (9) that just by plugging in the only values possible, that the compensated follower of Fig. 5B had a zero at –G and a pair or complex conjugate poles at

$$s_{p_1,p_2} = -\frac{G}{2} \pm \frac{G\sqrt{3}}{2}$$

as shown in Fig. 8, and it is not immediately apparent (a) that this does us any good, and (b) if the finding can be generalized. Strangely, the origin, the zero, and either pole lie at the corners of an equilateral triangle! It is on the basis of Fig. 6 and Fig. 7 that we decided that the phase response was greatly improved with little damage to the amplitude response.

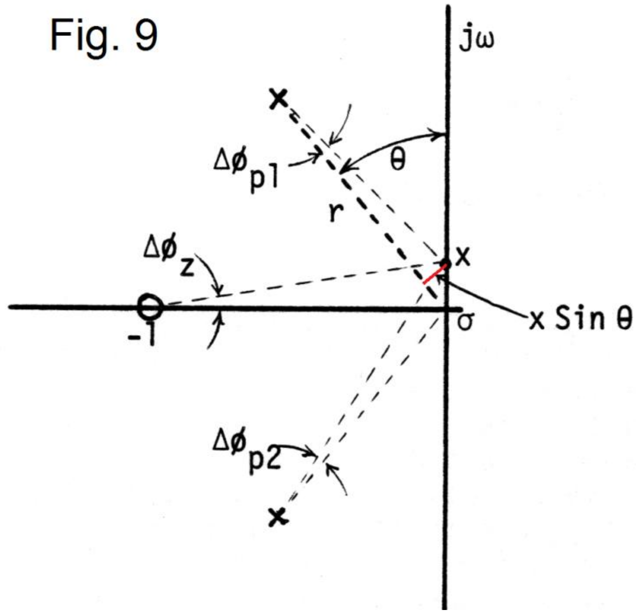
Fig. 8



We can turn the problem partly around to suppose that we have a zero (at –G or some other arbitrary position, perhaps at -1) and we thus know the result on the phase response. We ask if it is possible to find some positions of a pair or poles such that the shift off the two poles exactly cancels the shift off the zero; not for all frequencies, but at least such that there is zero derivative at ω=0.

This problem is posed by Fig. 9 where we have placed the zero at a normalized frequency of -1. We seek a distance r and an angle θ for the poles. Consider a small distance x along the jω axis. As seen from the zero at -1, this distance x subtends an angle, using the small-angle approximation for tangent, of x. We recognize that whatever this angle is, the angle off the two poles will be of opposite sign, and we want this (the sum for both poles) to cancel the angle off the zero. This might well seem unlikely, since for Fig. 9 we have chosen the distances actually found for Fig. 8, and r seems to be (and is) 1 in magnitude. Accordingly the sum of the angles off the two poles might appear to be too large. However, looking from the poles (either one) at the length x along the jω-axis we see not x but a tilted (smaller) version of x, which is x sin(θ), so the angle is x sin(θ)/r, and this occurs for both poles. To obtain zero incremental phase at ω=0, we thus need:

Fig. 9



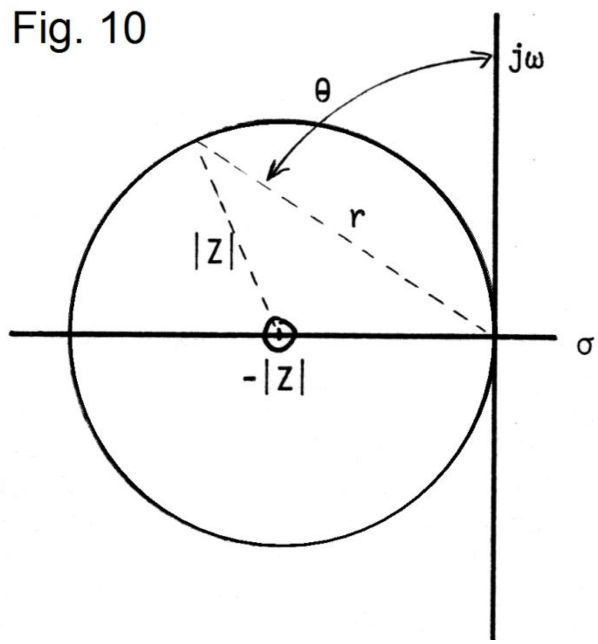
$$x - 2\,x \sin(\theta)/r = 0 \qquad (11)$$

Generalizing to the case where the zero is at a distance |Z|, our condition on r and θ is:

$$\mathbf{r = 2\,|Z|\,\sin(\theta)} \qquad (12)$$

This equation defines a circle, and indeed a circle which we can call magic. The circle is centered at -|Z| and it passes through s=0. Any pair of poles on this circle will give zero incremental phase at ω=0. Note that Fig. 8 meets these criterion with r = |Z| and θ=30°. Another choice that might be found would be -|Z| ± j |Z|, for r=$\sqrt{2}$ and θ=45°. It is also obvious that r=2 |Z| and θ=90° is also a solution, as the angle off the poles, both at -2 |Z| would be x/2.

Fig. 10



Our goal here is to have a simple criterion from which we can rather easily judge whether or not a pole pair is arranged, by choice of a configuration or of components, to have zero incremental phase.

THE INTEGRATOR AND THE INVERTER

– CONSIDERING PASSIVE AND ACTIVE COMPENSATIONS

THE INTEGRATOR – PASSIVE COMPENSATION

As we noted above, when a building-block network element becomes significantly non-ideal due to the frequency properties of a real op-amp, we can consider fixing it by passive or active means. In the case of the followers above (Fig. 5) we had no passive elements to monkey with – hence the active approach). Here we will move further on and look at both for the op-amp integrator and the inverting amplifier. We will keep in mind that we can identify success with a “magic circle” result. It seems to be the case that the integrator follows most easily from our findings on the follower, and this we will look at first. The equations given here will be sketchy at times. As mentioned, a more detailed description has been given in the earlier reports [4,5].

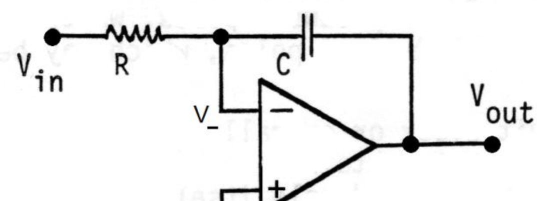


Fig 11A

Uncompensated Integrator

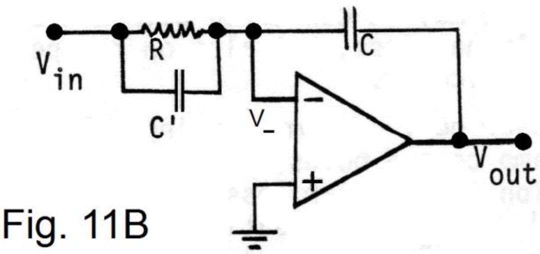


Fig. 11B

Passively Compensated Integrator

Fig. 11a shows the standard inverting integrator. Regardless of whether we consider the op-amp ideal or are using the (G/s) model, the voltage V_- is obtained by superimposed voltage dividers:

$$V_- = \frac{\left[V_{in}\left(\frac{1}{sC}\right) + V_{out}\,(R)\right]}{\frac{1}{sC} + R} \qquad (13a)$$

In the case of the ideal op-amp, this V_- is set to zero and we have:

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{-1}{sRC} \qquad \{\text{ IDEAL INTEGRATOR }\} \qquad (13b)$$

which has the pole at s=0 which we want. In the case of the real op-amp, we use equation (4) recognizing that V+ =0 (grounded) so

$$V_- = -(s/G)V_{out} \tag{13c}$$

Equating (13a) and (13c) we have:

$$T(s) = \frac{-1}{sCR(1+\frac{s}{G}+\frac{1}{GRC})} \qquad \text{\{ REAL OP AMP INTEGRATOR \}} \tag{13d}$$

so there is the desired pole at s_{p1}=0 and second pole at:

$$s_{p2} = -G - 1/RC \qquad \text{\{ REAL OP AMP INTEGRATOR \}} \tag{13e}$$

which is approximately –G in many cases.

Fig. 11B shows the case where we attempt to do a passive compensation. In particular, a capacitor C' is placed in parallel with the input resistor R. The goal is to achieve a zero term in the numerator. Once again we want to find V_-. The impedance of the parallel combination of R and C' is:

$$(R/sC')\,/\,(R+1/sC') = R\,/\,(1+sC'R) \tag{14a}$$

so we can write the equation for V_- as the superposition of voltage dividers:

$$V_- = \frac{(V_{in}\left(\frac{1}{sC}\right)+V_{out}\frac{R}{1+sC'R})}{(\frac{1}{sC}+\frac{R}{1+sC'R})} \tag{14b}$$

which again using equation (13c) can be solved for:

$$T(s) = \frac{-(1+sC'R)}{sCR[\frac{s(C+C')}{CG}+\frac{1}{RCG}+1]} \qquad \text{\{PASSIVE COMPENSATED INTEGRATOR\}} \tag{14c}$$

which has a zero at s_z=-1/RC' and a second pole (in addition to the pole at s=0) at:

$$s_{p2} = -\frac{(1+RCG)}{(C+C')R} \qquad \text{\{PASSIVE COMPENSATED INTEGRATOR\}} \tag{14d}$$

Setting $s_z = s_{p2}$, we can solve for C' exactly, as:

$$C' = 1/RG \qquad \text{\{PASSIVE COMPENSATED INTEGRATOR\}} \qquad (14e)$$

which is a shockingly simple result. We can apparently cancel the pole exactly. Perhaps you should try it yourself. Three points about the result can be made.

First, what we solved above is an exact solution, but it might seem conventional to try certain reasonable approximations. In particular, we noted above that the extra poles was at –G – 1/RC which we could take as –G if G>>1/RC. A second approximation would be to take C'<<C. Making both approximations, s_{p2} becomes –G, and we again get equation (14e).

The second point is that while the zero does in fact cancel the poles, it is not true that this causes the transfer function of equation (14c) to revert to that of equation (13b), the one for the ideal op-amp. It is an often-forgotten fact that the poles and zeros of a transfer function determine the transfer function only up to a multiplicative constant. [Multiply any transfer function by a constant and it still has the same poles/zeros.] So, if we plug C' from equation (14e) back into equation (14c), we get:

$$T(s) = \frac{-1}{sCR(1+\frac{1}{RCG})} \qquad \text{\{PASSIVE COMPENSATED INTEGRATOR\}} \qquad (14f)$$

The denominator term $\left(1 + \frac{1}{RCG}\right)$ is a constant and goes to 1 if RCG is >> 1. That is, if $\frac{1/RC}{G}$ is small, or G >> 1/RC. So things check. Recall as well that a constant in the transfer function of an integrator (the gain of an integrator) is the same as changing the time constant of an integrator [6] so it is of a lesser concern.

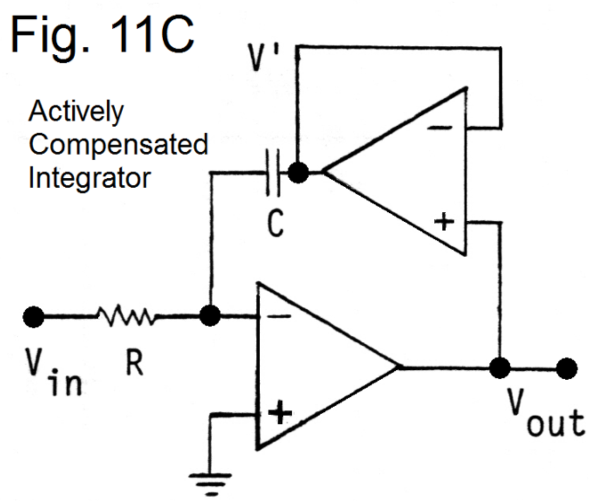
The third point relates to how well we know the actual value of G, and how well we know the ("tolerance" of the) passive components. That is, can we actually choose C' to cancel the extra pole? To the extent that we can be quite sure to have some error (perhaps 10%) in general, will we perhaps be better off trying active compensation?

None the less we are enthusiastic about exactly cancelling a pole. It seems to work for the integrator. Does it work in general? If not, why does it work here and not in other cases.

As we shall see here and as is obvious from the earlier versions of this presentation, it is not true in general. We get a compensation, but not cancellation. While we love to count capacitors (including the ones inside op-amps), and claim there should be the same number of poles, consider that it is possible to have two capacitors in a circuit and still have only one corresponding pole. Two capacitors in parallel (in series, or some other combination) are a trivial example. In the case of this passive compensated integrator, the capacitor C' seems to combine (add) with the capacitor C (see equations 14c and 14d). Indeed, the (-) node of Fig. 11B sees C and C' in parallel back to low-impedance sources. Still - strange?

THE INTEGRATOR – ACTIVE COMPENSATION

Fig. 11C shows an integrator configuration that uses a second op-amp for active compensation similar to Fig 5B for the follower. We can shortcut the analysis by using equation (7) from which we know that across a real op-amp follower we just get a transfer function of G/(s+G). Thus:



$$V' = V_{out}\frac{G}{s+G} \qquad (15a)$$

and as usual we find V- by the generalized voltage divider:

$$V_{-} = \frac{\left(V_{in}\frac{1}{sC} + V_{out}\frac{GR}{s+G}\right)}{\left(R+\frac{1}{sC}\right)} \qquad (15b)$$

and again using equation (13c) we find the transfer function:

$$T(s) = \frac{-G(s+G)}{sCR\left[\ s^2 + s\left(G+\frac{1}{RC}\right) + G\left(G+\frac{1}{RC}\right)\right]} \qquad \{\text{ACTIVE COMP. INTEGRATOR}\} \qquad (15c)$$

This shows a zero at –G, the desired pole at s=0, and two additional poles. Note that if we count the capacitors in Fig. 11C, we get the one obvious one and the two hidden inside the op-amps, so we expected the three poles. Note that if 1/RC<<G we find the two additional poles are exactly where they were for the actively compensated follower: see equations (8-10), and Fig. 8 (general form $s^2 + gs + g^2$). So we are not getting a pole cancellation here (as we had for passive compensation) but rather the “compensating array” of a zero and two poles.

It is probably not obvious that the poles of equation (15c) obey the magic circle requirement, but they do. Rather than try to solve the algebra here, it is useful to just compute the results (see Program 3 in the appendix). <u>We already know the result when 1/RC can be taken to be zero</u>, relative to G, and we know these positions are a favorable result. So the question is what happens <u>when 1/RC becomes larger</u>. When we calculate the roots of the second-order term in the denominator of equation (15c), we can plot these (see Fig. 11D). Further we know the real and imaginary parts and thus the angles and the radius r, and can verify the requirement of equation (12), and there is perfect agreement.

So the only thing that remains to be done is to look at how the poles move as a function of 1/RC, and this is what Fig. 11D reveals. We see the poles moving from their original position at $-\frac{G}{2} \pm \frac{G\sqrt{3}}{2}$ and move back around the circle as 1/RC gets larger. We are mostly interested in quite small values of 1/RC, like 0.1G or less, so we don't expect to venture too far around the circle, and we would have to check the frequency response magnitude if we try to go very far. But what we note is that this works – very well. [If 1/RC exceeds 3, the poles become purely real.]

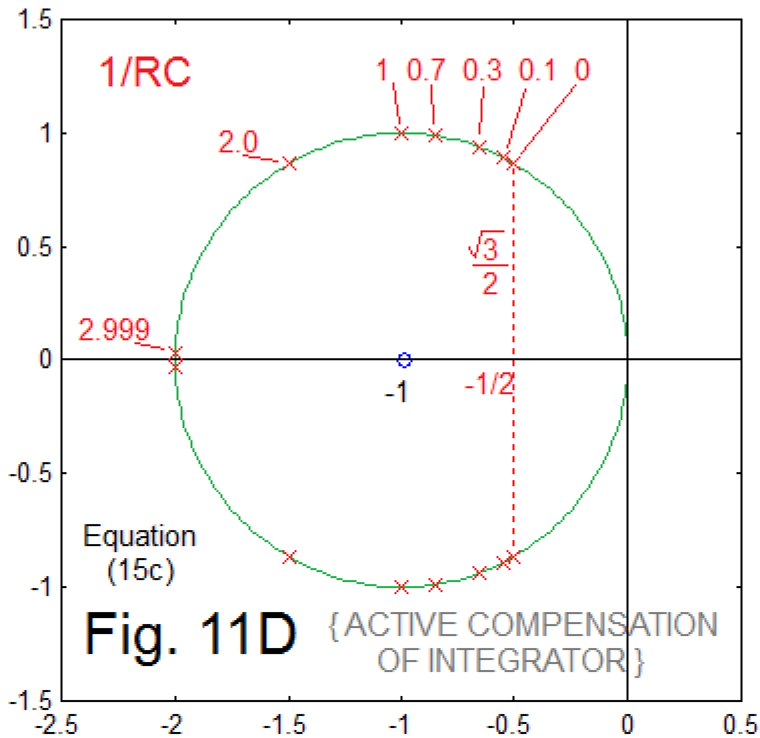


Fig. 11D {ACTIVE COMPENSATION OF INTEGRATOR}

THE INVERTER – PASSIVE COMPENSATION

We have had good success with the integrator, and it might seem that things would get simpler if we looked at the inverter. That is, we just replace the capacitor of the integrator with a resistor – which we might suppose would be simpler. Not really the case.

Fig. 12A shows the inverting amplifier. Again we just use the generalized voltage divider (superposition) to find V_-, and solve. We have expressed the feedback resistor as being some factor a times the input resistor. Thus the factor a is just the ideal op-amp gain (we don't need to review this). The voltage V- is given by:

$$V_- = \frac{V_{in}\, a + V_{out}}{a+1} \qquad (16a)$$

so using again equation (13c) we obtain:

$$T(s) = \frac{-aG}{s(a+1)+G} \qquad \text{\{REAL OP-AMP INVERTER\}} \qquad (16b)$$

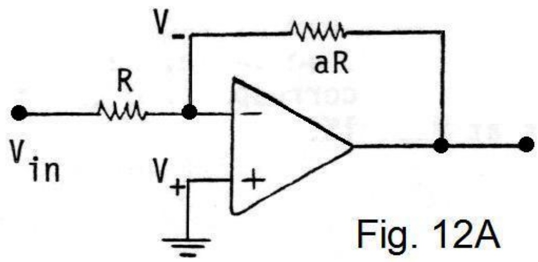


Fig. 12A

Inverting Amplifier Gain of a

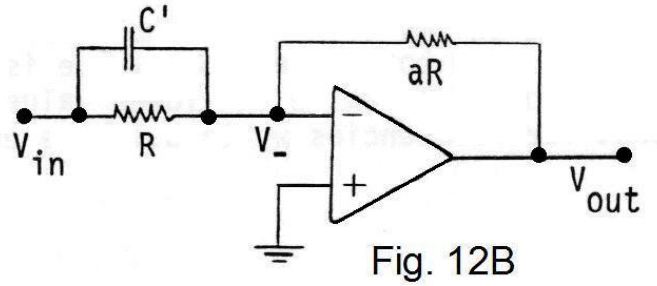


Fig. 12B

Passive Compensation of Inverting Amplifier

Equation (16b) has a pole at:

$$s_p = -G/(a+1) \qquad \text{\{REAL OP-AMP INVERTER\}} \qquad (16c)$$

This is the only pole present here – the pole of the op-amp itself. In the case of the unity-gain follower, the poles was at –G. If we have a unity-gain inverter (a=1) then the pole is at –G/2. So they are not the same.

This initial finding is closely related to the opening theme here – that when you are trying to make simple amplifiers with op-amps, when you try for more gain, you get less bandwidth. Here the pole moves in as the gain increases.

This is probably a good point to bring up the notion of “noise-gain.” The term itself, “noise-gain” is used in a number of different places in electronics with different meanings. Here we are defining noise-gain as the reciprocal of the feedback factor from the output back to the inverting input. This is a matter of looking at the appropriate voltage divider. In Fig. 12A, imagine that we have an output voltage V_{out} and we want to know what fraction of this gets back to V_-. We already know this, it’s R/(R+ aR) = 1/(1+a). That is, we recognize that we have agreed to apply superposition (linearity) to the voltage divider, and the feedback from V_{out} is that obtained by provisionally setting V_{in} to 0. So the noise gain is (1+a). This is larger than the gain of the inverter, which is a (or –a if you wish). The pole in equation (16c) is the original G divided by the noise gain. This will be a general way to find the pole, or check our specific calculations. Does it apply to the follower? For the follower, the feeback is 1 so the noise gain is 1, and the pole is at -G. What about the non-inverting amplifier (Fig. 3). There the feedback is $R_1/(R_2+R_1)$ so the noise gain (and the amplifier gain in this case) is $1 + R_2/R_1$, which we called K_{ni} in equation (3). Indeed, the pole was at $–G/K_{ni}$ in equation (6).

From this we also see that the inverting structure, for a gain of 1 has only half the bandwidth of the corresponding non-inverter (the follower). By the time the gain gets up to perhaps 10 or so, the difference between the two structures is proportionally much less. For both the bandwidth is approximately G divided by the gain.

Fig. 12B shows the proposed method of passive compensation of the inverting amplifier. Again we will try just putting a shunt capacitor across the input resistor. That worked perfectly in the integrator case above. The equation for V_- is just:

$$V_- = \frac{V_{in}aR + V_{out}\frac{R}{1+sC'R}}{aR + \frac{R}{1+sC'R}} \qquad (17a)$$

and we can still use equation (13c) to arrive at:

$$T(s) = \frac{\left(\frac{-G}{C'R}\right)(1+sC'R)}{s^2 + \frac{(1+a)s}{aC'R} + \frac{G}{aC'R}} \qquad \text{\{ PASSIVE COMPENSATED INVERTER \}} \qquad (17b)$$

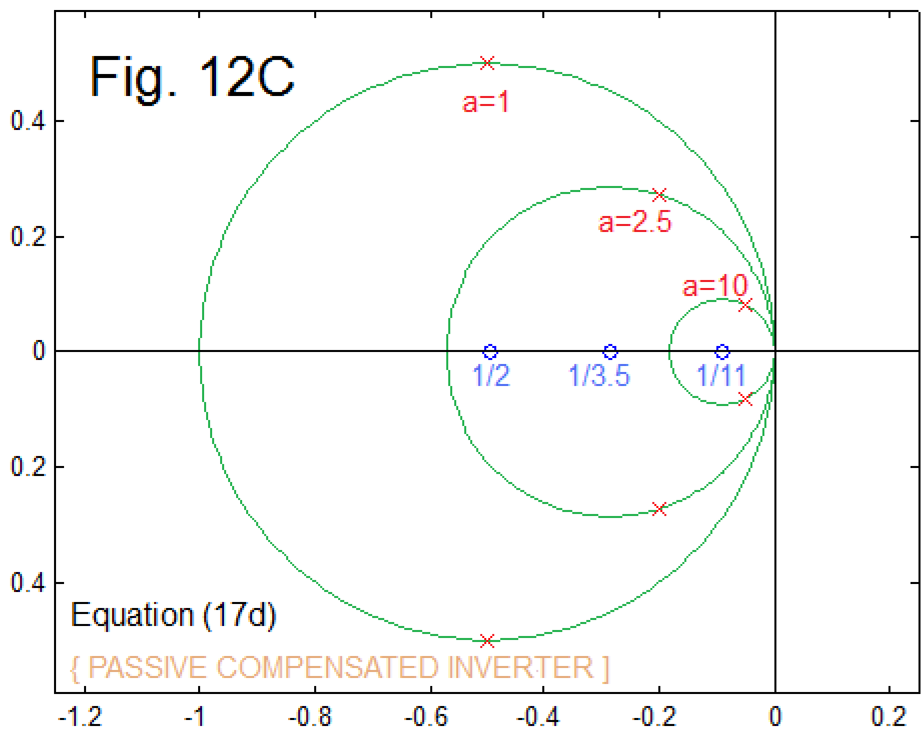
This has a zero at -1/RC' as we might have expected, and two poles. Here we can take the shortcut of guessing that we want to place the zero on top of the original pole. Thus:

1/RC' = G/(a+1) (17c)

so this gives us the value for C'R in equation (17b) and we can find the poles:

$$s_{p1,p2} = \frac{-G}{2a} \pm \left(\frac{jG}{2a}\right)\sqrt{\frac{3a-1}{a+1}} \qquad \text{\{ PASSIVE COMPENSATED INVERTER \}} \qquad (17d)$$

This does not automatically guarantee anything. In particular, we have not cancelled the one pole, but now have two complex conjugate poles. About all we can do is look for a magic circle result coming out of equation (17d), and we have it, as shown in Fig. 12C (see Program 4 of appendix for code used). In calculating and plotting poles, along with the corresponding magic circle, Fig. 12C is similar to Fig. 11D. But here there are

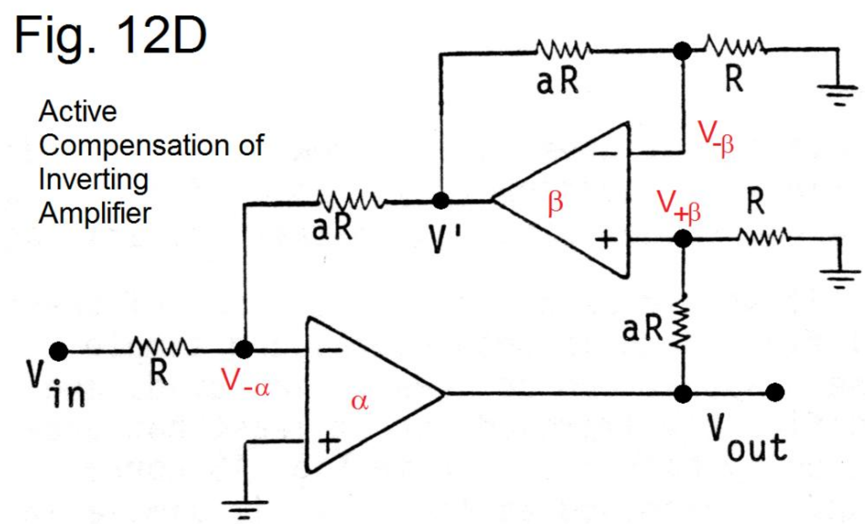


different circles, but always centered on the zero and passing through s=0, for different values of the gain parameter a. We show three different cases, for gains of a = 1, 2.5, and 10. The corresponding zeros are at 1/(a+1) = 1/2, 1/3.5, and 1/11, with the poles on the corresponding circle.

We note that the situation here, which we were guessing might be the same as that for the integrator (cancellation of the op-amp pole) is rather a compensating array, not cancellation.

THE INVERTER – ACTIVE COMPENSATION

The final of the four cases here is the active compensation of the inverting amplifier – putting a second op-amp in the feedback loop. In the case of the follower, and the integrator, this was just a unity gain follower (Fig. 5B, Fig. 11C) and this we tried for the inverter as well [4, 5] and we will not repeat that analysis here, as we showed there that it did not work, and instead showed how it was necessary to have a op-amp working with the same noise gain, in the loop. This is shown in Fig. 12D where we indeed have a unity gain from V_{out} back to V', not as a standard follower, but as a non-inverting amplifier with corresponding attenuator. So we still have in mind that we want to place a pole in the feedback loop that is in the same position as the original network. To get this, the feedback loop must have the same noise gain as the original op-amp. That is, there is a non-inverting amplifier in the feedback loop (the upper op-amp).



While we have done the non-inverting amplifier with a real op-amp above [Fig. 3 leading to equations (5) and (6)], we will put down some details here for clarity. Note that the two op-amps are labeled α and β:

$$V_{-\beta} = \frac{V'}{1+a} \tag{18a}$$

$$V_{+\beta} = \frac{V_{out}}{1+a} \qquad (18b)$$

Again using equation (13c) we solve for V' as:

$$V' = \frac{GV_{out}}{s(1+a)+G} \qquad (18c)$$

which is equivalent to equation (5) except here we have the input attenuator. Note that as G→∞ the upper stage is a follower. But note that it is now a follower with a pole at –G/(1+a), not at –G.

Now dealing with the bottom op-amp we have:

$$V_{-\alpha} = \frac{V_{in}a+V'}{1+a} = \frac{-s}{G}V_{out} \qquad (18d)$$

and plugging equation (18c) into equation (18d) we can solve for T(s) as:

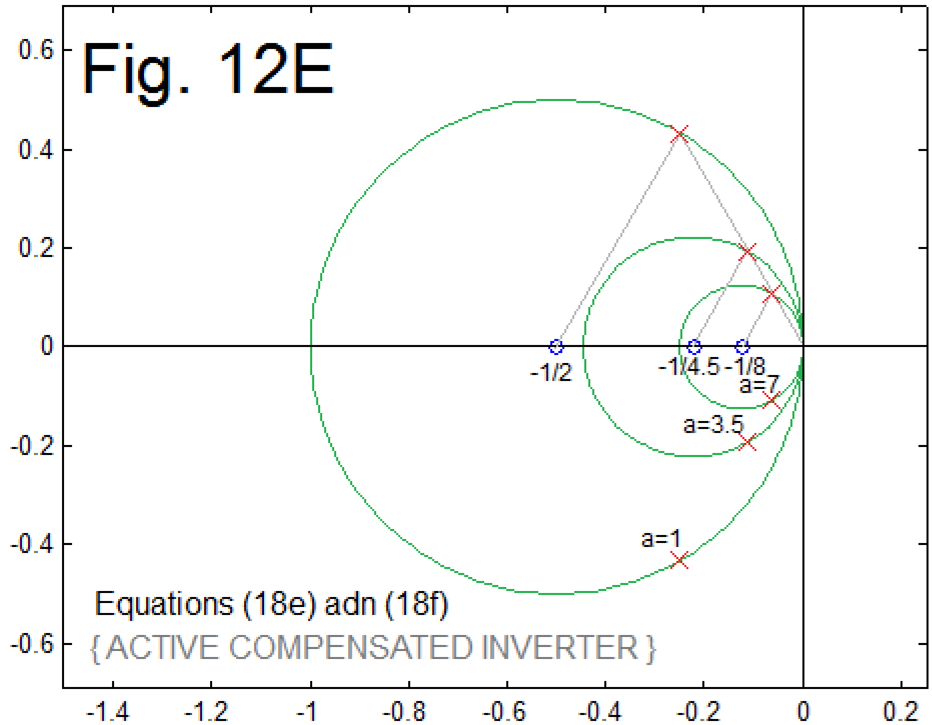
$$T(s) = \frac{-\left(\frac{aG}{(1+a)^2}\right)[\,s(1+a)+G\,]}{s^2+\frac{Gs}{1+a}+\frac{G^2}{(1+a)^2}} \qquad \text{\{ ACTIVE COMPENSATED INVERTER \}} \qquad (18e)$$

which has a zero at s_z = - G/(1+a) and poles at

$$s_{p1,p2} = \frac{-G}{2(1+a)} \pm \frac{jG\sqrt{3}}{2(1+a)} \qquad \text{\{ ACTIVE COMPENSATED INVERTER \}} \qquad (18f)$$

which we clearly see as a magic circle result (Fig. 12E).

We note that the array here has the same equilateral triangle geometry that we found was one of a continuum of solutions optimal for minimizing incremental phase, as we found for the active compensated inverter, equations (9) and (10), for Fig. 8, for the actively compensated integrator when 1/RC<<G (Fig. 11D), and which we will continue to find appearing. Plots of the poles and zeros of equations (18e) for several values of the gain parameter a are shown in Fig. 12E. This is another active compensated case, so perhaps we are not surprised. Later we shall see this exact same result appear in a passively compensated case, equations (21e) and (21f) for a non-inverter, as well as (as we might better expect), a similar scaled result for the actively compensated summer, equations (20a) and (20b). It seems that as long as you adhere to a few simple principles, things more or less work out automatically.



FOUR EXAMPLES FOUND ABOVE

Above we had two different network blocks (an integrator and an inverter) , and two different ways of compensation (passive and active) and this let to four study cases, each of which came out a little different. Before going on to some interesting design examples, we can makes sure we understand what we have found.

In the case of the integrator, the ideal integrator had a pole at s=0 (equation 13b), while the real op-amp integrator had a second pole at s=-G (equation 13e). Using passive compensation (Fig. 11B), it was possible to cancel the op-amp pole for a slight change in time constant (or gain), as in equation (14f). Using active compensation with a unity gain follower in the feedback loop, (Fig. 11C), it was not possible to cancel the pole, but a very favorable phase response was obtained using a magic circle array of a zero and two poles (equation 15c).

When we went on to inverter, the ideal inverter had no poles, but the real inverter had a poles at –G/(1+a), equation (16b). When we tried the passive compensation similar to that that worked so well with the integrator (Fig. 12B), a strange thing happened. Instead of no poles, we got two (the compensating capacitor added a second pole), equation (17b), but the magic circle result (a generalization of the active integrator compensation) was also found to be present, equation (17d). So that worked. For the active compensation of the inverter, we again put a unity gain circuit in the feedback loop (Fig. 12D), but this could not be just the usual follower, but had to be an attenuator/amplifier combination with the same noise gain as the original inverter. The result, equation (18e) had the magic circle array as the integrator, but adjusted for the gain (compare Figures 8, 11D, and 12C).

FURTHER EXAMPLES

INVERTING SUMMER - PASSIVE COMPENSATION

Many times circuit blocks are summers, not just amplifiers and/or integrators. Very similar techniques can be employed to compensate these for the effects of a real op-amp. Fig. 13A shows an inverting summer to which we have show two compensating capacitors C'. Here we undersand that the two voltages to be summed (inverted), V_1 and V_2, are to be considered zero impedance sources, as is the op-amp output V_{out}. Thus when calculating noise gain, we consider V_1 and V_2 to be zero, and they look like (neglecting the C' resistors for the moment) a single resistor R/2 going to ground. Hence the V_- node is 1/3 of V_{out}, and the noise gain is the reciprocal of this feedback factor, or 3. Without the compensation, the inverting summer has a pole at –G/3.

In analyzing the network with the capacitors C', we need to calculate the voltage at the node V_-. Without the capacitors, this would be $(V_1 + V_2 + V_{out})/3$. With the capacitors the result is similar, but we need to treat the input legs from V_1 and V_2 as impedances $Z = R/(1+sC'R)$. As before, we can use superposition of voltage dividers (see also Fig. 13B).

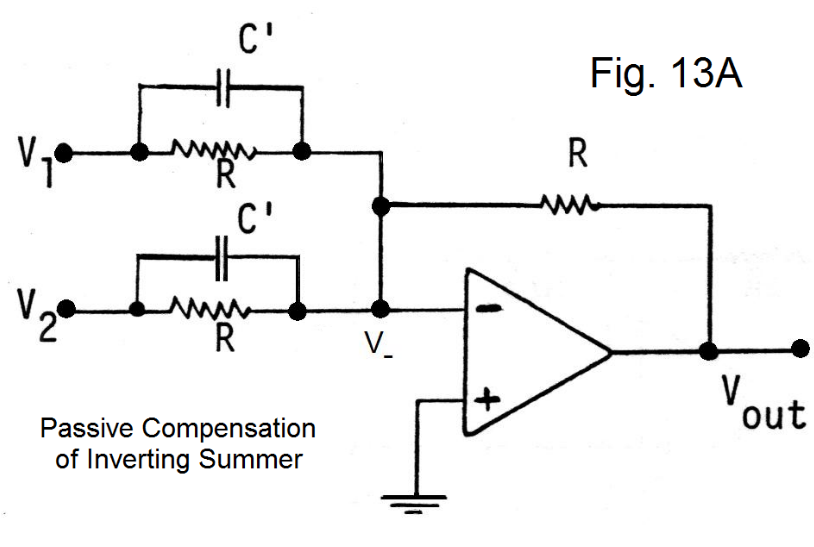
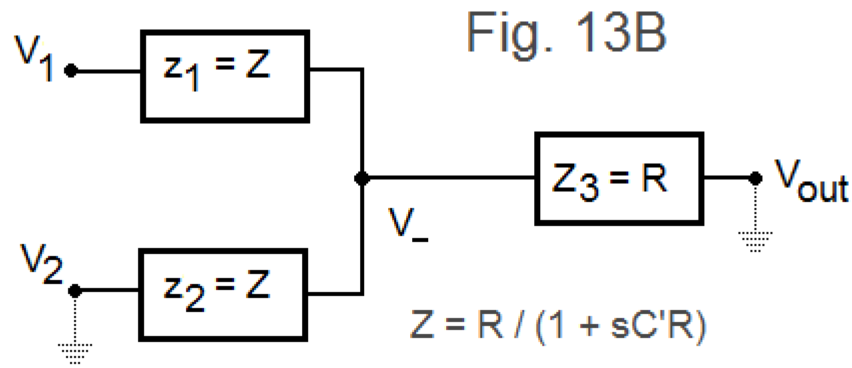


Fig. 13A

Passive Compensation of Inverting Summer

It is possible to spend a whole day (off and on) trying to get a workable form for this analysis, even when you have the supposedly right answer at hand. (Don't ask how I know this.) Accordingly, a general approach is discussed here [8]. As we said, the center node V_- is a superposition. For example, the contribution of V_1 to V_- is suggested in Fig. 13B where the V_2 and V_{out} voltages are zeroed (as suggested by the dotted ground symbol). This done, the partial voltage is:



$$V_- = V_1 \frac{[\, z_2 || z_3 \,]}{z_1 + [\, z_2 || z_3 \,]} = V_1 \frac{\frac{z_2 z_3}{z_2+z_3}}{z_1 + \frac{z_2 z_3}{z_2+z_3}} = V_1 \frac{z_2 z_3}{z_1 z_3 + z_2 z_3 + z_1 z_2} \qquad (19a)$$

where the symbol "||" indicates a parallel combination. There are two more similar terms for the contributions of V_2 and of V_{out}. <u>All three contributions have the same denominator</u> (the three products of the two impedances two at a time) and the numerator term is the product of the impedances in the lower leg of the voltage dividers. Using these results we have:

$$V_- = \frac{V_1 \frac{1}{(1+sC'R)} + V_2 \frac{1}{(1+sC'R)} + V_{out} \frac{1}{(1+sC'R)^2}}{\frac{1}{(1+sC'R)} + \frac{1}{(1+sC'R)} + \frac{1}{(1+sC'R)^2}} = \frac{-s}{G} V_{out} \qquad (19b)$$

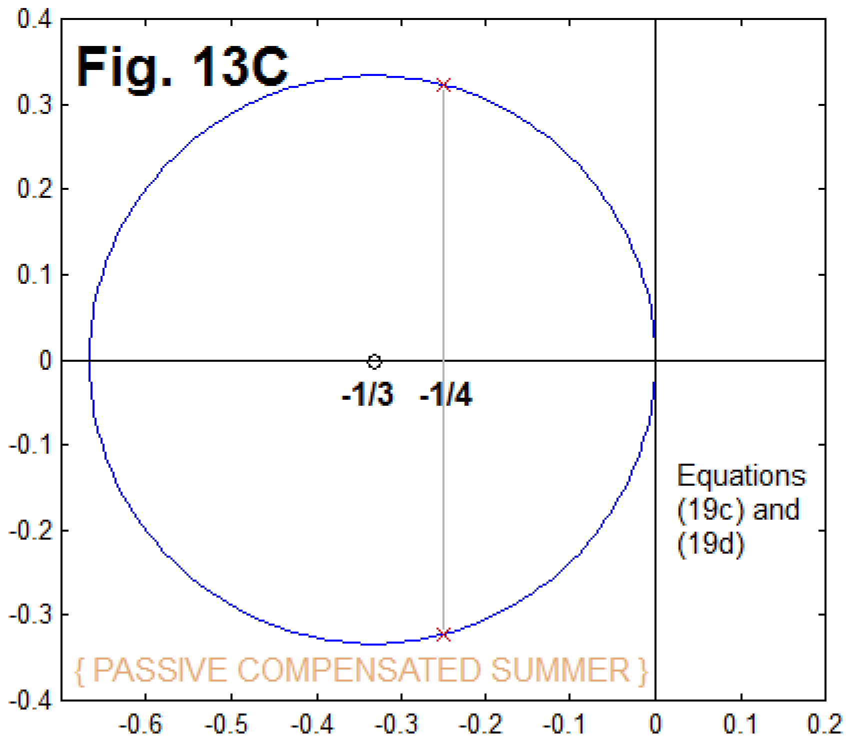
Equation (19b) is easily solved for T(s) which in this case is $V_{out}/(V_1+V_2)$ meaning that it is the result of a sum, as:

$$T(s) = \frac{V_{out}}{V_1+V_2} = \frac{\frac{-G}{2C'R}(1+sC'R)}{s^2 + \frac{3s}{2C'R} + \frac{G}{2C'R}} \qquad \text{\{ PASSIVE COMPENSATED SUMMER \}} \qquad (19c)$$

Our usual practice is to set the zero of equation (19c) to be on top of the original poles at –G/3, hence 1/RC' = G/3, and plugging this back into the denominator of equation (19c) yields poles at:

$$s_{p_1,p_2} = -\frac{G}{4} \pm \frac{jG}{4}\sqrt{\frac{5}{3}} \qquad \text{\{ PASSIVE COMPENSATED SUMMER\}} \qquad (19d)$$

and these are indeed on the circle centered at -1/3 as shown in Fig. 13C. So we have achieved one of our magic circle results.



INVERTING SUMMER - ACTIVE COMPENSATION

This example, shown in Fig. 14, adds just a bit that is actually new to the example of the inverting amplifier (three inputs, the gain a) - except notice that, again assuming zero impedance sources, the correct calculation of the noise gain. The feedback from the output to V_- would be a factor of (R/3)/(aR+R/3). Hence the noise gain is 1+3a. The pole is at -G/(1+3a). The transfer function of Fig. 14 is:

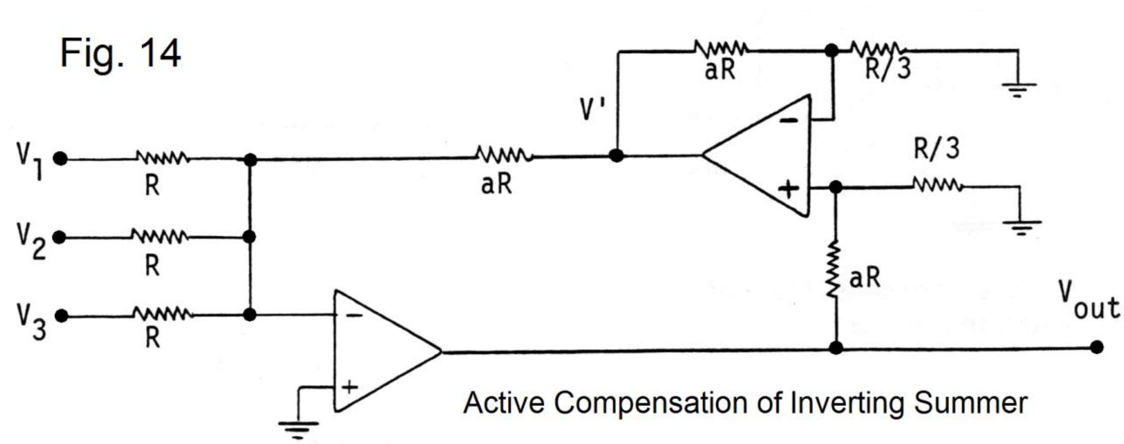
$$T(s) = \frac{V_{out}}{V_1+V_2+V_3} = \frac{\frac{-Ga}{(1+3a)^2}[s(1+3a)+G]}{s^2+\frac{Gs}{1+3a}+\frac{G^2}{(1+3a)^2}}$$ { ACTIVE COMPENSATED SUMMER } (20a)

This has a zero at the position of the original pole: s_z = -G/(1+3a). Remember that here we put the zero on the original poles by establishing the correct noise gain in the feedback loop, and hence the attenuator consisting of the aR and R/3 resistors. The resulting poles are at:

$$s_{p_1,p_2} = \frac{-G}{2(1+3a)} \pm \frac{jG\sqrt{3}}{2(1+3a)} \quad \text{\{ ACTIVE COMPENSATED INVERTING SUMMER \}} \quad (20b)$$

which are magic circle poles of the same nature as we have seen with other examples of active compensation.

Fig. 14



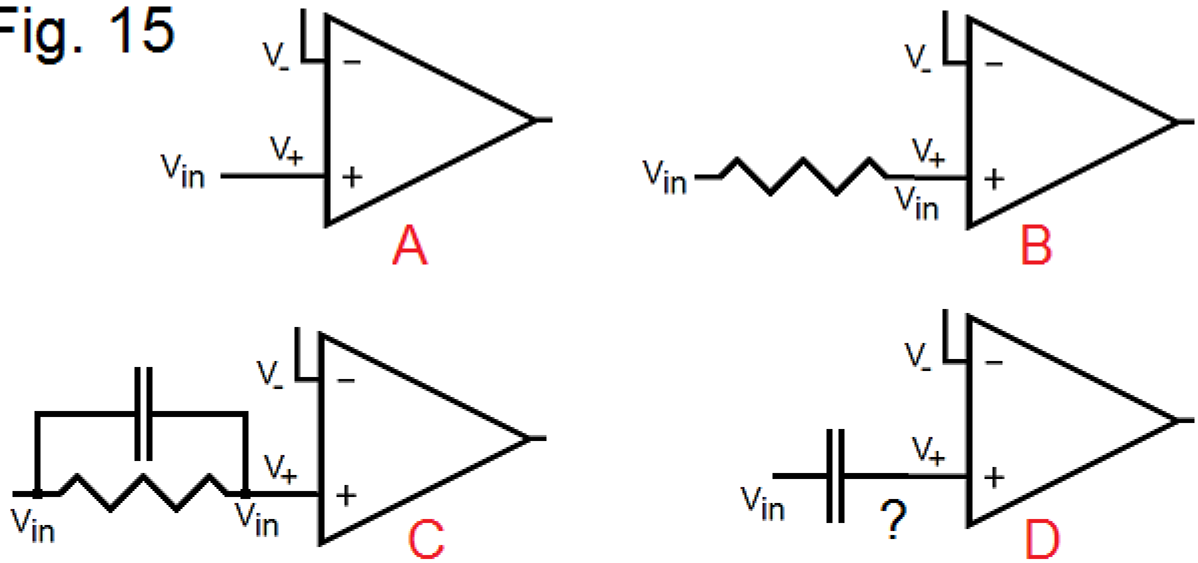
Active Compensation of Inverting Summer

NON-INVERTING STUFF

We started back in Fig. 3 and Fig. 5 with non-inverting circuits, but with those exceptions all of what we have done was inverting. Just below we need to look at the non-inverting amplifiers and summers. We probably should start here however with a reminder that impedances are really not significant if no currents flow. There is no voltage drop across an impedance unless current flows through it. In the case of a direct connection of an op-amp input to an otherwise passive network, no current flows into the op-amp. Fig. 15 illustrates some consequences of this fact. Note that are very familiar with the use of a shunt capacitor across a resistor, as in Fig. 15C. We have used this many times to place a zero in an overall response. Can we still use this?

Fig. 15A suggests the direct connection of a voltage V_{in} to an op-amp input V_-. In Fig. 15B we show the case where V_{in} is connected to V_+ through a series resistor. Now, it is true that the op-amp is assumed to draw no bias current, but this is NOT the main point. The point is that if a current were flowing through R, there is no place, no additional external paths, for it to go (unlike - in comparison - the summing node of an inverting structure). Thus V_+ is still V_{in} with the series resistor R in Fig. 15B. Likewise the parallel connection of Fig. 15C does not change the voltage in any way. So adding a capacitor shunt in this case changes nothing. What would happen if it were just the capacitor as in Fig. 15D? Well at this point the fact that the op-amp's input bias current is not really zero (it's likely some <u>very</u> small constant DC current) matters. The capacitor would eventually charge, and the op-amp output would pin. But the point is that we aren't going to get a zero by the shunting methods we used above.

Fig. 15

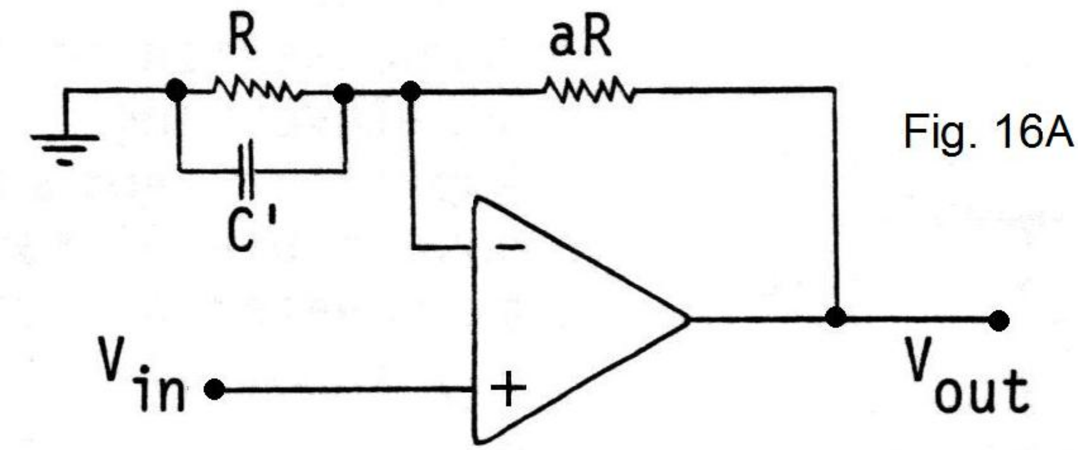


Likewise any purely passive network attached directly to an op-amp input will do whatever the passive does, and then transfers this voltage to the op-amp input (see for example Fig. 17 below). [I am reminded of the case seen several times in a student lab where a student would complain that the low-pass filter they had implemented was not rolling off, even at the max frequency of the function generator. Indeed. They had forgotten to plug in the power supply. The whole circuit, the entire breadboard, and the attached power supply all the way back to the floating unplugged supply cord was just driven up and down passively by the function generator, as monitored by the high-impedance scope input.]

With these cautions, it is also the case that non-inverting stuff can usually be compensated easily. A glance ahead to Fig. 16A and Fig. 17 shows that we just put shunt capacitors (just one) in the feedback leg. Indeed, in effect they were in a feedback leg with the inverting structures. We were not so much adding a zero to the input as we were putting a pole in the feedback.

NON-INVERTING AMPLIFIER - PASSIVE COMPENSATION

Back in Fig. 3 and Fig. 4, the frequency limitations of the non-inverting amplifier were discussed. This is certainly typical of what we deal with when designing AC amplifiers. Fig. 16A shows the method of passively compensating this non-inverting amplifier. The easiest thing here is to note that the V- input is determined purely by the feedback link from the output.



Passive Compensation of Non-Inverting Amplifier

$$V_{-} = \frac{V_{out}}{(1+a) + asC'R} \tag{21a}$$

which is the low-pass (pole) in the feedback loop we expected. The transfer function is found in the usual way as:

$$T(s) = \frac{\frac{G}{C'R}\left[\frac{1+a}{a} + sC'R\right]}{s^2 + \frac{1+a}{aC'R}s + \frac{G}{aC'R}}$$ { PASSIVE COMPENSATED NON-INVERTER } (21b)

which has a zero at:

$$s_z = -\frac{1+a}{a}\frac{1}{RC'}$$ { PASSIVE COMPENSATED NON-INVERTER } (21c)

Recall that the uncompensated real op-amp for this case had a pole at –G/(1+a) and so we want to place the zero of equation (21c) at that value, hence:

$C' = (a+1)^2/aRG$ (21d)

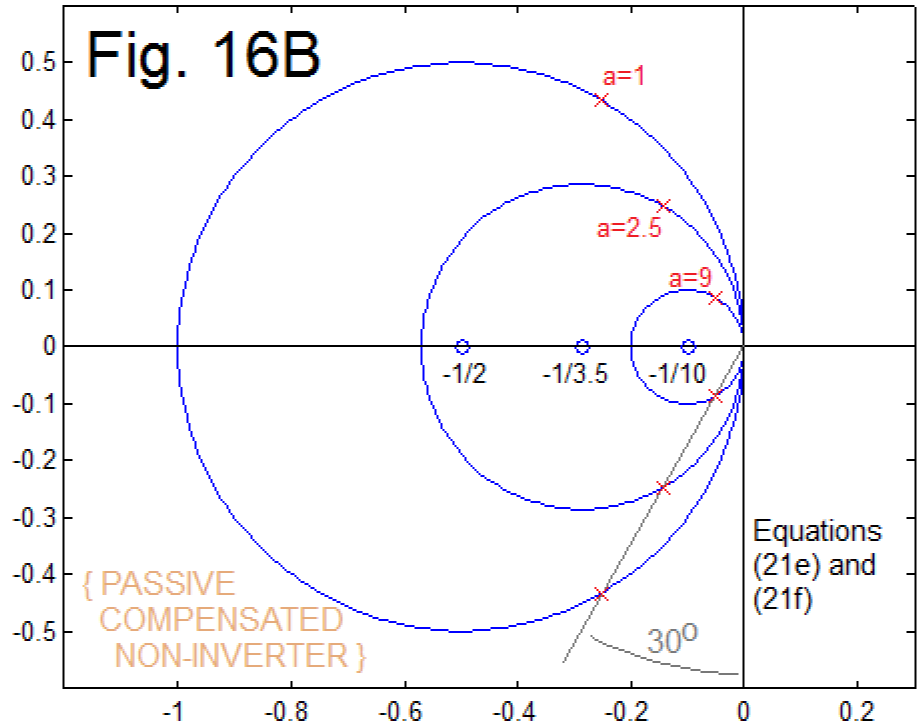
and with this choice, perhaps surprisingly, equation (21b) simplifies to:

$$T(s) = \frac{\frac{G^2}{1+a}\left[1 + \frac{(1+a)s}{G}\right]}{s^2 + \frac{Gs}{1+a} + \frac{G^2}{(1+a)^2}}$$ { PASSIVE COMPENSATED NON-INVERTER } (21e)

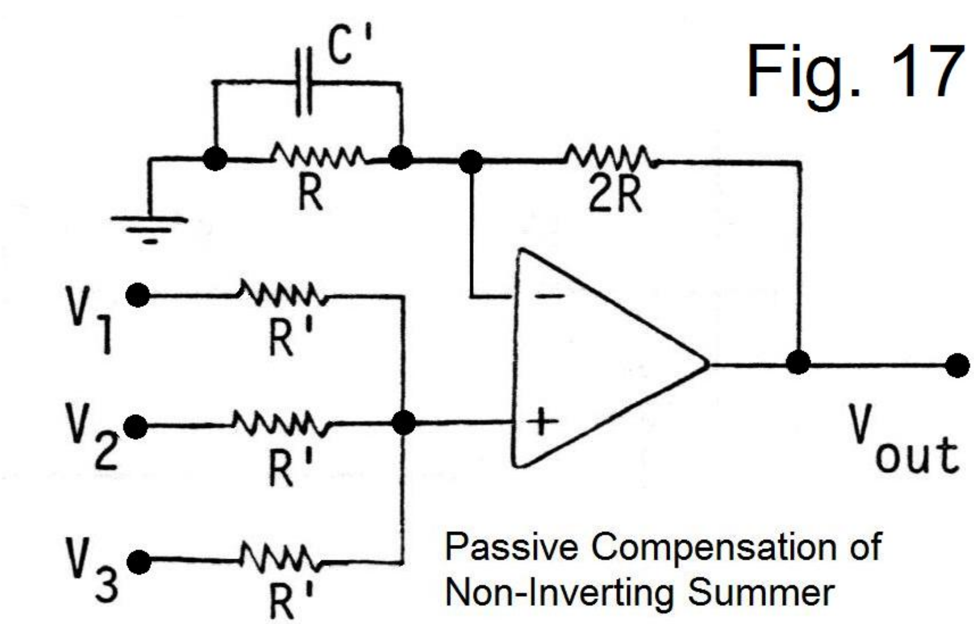
and thus the poles of equation (21b) are, as in equation (18e), at:

$$s_{p_1,p_2} = \frac{-G}{2(1+a)} \pm \frac{jG\sqrt{3}}{2(1+a)} \quad \text{\{ PASSIVE COMPENSATED NON-INVERTER \}} \qquad (21f)$$

These poles have the favorable magic circle relationship that we have come to expect. Fig. 16B shows the plot of these poles for a=1, a=2.5, and a=9. This plot seems similar to Fig. 12C, the corresponding passively compensated inverter. One difference is that here the relative positions of the poles and zeros are the same. To better see this, we have added a straight line through the poles (30°) in the third quadrant. Compare also with equations (20a(m (20b), (18c), (18f) and Fig. 12E.



The circuit of Fig 17 can be seen as a special case of Fig. 16A: the case of a=2 (a gain of 3) and with three inputs instead of just 1. It simply sums the three. The voltage V_+ is just the average of the three V_1, V_2, and V_3. This multiplied by 3 is the sum. This is an example of doing something before we apply a voltage to the op-amp input. The op-amp does not care.



NON-INVERTING AMPLIFIER - ACTIVE COMPENSATION

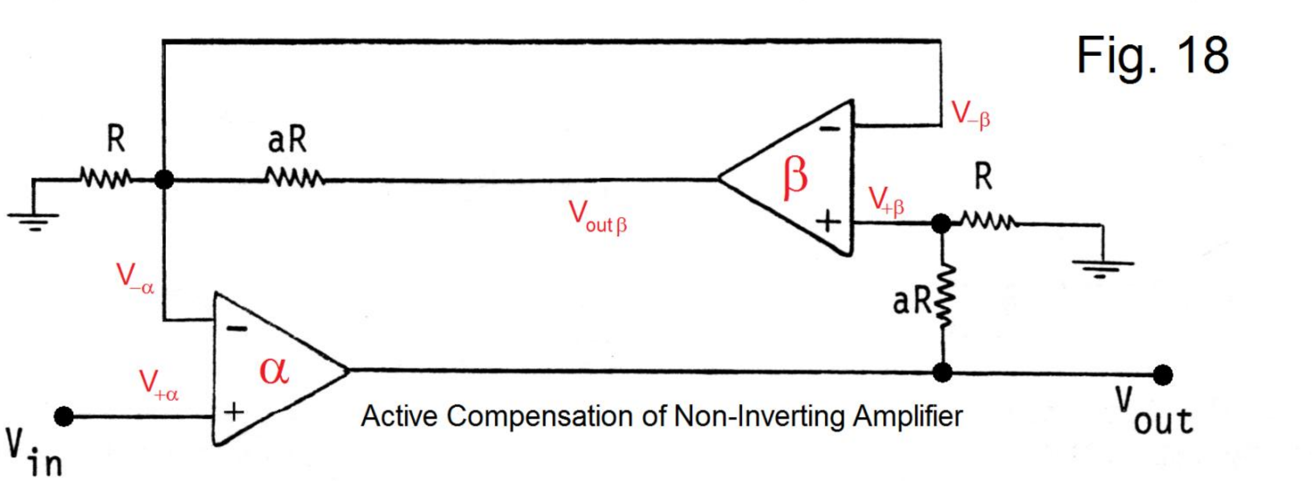


Fig. 18 shows the active compensation of the non-inverting amplifier. Compare this to Fig. 5B, which is the special case of a=0 (also remove the R resistors), the compensated follower. Fig. 12D, the actively compensated inverting amplifier is also another comparison to make. The Fig. 18 non-inverter has an advantage in using the same attenuator (the one on the left) twice. We first apply the case of the real non-inverting amplifier to β and have:

$$V_{out\beta} = V_{out}\frac{G}{s(1+a)+G} \tag{22a}$$

And then applying the real op-amp equation to α we have:

$$V_{out} = \frac{G}{s}\left[V_{in} - \left(\frac{1}{1+a}\right)\frac{GV_{out}}{s(1+a)+G}\right] \tag{22b}$$

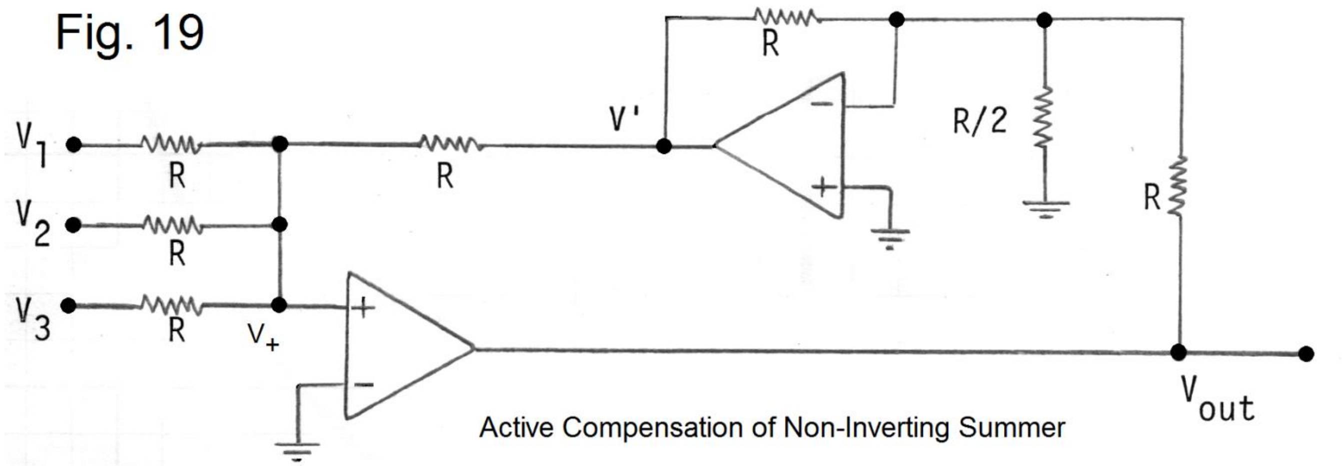
which solves for the transfer function:

$$T(s) = \frac{\left(\frac{G}{(1+a)}\right)[\,s(1+a)+G\,]}{s^2+\frac{Gs}{1+a}+\frac{G^2}{(1+a)^2}} \quad \{\text{ ACTIVE COMPENSATED NON-INVERTER }\} \tag{22c}$$

This is identical to equation (18e) for the active compensated inverter, except for a (-) sign and the additional gain of a/(1+a) in equation (18e). Equation (22c) thus has the same poles: equation (18f).

NON-NVERTING SUMMER – ACTIVE COMPENSATION

Fig. 19



Active Compensation of Non-Inverting Summer

Once we have decided to use active compensation, we agree to use a second op-amp in a feedback loop, and we can thus make this an inverter. This means that we can still use a summing node as a summer, but it will become the (+) input rather than the usual (-) summing node. (At least two Internet sources using this type of circuit have it wrong – be careful.) We then have only to match the noise gains, and this is tricky. First, note that the gain of the upper op-amp inverter is -1, as we desire. This matches the gain of 1 we want for the summer overall. So that's done. However, Consider how the voltage V' is fed back to the (-) input of the upper op-amp. The factor would be 1/2 if we did not include the resistor R/2. Notice that the resistor R/2 is connected between a summing node and ground. As an ideal op-amp, it is doing nothing. With the real op-amp, it is increasing the noise gain (adjusting it to the necessary match to the lower stage). With the R/2 resistor, only 1/4 of V' is fed back, and this matches the attenuation of V' as fed to the (+) input of the lower op-amp, because there are three resistors R to the input voltages, V_1, V_2, and V_3. If you prefer, the R/2 resistor is the two extra R resistors needed to balance the upper stage.

This understood, we note that:

$$V' = \frac{-GV_{out}}{(G+4s)} \qquad (23a)$$

and V_+ of the lower op-amp really is just the average:

$$V_+ = (V_1 + V_2 + V_3 + V')/4 \qquad (23b)$$

So using the real op-amp equation (4) we solve for the transfer function:

$$T(s) = \frac{\left(\frac{G}{16}\right)(G+4s)}{s^2+\frac{Gs}{4}+\frac{G^2}{16}} \quad \{ \text{ ACTIVE COMPENSATED NON-INVERTING SUMMER } \} \qquad (23c)$$

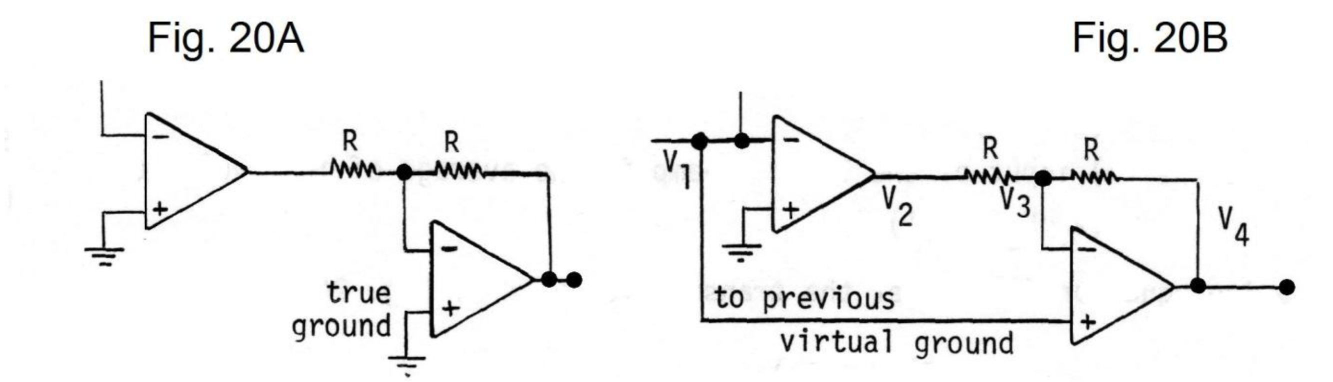
which has the magic circle poles:

$$s_{p_1,p_2} = -\frac{G}{8} \pm \frac{jG\sqrt{3}}{8} \quad \text{\{ ACTIVE COMPENSATED NON-INVERTING SUMMER \}} \qquad (23d)$$

Possibly our only objection here would be that the entire array is collapsed inward toward s=0 relative to some other cases. The lesson here is that while our summer is low gain (gain of 1) the noise gain is higher (it's 4) and thus we expect less bandwidth. However, it was the excess phase that we hoped to reduce, and that probably works just fine still.

A PHASE-FREE INVERSION !

It is far from uncommon to find that at some point we really do need an inverted signal and we fear that having just neatened things up, we are not going to undo it. After all, the inverter would seem to give us a pole at –G/2, so we would need to compensate for that if it is going to be a problem. I very neat and unexpected solution was pointed out by Brackett & Sedra [6] where a phase-free inversion is sometimes possible by obtaining a ground potential by using a virtual ground from a previous stage (the one you want to invert). The idea is shown in Fig. 20A as replaced by Fig. 20B.



Looking at Fig. 20B, the voltage V_2 is obtained from the first op-amp by:

$$V_2 = -\frac{G}{s} V_1 \qquad (24a)$$

and we see that:

$$V_3 = \frac{V_2+V_4}{2} \qquad (24b)$$

and thus:

$$V_4 = \frac{G}{s}(V_1 - V_3) \qquad (24c)$$

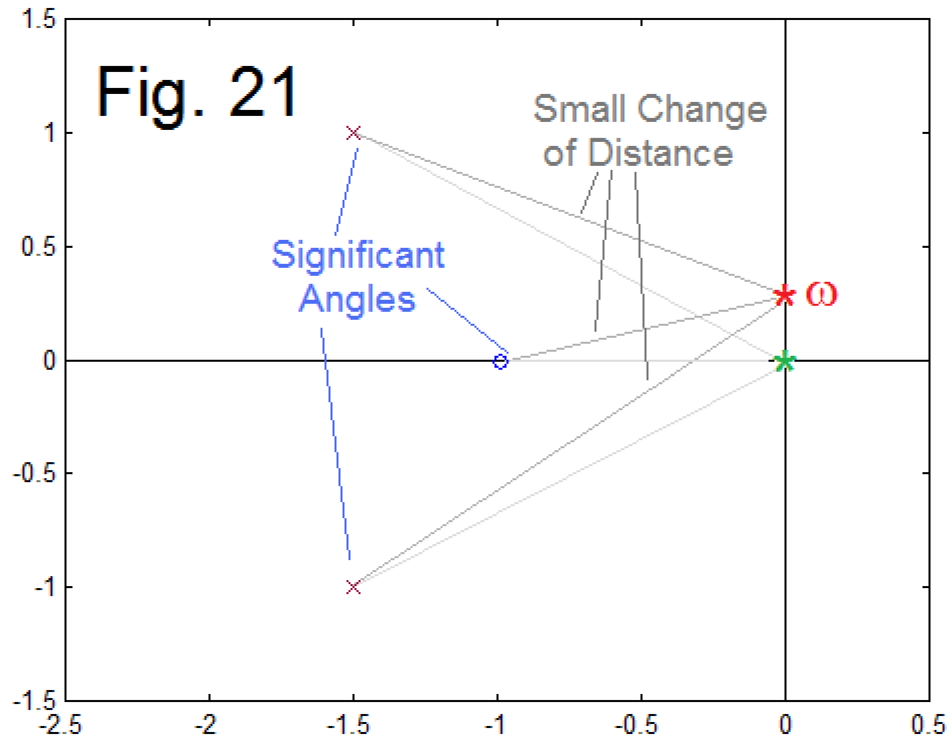
yields:

$$\frac{V_4}{V_2} = -\frac{(1+\frac{G}{2s})}{(1+\frac{G}{2s})} = -1 \qquad (24d)$$

which is the unexpected but very useful result.

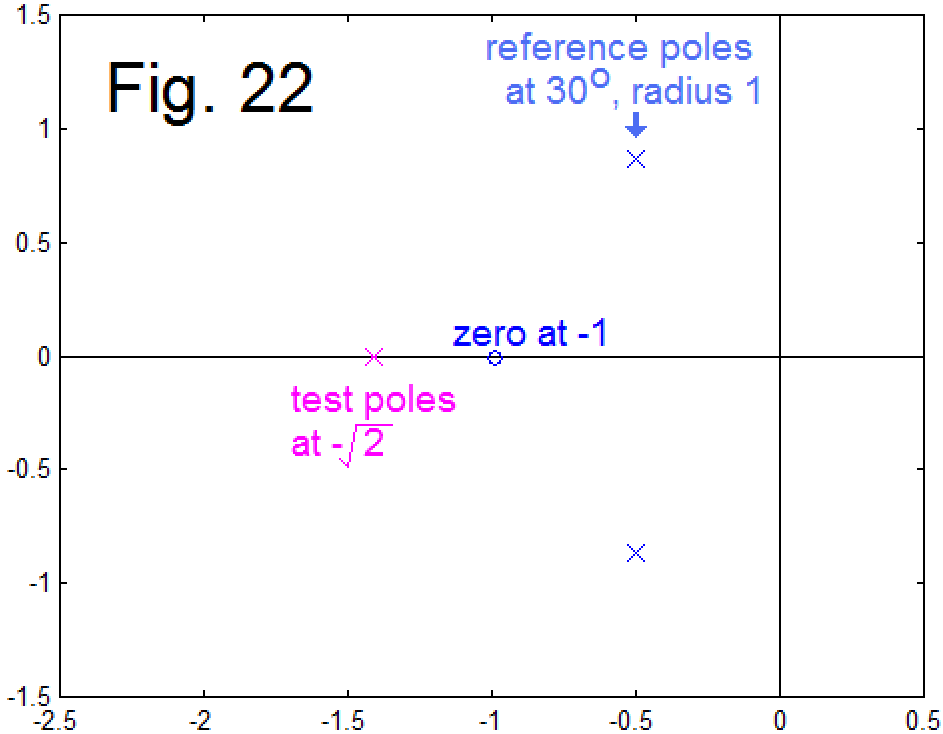
BUT WHY ALL THE ATTENTION TO PHASE - AND NOT TO MAGNITUDE?

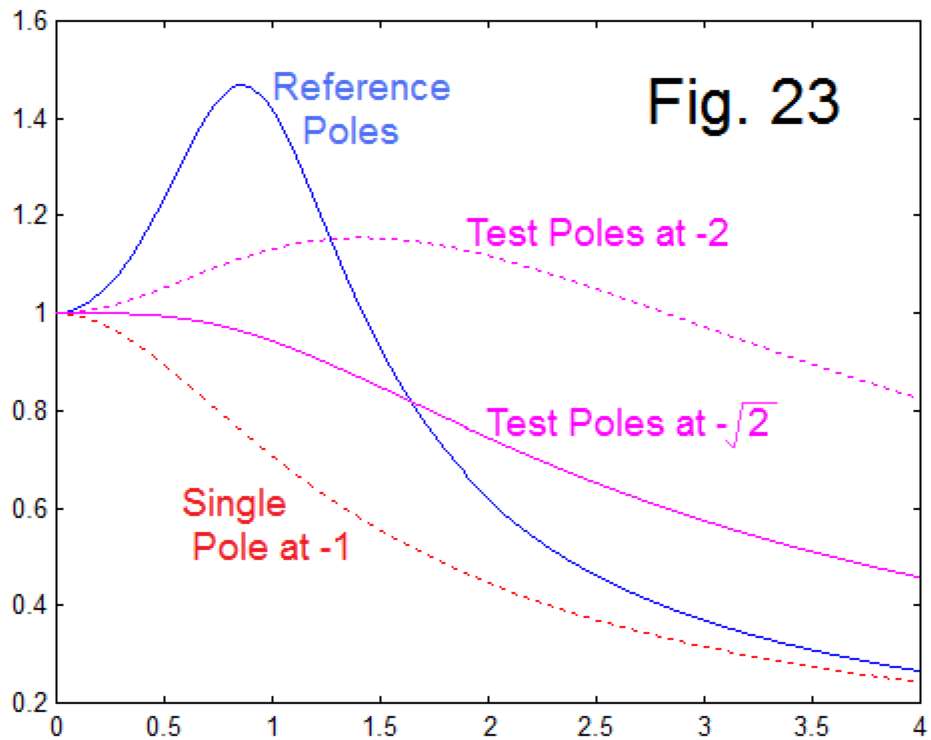
Above we have been explicit that we have been trying to improve a phase response (make it minimal) and perhaps we should say something about magnitude response. The first thing to add is perhaps, as we suggested, we don't care that much for phase response except when we go about adding two signals of the same frequency together, at which time as the resulting magnitude can vary all over the place with changes of relative phase. The second thing is that while we have tried to "correct" phase response, the same actions have tended to at least improve magnitude response directly. This is suggested in Fig. 21, a view in the s-plane.



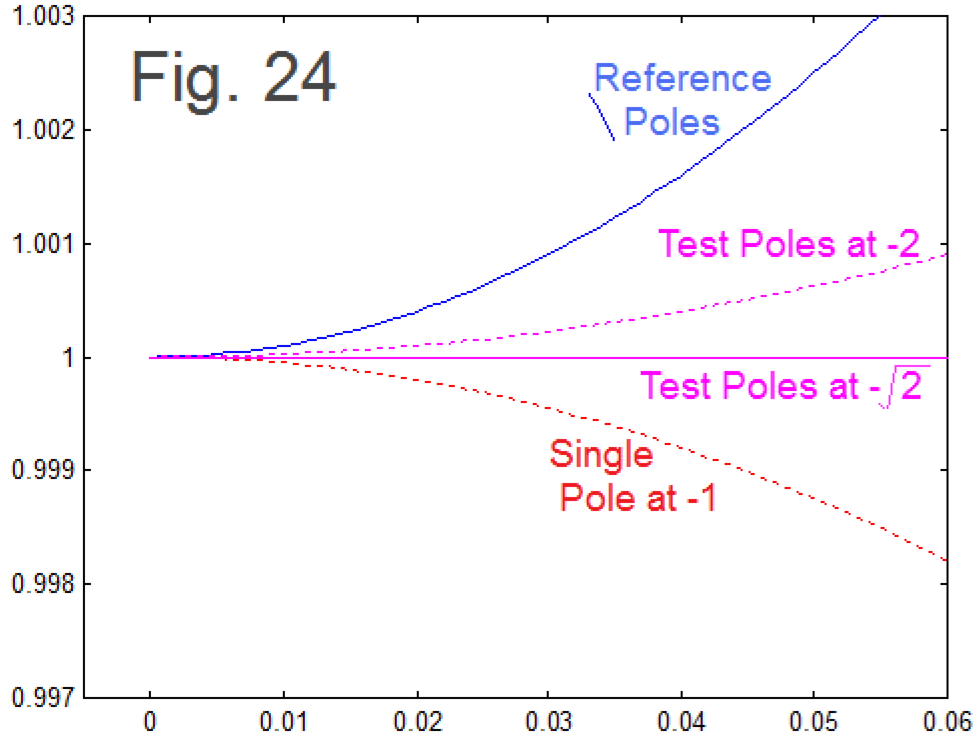
Here we have three positions in the left half plane. One is at s=-1, and the other two are conjugate positions at s=-1.5 ±j, and these are chosen as a somewhat arbitrary example. We will compare the array shown, a zero at s= -1 and poles at s= -1±j to the case where there was a single pole at -1 (a typical nominal real op-amp response). We view the frequency response as being the effects of the poles/zeros as frequency (ω) increases from s=0, moving up the jω-axis, from the green star to the red star, which is perhaps drawn larger than we might expect in a real case. The phase is the angle as seen from the singularity (pole or zero, with opposite signs) or "subtended" by the length ω . The magnitude response is proportional to the distance to the zeros divided by the distances to the poles.

When there is only a pole at s=-1, note that the angle starts to increase with increasing ω, with a non-zero initial slope (see what follows). In contrast, the distance increases as we move from 0, but starting with zero slope. This means that the phase error due to the pole is going to be the more problematic, at least initially. It was our purpose all along to replace the pole at s=-1 with an array: a zero at s=-1, and two conjugate poles. Our finding of "magic circle" positions represented success in achieving an initial phase slope of zero, the same as we already started with for the magnitude response (the lengths). The question remains - in fixing the phase, how badly have we messed up the magnitude response. Or, have we perhaps improved the magnitude response as well, although we may not have optimized it. Fig. 22 shows some actual choices of poles/zeros.





In Fig. 22, we show the zero at s=-1 and "reference poles" at $s = -\frac{1}{2} \pm \frac{j\sqrt{3}}{2}$ which has been a frequently-observed "magic circle" result which optimizes phase. We notice that the corresponding magnitude response of Fig. 23 has a pronounced bump of nearly 1.5 as we approach a frequency of 1. This we immediately compare to the dashed red curve in Fig. 23 which represented the single pole at s=-1. Thus the bump is at least an increase of overall bandwidth. As noted, in practical applications we expect frequencies that may only go up to perhaps 0.1, so we are well below the bump. For this reason, the zoom-in of Fig. 24 is very useful. As observed above, we are roughly speaking substituting a gentle rise for a gentle dip (blue for dashed red).



One key to making the poles/zeros have less influence is to keep the poles/zeros as far away as possible. In a sense, this is what we are doing when we choose an op-amp with a large GBP relative to the frequencies in our application. If we wish to have a magic-circle solution, and have the poles as far away as possible, we would put them both at s=-2. This is obvious considering a small angle approximation. In fact, we have not seem this result come out naturally, except it is approached in Fig. 11D. Indeed, in looking at Fig. 23 and/or Fig. 24, putting the test poles both at s=-2 gives an improved result (dashed magenta curves). So, keeping the incremental phase at zero, this would be the best we could expect. It is not clear how we would reutinely achieve this, if it can be done at all.

If our goal is to actually achieve as flat a response as possible, we might expect to give up on the exact phase optimization. Is there some other place to put the poles? Yes, if we put them both at $s = -\sqrt{2}$ we get the very flat response shown in Fig. 23 and especially in Fig. 24 (solid magenta curves). The geometric interpretation of frequency response shows why this value is best. However, it is not clear how to achieve this. Possibly it can't be done?

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http://electronotes.netfirms.com/ENWN1.doc

[2] "Working with Finite Gain-Bandwidth-Product Op-Amps – 4: Integrators," Electronotes Application Note 225, July 25, 1081

[3] "Active Compensation of the Inverting Integrator", Electronotes Application Note 266, Dec. 20, 1982.

[4] B. Hutchins, "Compensation of Linear Circuit Blocks for the Effects of Real Operational Amplfiers," ***Electronotes***, Vol. 15, Special Issue F, July 1986, pp 47-65

[5] B. Hutchins, "Analog Signal Processing, Chapter 7", ***Electronotes***, Vol. 20, No. 195, July 2000
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[6] "A Note on the Gain and Time Constant of Integrators," Electronotes Application Note No. 253, Mar 2003.
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[7] Brackett, P. and A. Sedra, "Active compensation for high-frequency effects in op-amp circuits with applications to active RC filters," ***IEEE Trans on Circuits and Systems***, Vol. CAS-23, (1976) pp 68-72

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APPENDIX 1 – EXPERIMENTAL TEST

I once worked with a student who was remarkable in that he was never inclined to test (by actual experiment) anything. He developed the theory and that was it. As I remember, this never got him into any trouble.* I was and am inclined to do things the other way around – at least you do test some ideas bit by bit as you proceed with a theory. So I remember in general testing some of the ideas presented above during the first (1986) iteration, but I had no record as such, published in our Electronotes stuff, or certainly not recoverable from my papers.

So after doing the old theory and a bit of new stuff here, I thought I ought to try to breadboard a few circuits. As I thought about this, I did anticipate a few thorny issues. A at the same time I recalled the way to get around them, so I do in a sense recall doing the experiments. Note that, <u>in theory, there is nothing difficult about the experiment!</u> In practice, probably most experiments have at least minor pitfalls.

The first issue is that we apparently would need to work here well above the audio range, so we can't be as sloppy at we might have preferred. Pushing that MHz button on the function generator is likely unfamiliar. I recalled that this meant that one was well-advised to work at higher gains so that bandwidths were less. Now, working at high gains also means smaller signals, not only at the input, but at the output so as not to encounter slew-limiting as well as bandwidth limitations. High gain in turn means noise and DC offset issues. So I decided to use the active compensated non-inverting amplifier with a gain of 101. The experimental setup is basically that of Fig. 18, and is shown in Fig. 25A. All I have added here is the input attenuator, which knocks the gain from V_{in} to V_{out} down from 101 to just 1. The circuit is thus unity gain overall. This means that I can keep V_{out} reasonably small without having to make V_{in} super-small. Turning down the output amplitude from the function generator can only be done just so far – hence the attenunator is convenient if not essential. You live with the noise and the DC drift.

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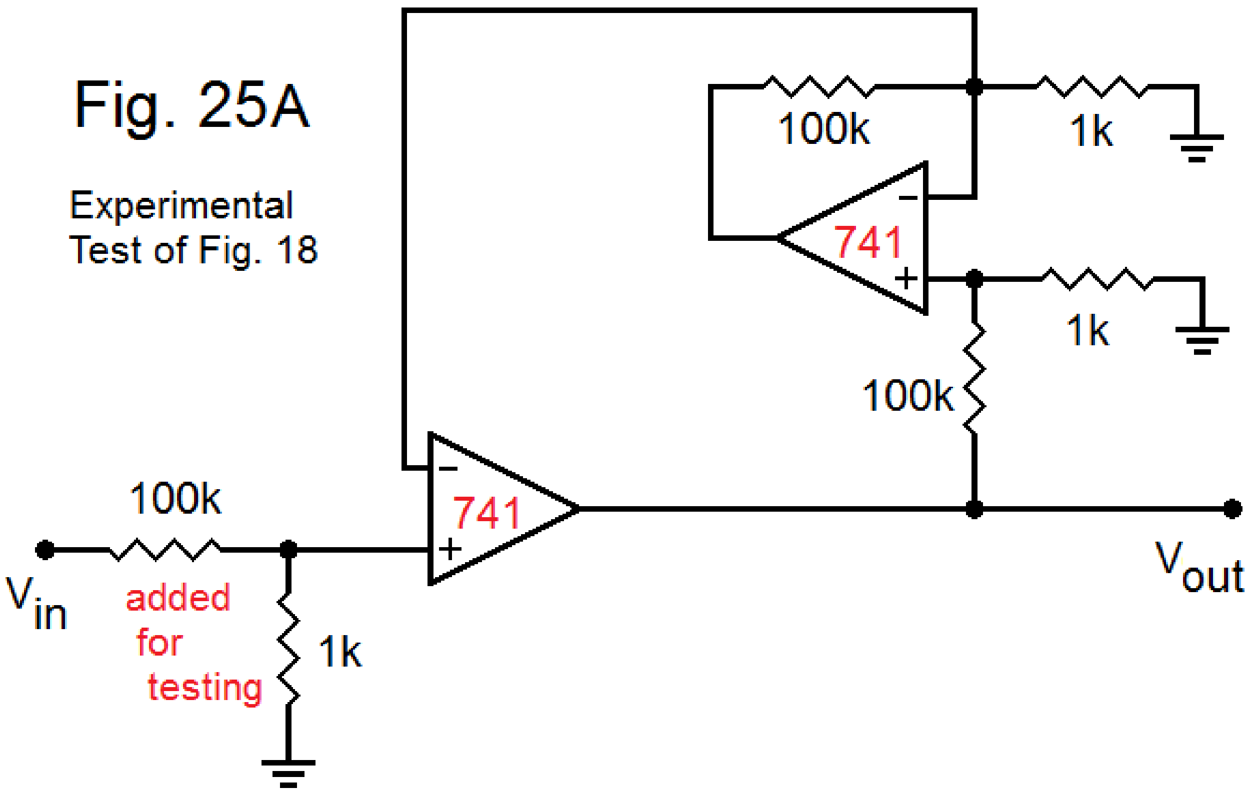
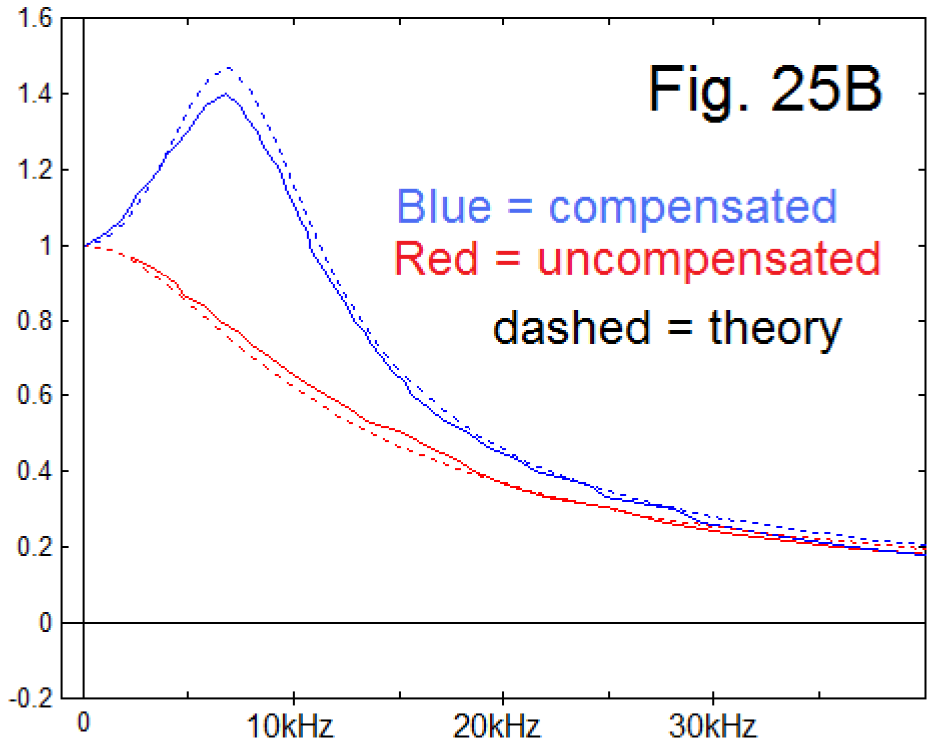
* This student was famous as well for pushing deadlines. With his thesis due at the graduate office by 4:30, at about 4:15 there were three of us at three different Xerox machines making copies and running for signatures. At 4:20 he <u>literally ran out the door</u> – the graduate office being only diagonally across from our building. He made it. All this is amusing enough, but when he came back, he proudly announced, as though it were a great discovery: "There wasn't any line." Perhaps, just perhaps, everyone else had been there hours ago, if not days ago! Readers of this newsletter may well recall the story of another Cornell grad student who had a bit of trouble getting thesis material handed in on time. In his case, he was supposed to have the material on his advisor's desk by 5 PM. With minutes to spare, he jumped on the elevator, and with nothing else to do, kept jumping as the elevator rose, to experimentally determine its resonant frequency (here was a man who valued experiment). He caused it to stall, and missed the deadline, for which he was forgiven by those who perhaps were not given the full details of his actual involvement. That was of course, Bob Moog.



Note that the rest of the circuit is the same as Fig. 18 except is drawn a bit differently with the shared attenuator looped around clockwise. The use of a gain of 101 here means that the frequencies are restricted to a few tens of kHz (1 MHz/101), which makes measurements much easier, as noted above. Measuring the uncompensated amplifier is a matter of just separately using the four resistors and op-amp on theupper right of Fig. 25A. This in turn allows us to make a measurement of the GBP by running the frequency up so that the gain drops to something like 1/5 and multiplying 101/5 by that frequency. Experimentally the GBP was about 800 kHz, a bit below the data sheet value of 1 MHz, and this was found for both real 741s and for LF13741s. This value was used in the theoretical calculation.

The results are shown plotted in Fig. 25B. The red curves are the uncompensated single-op-amp circuit, the solid curve being the experimental measurements while the dashed curve is the single-pole roll-off [essentially back to equation (5) but allowing for the input attenuation of 1/101 used here). The agreement between theory and experiment is not bad. The blue curves shown the results for the full circuit of Fig. 25A, and again we have reasonable agreement between theory and experiment. The theoretical curve is equation (22c) divided by 101 as an attenuator.



APPENDIX 2 - THE MAGIC CIRCLE

Above we have used equation (12) as the statement of a favorable array of one zero and two poles, and it has been used as a verification of the success of a method, and not as a design method. In particular, it is not a "design equation".

The "design methodology" above is to place a zero in the response, at exactly the point in the s-plane where the original uncompensated circuit had a pole, as caused by the inherent op-amp pole. This we did either passively (with a shunt capacitor) in an input leg (or similar) or actively by putting a pole (as a second op-amp) in the op-amp feedback loop. Then we looked at what we got. Equation (12) was part of the verification, not the design. The design was placing a zero in the passive case, or matching "noise-gain" in the active case.

Note that equation (12) has three parameters: r, θ, and |z|. [Alternatively, it could have been the real and imaginary parts of the poles, and |z|.] So it is an equation with three variables, much as A + B + C =13 is an equation, with an infinity of solutions (on a plane). If we chose A=4, for example, we still have an infinity of solutions, now the line B+C=9. If we were to choose two of the variable, we could calculate the remaining one.

Equation (12) is somewhat curious however. Note that if we chose the poles (two parameters) we can uniquely find |z|. On the other hand, choosing |z| (only one parameter) does not determine the poles.   Instead we still have an infinity of solutions (still have two unknowns), but poles restricted to a circle (rather than a straight line).

When we solve an equation or follow a design procedure it is usual to think in terms of things that are given, and things to be calculated.   As we said just above, what we have is a methodology and not a design formula. Further, what is given is the original pole of the op-amp. <u>but now as it is moved (a lot) by the associated network</u>. Thus we distinguish the op-amp pole (the built-in compensation) from the op-amp caused pole now found in the network that uses the op-amp. Our procedure is to take this pole information for the original uncompensated network using the <u>real</u> op-amp (not an ideal op-amp), and first convert it into the information on a zero, a zero we want to find in the final compensated network. Very simply, we want a <u>zero exactly where the pole of the associate network was found</u>. So, while equation (12) is written for the position of the <u>final zero</u>, it can equally well or better be thought of as the associated <u>original network pole</u>.

Something is missing though.   How does <u>one number</u> (the original real pole) become <u>two numbers</u> (the real and imaginary parts of two poles)? On a specific circle – yes – but why at a particular angle? Reversing out thoughts, if we choose a pair of complex conjugate poles and calculate the one real pole [that is, actually |z| using equation (12)], why is equation (12) “the boss”?

One clue is the network, while G comes in of course, we have parameters such as a gain factor a and the time constant RC’ of the shunt. It has to be the associated passive network. Observe that when these parameters appear above, they are in the denominator polynomial and thus in the pole equations. Lets consider once again the inverting amplifier (Fig. A2-1), which is really just Fig. 12B again.

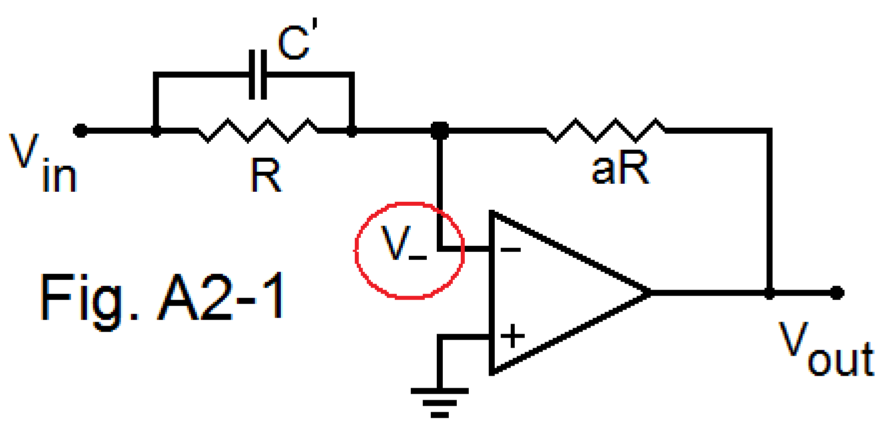


Fig. A2-1

For the moment, we just want to see the effect of the shunt capacitor C' on the voltage V_- as it is passed to the inverting input. We recall that the impedance of C' in parallel with R is R/(1+sC'R). Suppose first that the op-amp is ideal. We have just an ideal inverter, so:

$$T(s) = \frac{-aR}{\frac{R}{1+sC'R}} = -a(1+sC'R) \tag{A2-1}$$

This is a strange transfer function in that we are certainly not accustomed to finding one with no pole(s). This has only the zero at -1/C'R. We are accustomed, however, to finding a zero as placed by this shunt capacitor. But the finding is artificial – this is after all set to be an ideal op-amp. V_- had no choice except to be zero.

There is a big difference if we do not require V_- to be a virtual ground, as it never really is. We will be using superposition as we have used extensively above, but here with a care to looking at the separate paths for whether they contain poles and zeros. It is convenient to use the two circuits of Fig. A2-2 and A2-3.

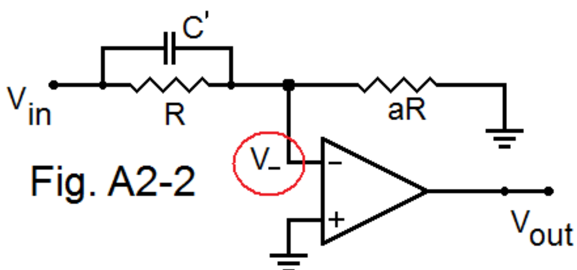


Fig. A2-2

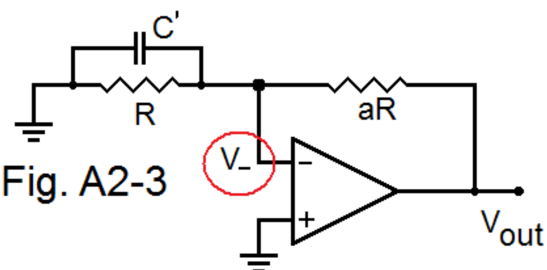


Fig. A2-3

The path form V_{in} to V_- shown in Fig. A2-2 is the voltage divider:

$$\frac{V_-}{V_{in}} = \frac{aR}{aR + \frac{R}{1+sC'R}} = \frac{\left(\frac{1}{RC'}\right)(1+sC'R)}{s+\frac{(a+1)}{a}\frac{1}{RC'}} \tag{A2-2}$$

while the path form V_{out} to V_- shown in Fig. A2-3 is the voltage divider:

$$\frac{V_-}{V_{out}} = \frac{\frac{R}{1+sC'R}}{aR + \frac{R}{1+sC'R}} = \frac{\left(\frac{1}{aRC'}\right)}{s+\frac{(a+1)}{a}\frac{1}{RC'}} \tag{A2-3}$$

We thus see that the path from Vin to V- has a zero, while the path from Vout back to V- does not, but both have the same pole. Both use the same series of impedances, so we expect the same poles. We do not need to redo the remainder of the calculation here, but when we do, we get equation (17b) as before. Notice that all three of the parameters, the G of the op-amps, the resistor spread a, and the RC' time constant are involved in the calculation of the poles.

APPENDIX 3 - MATLAB PROGRAMS

The programs below were used to help generate many of the figures in this report, as noted in the comments at the top of the program code. These are included just to compete the record. The code has not been optimized or refined.

************************ Program 1 ***

```
 %   op-amp.m      makes Fig. 2 and Fig. 4 of EN#214
f=1:2:25000;
fp=10
GBPHZ=100000

sq=f.^2/fp^2;
d=sqrt(1+sq);
T=1./d;
T=T*GBPHZ;
figure(1)
loglog(f,T)
hold on
plot([fp fp],GBPHZ*[0.0000001 10],'r')
plot([1 fp],GBPHZ*[0.707 0.707],'c')
axis([-2000 52000 GBPHZ*10^(-4) GBPHZ*1.5])
hold off

GBPHZ=1000000
alpha=3000;
sqq=((f.^2)*(alpha^2))/(GBPHZ^2);
dd=sqrt(1+sqq);
T3000=alpha./dd;

figure(2)
loglog(f,T)
hold on
loglog(f,T3000,'g')
plot([fp fp],GBPHZ*[0.0000001 fp],'r')
plot([333 333],GBPHZ*[0.0000001 10],'r:')
plot([1 fp],GBPHZ*[0.707 0.707]/10,'c')
plot([1 GBPHZ/alpha],alpha*[0.707 0.707],'c')
axis([-2000 52000 GBPHZ*10^(-5) GBPHZ*.2])
hold off
figure(2)

sq=333^2/fp^2;
d=sqrt(1+sq);
T10333=1/d;
T10333=T10333*GBPHZ

alpha=3000;
sqq=((333^2)*(alpha^2))/(GBPHZ^2);
dd=sqrt(1+sqq);
T3000333=alpha/dd

T10333/T3000333
```

************************** Program 2 ***

```
%  opampcomp.m   makes 6A, 6B, 7A, and 7B of EN#214
z=-1
p1=-1/2+j*sqrt(3)/2
p2=-1/2-j*sqrt(3)/2

n=poly(z)
d=poly([p1 p2])
w=[0:.001:4];
H=freqs(n,d,w);

p0=-1
n0=1
d0=poly(p0)
H0=freqs(n0,d0,w);

figure(1)
plot(w,abs(H))
hold on
plot(w,abs(H0),'r:')
plot([1 1],[-1 3],'k:')
plot([0 5],[0 0],'k')
plot([0 0],[-1 3],'k')
plot([0 1],[sqrt(2) sqrt(2)],'g:')
plot([0 1],[1/sqrt(2) 1/sqrt(2)],'g:')
axis([-.2 4.2 -.2 2.0])
hold off

figure(2)
plot(w,180*angle(H)/pi)
hold on
plot(w,180*angle(H0)/pi,'r:')
plot([1 1],[-100 10],'k:')
plot([0 5],[0 0],'k')
plot([0 0],[-100 10],'k')
plot([0 1 ],[-45 -45],'g:')
axis([-.2 4.2 -100 10])
hold off
figure(1)

w(1001)
abs(H(1001))
abs(H0(1001))
180*angle(H(1001))/pi
180*angle(H0(1001))/pi

 figure(3)
plot(w,abs(H))
hold on
plot(w,abs(H0),'r:')
plot([1 1],[-1 3],'k:')
plot([0 5],[0 0],'k')
plot([0 0],[-1 3],'k')
plot([0 1],[sqrt(2) sqrt(2)],'g:')
plot([0 1],[1/sqrt(2) 1/sqrt(2)],'g:')
axis([-.02 0.12 0.98 1.02])
hold off
```

```
figure(4)
plot(w,180*angle(H)/pi)
hold on
plot(w,180*angle(H0)/pi,'r:')
plot([1 1],[-100 10],'k:')
plot([0 5],[0 0],'k')
plot([0 0],[-100 10],'k')
plot([0 1 ],[-45 -45],'g:')
axis([-.02 0.12 -10 2])
hold off

w=[0:.05:400];
H=freqs(n,d,w);

p0=-1
n0=1
d0=poly(p0)
H0=freqs(n0,d0,w);

figure(5)
loglog(w,abs(H))
hold on
plot(w,abs(H0),'r:')
plot([1 1],[-1 3],'k:')
plot([0 400],[0 0],'k')
plot([0 0],[-1 3],'k')
plot([0 1],[sqrt(2) sqrt(2)],'g:')
plot([0 1],[1/sqrt(2) 1/sqrt(2)],'g:')
axis([-.2 420 -.0002 2.0])
hold off

figure(1)
```

******************** Program 3 **

```
% comptest.m    makes Fig. 11D of EN#214

G=1
oneoverRC=0.001
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
a=(180/pi)*atan(real(poles)./imag(poles))
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
figure(1)
ca=0:pi/100:2*pi;
c=exp(j*ca)-1;
plot(real(c),imag(c),'g')
hold on
plot(-1,0,'ob')
plot(poles,'rx')

oneoverRC=0.1
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
polesnorm=poles./abs(poles)
a=(180/pi)*atan(real(poles)./imag(poles))
```

```
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
plot(poles,'rx')

oneoverRC=0.3
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
polesnorm=poles./abs(poles)
a=(180/pi)*atan(real(poles)./imag(poles))
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
plot(poles,'rx')

oneoverRC=.7
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
polesnorm=poles./abs(poles)
a=(180/pi)*atan(real(poles)./imag(poles))
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
plot(poles,'rx')
oneoverRC=1
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
polesnorm=poles./abs(poles)
a=(180/pi)*atan(real(poles)./imag(poles))
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
plot(poles,'rx')

oneoverRC=2
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
polesnorm=poles./abs(poles)
a=(180/pi)*atan(real(poles)./imag(poles))
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
plot(poles,'rx')

oneoverRC=2.999
poles=roots([1 G+oneoverRC G*(G+oneoverRC)])
polesnorm=poles./abs(poles)
a=(180/pi)*atan(real(poles)./imag(poles))
a=(pi/180)*a
r=abs(poles)
test=2*G*sin(a)
plot(-1,0,'ob')
plot(poles,'rx')
plot([0 0],[-3 3],'k')
plot([-3 1],[0 0],'k')
plot([-.5 -.5],[-sqrt(3)/2 sqrt(3)/2],'r:')
axis('square')
axis([-2.5 0.5 -1.5 1.5])

hold off
figure(1)
```

*********************************** Program 4 **

```
%comptest2.m   makes Fig. 12C of EN#214

G=1
a=10
R=10000
Cprime=(a+1)/(R*G)
poles1=roots([1 (1+a)/(a*Cprime*R) G/(a*Cprime*R)])
r=G/(a+1)
ac=0:pi/100:2*pi;
c=r*(-1+exp(j*ac));
figure(1)
plot(real(c),imag(c),'g')
hold on
plot(-r,0,'ob')
plot(poles1,'xr')
plot([0 0],[-3 3],'k')
plot([-3 3],[0 0],'k')
axis([-3*r 0.5*r -1.5*r 1.5*r])
axis('equal')
hold off
figure(1)
%
pole11=poles1(1)
pole21= -(1+a)/(2*a*R*Cprime) + (j/(2*a*R*Cprime))*sqrt(4*G*R*Cprime*a - (1+a)^2)
pole31=-G/(2*a) + (j*G)/(2*a)*sqrt((3*a-1)/(a+1))
%
% Multiple Plots
figure(2)
G=1
a=10
R=10000
Cprime=(a+1)/(R*G)
poles1=roots([1 (1+a)/(a*Cprime*R) G/(a*Cprime*R)])
r=G/(a+1)
ac=0:pi/100:2*pi;
c=r*(-1+exp(j*ac));
plot(real(c),imag(c),'g')
hold on
plot(-r,0,'ob')
plot(poles1,'xr')
plot([0 0],[-3 3],'k')
plot([-3 3],[0 0],'k')
%
%
G=1
a=2.5
R=10000
Cprime=(a+1)/(R*G)
poles1=roots([1 (1+a)/(a*Cprime*R) G/(a*Cprime*R)])
r=G/(a+1)
ac=0:pi/100:2*pi;
c=r*(-1+exp(j*ac));
%
```

```
plot(real(c),imag(c),'g')
plot(-r,0,'ob')
plot(poles1,'xr')
plot([0 0],[-3 3],'k')
plot([-3 3],[0 0],'k')
%
%
G=1
a=1
R=10000
Cprime=(a+1)/(R*G)
poles1=roots([1 (1+a)/(a*Cprime*R) G/(a*Cprime*R)])
r=G/(a+1)
ac=0:pi/100:2*pi;
c=r*(-1+exp(j*ac));
%
plot(real(c),imag(c),'g')
hold on
plot(-r,0,'ob')
plot(poles1,'xr')
plot([0 0],[-3 3],'k')
plot([-3 3],[0 0],'k')
axis([-2.5*r 0.5*r -1.5*r 1.5*r])
%
axis('equal')
hold off
figure(2)
```

******************************** Program 5 **

```
% passcompinvsum.m   makes Fig. 13C of EN#214
p=roots([1 1/2 1/6])
magz=(1/3)
r=abs(p)
rcheck=2*magz*cos(angle(p'))
ca=0:pi/100:2*pi;
c=-1/3+1/3*exp(j*ca);
figure(1)
plot(c)
hold on
plot(p,'xr')
plot(-1/3,0,'ok')
plot([0 0],[-2 2],'k')
plot([-2 2],[0 0],'k')
axis('equal')
axis([-0.7 0.2 -.4 .4])
hold off
figure(1)
```

******************************** Program 6 ***

```
% EN214Fig16.m   Makes Fig. 16B of EN#214
a=1
G=1
p1=-G/(2*(1+a)) + j*G*sqrt(3)/(2*(1+a))
p2=p1'
sz=-G/(1+a)
r=abs(sz)
ca=0:pi/100:2*pi;
c=-r+r*exp(j*ca);
figure(1)
plot(c)
hold on
plot([-2 2],[0 0],'k')
plot([0 0],[-2 2],'k')
plot(sz,0,'ob')
plot([p1 p2],'xr')
axis('equal')

 a=2.5
G=1
p1=-G/(2*(1+a)) + j*G*sqrt(3)/(2*(1+a))
p2=p1'
sz=-G/(1+a)
r=abs(sz)
ca=0:pi/100:2*pi;
c=-r+r*exp(j*ca);
figure(1)
plot(c)
hold on
plot([-2 2],[0 0],'k')
plot([0 0],[-2 2],'k')
plot(sz,0,'ob')
plot([p1 p2],'xr')
axis('equal')
axis([-1.2 0.3 -0.6 0.6])

a=9
G=1
p1=-G/(2*(1+a)) + j*G*sqrt(3)/(2*(1+a))
p2=p1'
sz=-G/(1+a)
r=abs(sz)
ca=0:pi/100:2*pi;
c=-r+r*exp(j*ca);

figure(1)
plot(c)
hold on
plot([-2 2],[0 0],'k')
plot([0 0],[-2 2],'k')
plot(sz,0,'ob')
plot([p1 p2],'xr')
axis('equal')
axis([-1.2 0.3 -0.6 0.6])
hold off
figure(1)
```

****************************** Program 7 **

```
%magtest2.m    makes Fig. 22, Fig 23, and Fig. 24 of EN#214
%  begin with Told=zeros(1,4001)
%  test poles realp, imagp
%  note: output Told required for next iteration
%  magenta is result of test poles, dotted magenta is last trial Told
%  intersting results put poles both at -2, or at - sqrt(2)

function Told=magtest2(Told,realp,imagp)
clf
r=1
theta=30
w=0:.001:4;
T0=1./sqrt(1+w.^2);
theta=theta*pi/180;

figure(1)
% reference poles at 30 degrees
p1=-r*sin(theta)+j*cos(theta)
p2=-r*sin(theta)-j*cos(theta)
plot([p1 p2],'xb')
hold on
plot([0 0],[-10 10],'k')
plot([-10 10],[0 0],'k')
% holding pole/zero plot

Tz=abs(1+j*w);
Tp1=abs(p1+j*w);
Tp2=abs(p2+j*w);
T=Tz./(Tp1.*Tp2);

% test poles
p1=realp+j*imagp
p2=realp-j*imagp
figure(1)
plot([p1 p2],'xm')
plot(-1,0,'ob')
hold off
axis([-2.5 0.5 -1.5 1.5])

Tz=abs(1+j*w);
Tp1=abs(p1+j*w);
Tp2=abs(p2+j*w);
T1=Tz./(Tp1.*Tp2);

T1=T1/T1(1);

% now plot magnitudes
figure(2)
plot(w,T0,'r:')
hold on
plot(w,T,'b')
plot(w,T1,'m')
plot(w,Told,'m:')

hold off
```

```
% same plot - different axis
figure(3)
plot(w,T0,'r:')
hold on
plot(w,T,'b')
plot(w,T1,'m')
plot(w,Told,'m:')
hold off
axis([-.005 .06 .997 1.003])
figure(3)

%
Told=T1;
```

*   *   *   *   *   *   *   *   *   *