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When Analog Signal Processing was written more than 10 years ago, I did not suppose that Chapter 7, the chapter on sensitivity would be the longest (by far) in the text. In fact, if one thinks of a chapter on passive sensitivity followed by one on active sensitivity, we could avoid the long chapter, but as stated then (and true now) we want the reader to think in terms of both, simultaneously, as they often trade off against each other. All and all, this is an important chapter.

In keeping with our past few issues that included chapters of ASP, we are adding some additional analog material. In this issue, it was a simple choice as to what we needed to do. This was the matter of clearing up a loose end omitted from Chapter 7. If you wish, think of it as an extension of Chapter 7 - read Chapter 7 first, and then the new discussion.

ASP will conclude next issue, with Chapters 8, 9, and 10, to include voltage-controlled filters, delay line filters (in a sense, an introduction to digital filters), and analog adaptive filters.

ACTIVE SENSITIVITY IN A COMPENSATED STATE-VARIABLE FILTER

-by Bernie Hutchins

INTRODUCTION

The installment of Analog Signal Processing in this issue is Chapter 7, which deals with passive and active sensitivity. There is a good deal of practical information in this chapter, but in another sense, we have left something out from a theoretical viewpoint, largely as a consequence of algebraphobia. This we will try to remedy here.

Specifically, two approaches to active sensitivity problems have been used. In the first approach, we solve for the transfer function of a particular network using a model of a real (rather than an ideal) op-amp. The "G/s" model for an op-amp has proven useful:

$$Vout = (G/s)(V_+ - V_-) \tag{1}$$

In this approach, a standard, ideal, one op-amp, second-order network (e.g., Sallen-Key or M.F.I.G.), becomes third-order. Typically we find one extra pole

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CHAPTER 7

PASSIVE AND ACTIVE SENSITIVITY

7-1 Introduction

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7-5 Active Sensitivity of Second-Order Filters

7-6 Sensitivity of State-Variable Filters

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7-1 INTRODUCTION:

The area of active filtering, like many others we encounter in engineering, is one where adequate attention must be paid to the differences between theory and practice. Here we are concerned with how well an actual realization of an active filter will match its theoretical counterpart. There are several aspects to this problem that should be considered. First, there are considerations of "passive sensitivity." Any actual construction of an active filter will differ in performance from the ideal calculated response because the R and C component values will not be exact, but will have a "tolerance" associated with their nominal value. Secondly, the op-amps used in actual construction will be non-ideal, and this has an effect on performance which can be particularly important at high frequencies and/or for high Q sections.

Passive sensitivity is basically a measure of how much a variation in a circuit R or C will change a performance parameter such as the cutoff frequency or the Q of the filter. In general, we look for passive sensitivity values that are zero or small constants, and dislike values that increase with Q, or worse as Q^2. Since we do not know ahead of time exactly what the components will be, but only know them statistically, sensitivity calculations are used to tell us basically how good or how bad things might be. In any one case, variations of performance from nominal might be very small, or component variations might have offsetting effects that would leave the performance near nominal. On the other hand, the individual component variations might "conspire" to make things very bad. Passive sensitivity calculations are thus often used as an overall guide to the relative merits of different configurations. They may also be used, however, in cases where individual components in separate filters need to be "tweaked" in order to tune the particular filter unit. In such cases, the necessary adjustment to a particular component's value, so as to adjust the performance parameter, can be obtained from the passive sensitivity value for that component.

Passive sensitivity calculations may require some recalculation of the filter's transfer function and design equations. This is because, nominally, we may have set two resistors equal, for example, and we now need to assume that they have (at least slightly) different values, R_1 and R_2, say. This is necessary because even nominally equal resistors or capacitors are found in different positions in the network, and their effects on performance may differ. We need to identify which of the two resistors, R_1 or R_2, which are nominally both equal to R, we are talking about. In general, we try to express sensitivities as functions of the Q of the filter. Accordingly, calculations of passive sensitivity are somewhat of an "art" and not just a matter of taking partial derivatives. Proper attention should be paid to the way things are done.

Active sensitivity is somewhat less statistical, and somewhat more subtle. In going from ideal op-amps to real op-amps, we need to start all over in our calculations. Instead of assuming that the op-amp gain is infinite (that $V_+=V_-$), we assume that the gain is approximately G/s, where G is the "gain-bandwidth product" in rad/sec. In general, the factor of 1/s increases the filter's order by one. A second-order filter thus becomes a (rather annoying) third order. While the extra pole, introduced by the op-amp, may be relatively far away, there is usually an associated perturbation of the nominal poles which can be very important. At times we can allow for this change of performance by "overdesigning" the filter in the first place, and then allowing it to "drop back" to something close to nominal. At other times, the individual circuit blocks can be individually compensated. Note that it is not usually the finite value of G that is the problem, but rather the fact that the gain is frequency dependent as G/s. Note that there is also a "passive sensitivity" associated with G as it varies among individual op-amps, and we sometimes need to be aware of this.

Any design must be considered for both its passive and its active sensitivities. Both must be satisfactory. Because low levels of passive sensitivity are often related to extensive negative feedback, requiring large gains from the active elements, low passive sensitivity may be associated with high active sensitivity. Choice of a useful configuration may thus well be a question of finding the correct balance between passive and active sensitivity, also considering matters of design ease and tuning. In addition, a complete design should also consider the extra cost of components with more favorable tolerance values, and of higher G op-amps, if these are needed.

While active sensitivity is generally concerned with the non-ideal, non-infinite, non-frequency-independent gain of the op-amp, most of the other non-ideal parameters of real op-amps are of less concern. One exception may be the finite slew rate of an op-amp. This slew limiting may result in a particular phenomenon known as "jump resonance" which is particularly noticable at higher frequencies and high output levels in high-Q filters. This phenomenon, due to the effective non-linearity due to slew rate limiting, manifests itself in terms of instantaneous jumps in output level, and an associated double valued frequency response curve. Such a situation must be avoided by getting a faster op-amp or by cutting the amplitude levels.

In this chapter, we will look at passive sensitivity first. Then we will look at the real op-amp, and the way the finite gain-bandwidth product affects the design of amplifiers. This op-amp model is then applied to second-order filters, and the poles of the resulting third-order filters can be determined. Finally, compensation methods for op-amp amplifiers and integrators will be studied.

7-2 PASSIVE SENSITIVITY

Active filters employ passive components, resistors and capacitors, in addition to active elements such as op-amps. Both the passive and the active elements may be non-ideal to a degree, and be non-nominal to a degree; in the case of a practical realization. For a complete study of an active filter design, both the passive and the active variations must be considered. Here we will be looking at passive sensitivity, considering the resistors and the capacitors to be ideal, but not nominal in value. Associated with actual "off the shelf" examples of passive components is a degree of uncertainty about the actual value as compared to the nominal value stamped on the component. When we look for an active filter structure with low passive sinsitivity, we are looking for a design where these expected variations from nominal (known as "tolerances") are of relatively minor importance.

When we complete the design of a filter, we generally have the components determined by calculator or computer to more decimal places than we could expect to make use of. For example, a resistor might be calculated to 21.145692 k. We don't expect to obtain this value when building our circuit, but would rather look to see what close values a manufacturer offers. In this case, a value of perhaps 22k would be available with ±5% tolerance. However this tolerance value alerts us to the fact that even though we calculated a value of 21.145692k, and have agreed to settle for 22k, we will get yet a third value that is within 5% of 22k, perhaps 22.337576k, which we will not even know unless we accurately measure each and every individual resistor to be used.

Note that if we did measure each and every component in a filter to be built, we could then plug these values back into our design equations and find out how much the performance varies from nominal. For example, we might be designing a bandpass for a center frequency of 1000 Hz and a Q of 100. If we then measure the values of the components that are actually to be used, we could calculate back and perhaps find a center frequency of 980 Hz and a Q of 78 would be achieved. This might or might not be satisfactory, depending on the application. Alternatively, the filter could simply be built and tested. At times, a combination of these sort of individual measurments and evaluations can be useful, but in general we can not justify the efforts. Instead, we look for structures that are already insensitive to component variation, so that off-the-shelf components will almost always result in a satisfactory realization which is under our control. This is where passive sensitivity calculations come in.

First we should be clear that although a passive component may be within 5% of a nominal value, it is not the case that a performance parameter which depends on this component, will also be within 5%. It might be significantly less, and it could well be much much worse. The reasons for this will be more apparent when we look at the mathematics of sensitivity, but for now, note that it is true.

We might know for example that the characteristic frequency of a filter depends on a particular resistor R*. Heuristically we might then take the formula for center frequency and see how much a 1% change in R* above, and then below nominal would change the center frequency. If it is significantly less than 1%, we would be pleased so far. If it is significantly larger than 1%, we would be worried. In fact, sensitivity is a measure of a percentage change of a performance parameter to a percentage change in a component, as will be seen in the mathematical formulation. Passive sensitivities will be used as a comparative means for evaluating active filter networks, and in a few cases for precise tuning methods. Note that this is somewhat different from what we call a "worst case" analysis. Low (or high) sensitivity is a figure of merit that serves as a basis for further considerations.

Mathematically we can look at sensitivity by starting with a performance parameter X (such as characteristic frequency of filter "Q") that depends on some passive component y (such as resistors and capacitors) by some design equation of the form:

$$X = X(y, \ldots\ldots) \qquad (7\text{-}1)$$

where y,...., etc. are additional passive components not under consideration yet. Note that this design equation depends on the particular active filter configuration used. The passive sensitivity is then defined as:

$$S_y^X = \frac{y}{X}\frac{\partial X}{\partial y} \qquad (7\text{-}2)$$

where S^X_y is "the sensitivity of X with respect to y", and note that a partial derivative is taken, thus concentrating on the dependence of X on y, effectively holding the other components constant.

While equation (7-2) is exactly what we use to calculate sensitivity, note that (7-2) can be written approximately as:

$$S_y^X \simeq \frac{y}{X}\frac{\Delta X}{\Delta y} = \frac{\Delta X/X}{\Delta y/y} \qquad (7\text{-}3)$$

which indicates that we are talking about a fractional (or percentage) variation in the parameter, relative to a fractional (percentage) change in a component.

An obvious "benchmark" for sensitivity is therefore a value of magnitude 1. Sensitivities of magnitude less than 1 indicate that the performance parameter varies "slower" than the component itself, which is a generally good situation. Sensitivities of magnitude greater than 1 indicate that the parameter varies faster than the component itself, which is generally bad. Note that for most useful filter structures, we would expect to see most or all the sensitivities significantly less than magnitude 1.

The sensitivity often depends simply on an exponent in equation (7-1). If for example X depends on y as y^m:

$$X = ky^m \qquad (7\text{-}4)$$

where k is a constant representing the other components, then:

$$S_y^X = m \qquad (7\text{-}5)$$

So it is often possible to write sensitivities by inspection. Note that it is common to find a characteristic frequency that depends on $1/\sqrt{y}$ so $S_y^{f_c} = -1/2$ is commonly seen.

It is certainly not the case that all sensitivities come out as constants, as some may well depend on other passive components in the network. In such a case, a numerical value can be obtained by putting in the nominal values of these components. Note that these components usually come in as the "(y/X)" normalization of the sensitivity calculation in equation (7-2), and will generally appear when the design equations include sums and/or differences.

In cases where the sensitivities are not constants, it is often instructive to group the various passive components so that performance parameters are obtained. In this way, it is often the case that sensitivities can be written in terms of filter Q. Since we often expect few if any sensitivity problems when the filter Q is low, having a sensitivity in terms of Q is very useful. For example, if a filter sensitivity is proportional to Q, we know that this sensitivity will be 10 times worse at a Q of 10 than it is at a Q of 1. Such results would suggest that that particular filter might be unsatisfactory in many applications. Note that high-Q filters occur not just as bandpass filters, but as individual second-order sections of filters that are as innocent looking as high-order Butterworth.

In theory, the calculation of passive sensitivities is a simple matter. One first ontains the expression for a filter's performance parameter in terms of the passive elements that determine that particular parameter. For example, the parameter X may depend on passive elements y,........ for which we write X = X(y,........). One then calculates the sensitivities by the use of equation (7-2). Typically we are looking at the sensitivity of the characteristic frequency ω_0 and the Q to each and every R and C in the network, and to various gain factors. In order to take the partial derivatives, any passive elements that are nominally the same value, must be separately identified. This may require the rederivation of the transfer function and the "design equations" for ω_0 and for Q, depending at the point where nominally equal components were given the same identifying symbol. For example, we may have derived T(s) and equations for ω_0 and for Q by carrying three resistors separately as R_1, R_2, R_3, and then simplified these equations for the special case where the resistors are equal, setting $R_1=R_2=R_3=R$. If so, the partial derivatives needed for equation (1) can be obtained from the original unsimplified equations. If however we had set $R_1=R_2=R_3=R$ before deriving T(s) and the design equations, then asking for a partial derivative such as $\partial Q/\partial R$ is meaningless, as we don't know which of the resistors "R" we are talking about, and it usually matters. In such a case, we would have to find the more general derivation, before any special case was selected

On the other hand, once the derivative is taken, it is useful, and the usual practice, to simplify the resulting expression in terms of the special case. In this way, we get a sensitivity expression that relates easily to the case we are really interested in. It is usually possible to simplify the expressions down to constants, or as expressions in terms of the nominal Q of the network.

In taking partial derivatives, it is wise not to multiply out any terms until necessary, as often major blocks of algebra will cancel right out. Thus while a partial derivative may look messy, the multiplication of this derivative by y/X in equation (7-2) can leave something very simple. Another very useful tip is to realize that:

$$S_y^Q = -S_y^{1/Q} = -S_y^D \qquad (7\text{-}6)$$

so that if we have a complicated expression for the damping D, it is not necessary to invert it for Q = 1/D, if we don't want to. We just take the sensitivities of D, and add a minus sign.

As an example, consider the multiple-feedback infinite-gain low-pass circuit which has a transfer function:

$$T(s) = \frac{-(R_3/R_1)\,(1/R_2R_3C_1C_2)}{s^2 + \underbrace{\frac{s}{\sqrt{R_2R_3C_1C_2}}}_{s\ \omega_o}\underbrace{\sqrt{C_2/C_1}\left[\sqrt{R_2/R_3} + \sqrt{R_3/R_2} + \sqrt{R_2R_3}/R_1\right]}_{D=1/Q} + \underbrace{1/R_2R_3C_1C_2}_{\omega_o^{\,2}}} \qquad (7\text{-}7)$$

where T(s) has been put in a form where the design equations are easily identified. We will consider the case where $R_1=R_2=R_3$, in which case the damping $D = 3\sqrt{C_2/C_1}$. Instead of taking sensitivities of Q, we will use the suggestion of equation (7-6) and work from D.

As a first example, consider the sensitivity of ω_o to resistor R_2. According to equation (7-2) we have:

$$S^{\omega_o}_{R_2} = \frac{R_2}{\omega_o}\frac{\partial\omega_o}{\partial R_2} = (R_2/\omega_o)\frac{\partial}{\partial R_2}\left[R_2^{-\frac{1}{2}}R_3^{-\frac{1}{2}}C_1^{-\frac{1}{2}}C_2^{-\frac{1}{2}}\right]$$

$$= (R_2/\omega_o)(-\tfrac{1}{2})R_2^{-3/2}R_3^{-\frac{1}{2}}C_1^{-\frac{1}{2}}C_2^{-\frac{1}{2}} = (R_2/\omega_o)(-1/2)R_2^{-1}\left[R_2^{-\frac{1}{2}}R_3^{-\frac{1}{2}}C_1^{-\frac{1}{2}}C_2^{-\frac{1}{2}}\right]$$

$$= -1/2 \qquad (7\text{-}8)$$

Notice the advantage of writing the square roots as exponents of +1/2 or -1/2, and note how the result cancels down to a very simple result.

As a second example, consider $S^{Q}_{C_1}$ which we will find as $-S^{D}_{C_1} = (C_1/D)\frac{\partial D}{\partial C_1}$. In a way very similar to the first example, this comes out to -1/2, and the corresponding sensitivity of Q to C_1 is thus +1/2. This further illustrates the very useful idea of using symmetry. In the case of sensitivities of ω_o, each of R_2, R_3, C_1, and C_2 occupies an equivalent or symmetric position in the design equation. We could carry out the equivalent calculations of equation (7-8) for R_3, C_1, and C_2, but this would be a waste of time, as they would clearly also be -1/2, as in the first example. To expand the idea of symmetry even further, in this second example we looked at the sensitivity of D to C_1, C_1 having an exponent of -1/2 in the equation for D, and the sensitivity therefore being -1/2, as was the corresponding case of ω_o sensitivity. In general, when a passive component appears only once, we may well be able to write down the sensitivity from a symmetric case, or just by inspection. The case of sensitivities of +1/2 and -1/2 resulting from square root terms is very common.

A somewhat more complicated case will better illustrate a more general case. Consider the sensitivity of Q to R_3. We will look at the sensitivity of D to R_3 and add a minus sign to the result for the Q sensitivity.

$$S^{D}_{R_3} = \frac{R_3}{D}\frac{\partial D}{\partial R_3} = (R_3/D)\frac{\partial}{\partial R_3}\left[\frac{C_2^{\frac{1}{2}}}{C_1^{\frac{1}{2}}}\left(R_2^{\frac{1}{2}}R_3^{-\frac{1}{2}} + R_3^{\frac{1}{2}}R_2^{-\frac{1}{2}} + R_2^{\frac{1}{2}}R_3^{\frac{1}{2}}R_1^{-1}\right)\right]$$

$$= (R_3/D)\,\frac{C_2^{\frac{1}{2}}}{C_1^{\frac{1}{2}}}\left[R_2^{\frac{1}{2}}(-\tfrac{1}{2})R_3^{-3/2} + (\tfrac{1}{2})R_3^{-\frac{1}{2}}R_2^{-\frac{1}{2}} + R_2^{\frac{1}{2}}R_1^{-1}(\tfrac{1}{2})R_3^{-\frac{1}{2}}\right] \qquad (7\text{-}9)$$

This we can simplify about our special case where $R_1=R_2=R_3$ as:

$$S^{D}_{R3} = \frac{R}{D}\,\frac{1}{2}\,\frac{C_2^{\frac{1}{2}}}{C_1^{\frac{1}{2}}}\left[-R^{-1} + R^{-1} + R^{-1}\right] = \frac{R}{D}\,\frac{1}{2}\,\frac{1}{3}\,(C_2/C_1)^{\frac{1}{2}}(3/R)$$

$$= \frac{R}{D}\,\frac{1}{2}\,\frac{1}{3}\,D\,\frac{1}{R} = 1/6 \qquad (7\text{-}10)$$

where we have used the special case, nominal value of $D = 3\sqrt{C_2/C_1}$. The sensitivity of Q to R_3 is thus -1/6.

Not all sensitivities will come out as constants, although constants are certainly welcome results, representing favorable sensitivities if their magnitudes are 1 or smaller. For example, in the Sallen-Key low-pass with equal resistors and equal capacitors, the damping term is:

$$D = \left[\frac{(1-K)}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right]/\omega_0 \tag{7-11}$$

From this we can get the sensitivity of D to K as:

$$S_K^D = \frac{K}{D}\frac{\partial D}{\partial K} = \frac{K}{D}(-1/R_2C_2)/\omega_0 \tag{7-12}$$

However, nominally $R_1=R_2=R$, $C_1=C_2=C$, $\omega_0=1/RC$, and $D=3-K$ so that $Q=1/D$ and $K = (3Q-1)/Q$. Thus:

$$S_K^Q = 3Q - 1 \tag{7-13}$$

where sensitivity has been written in terms of the nominal value of Q.

Fig. 7-1 shows the common single-amplifier low-pass configurations which we have looked at, along with their nominal component choices, and their passive sensitivities. We note some general differences between these cases which are essentially typical of the particular configuration approach. Note for reference the M.F.I.G. low-pass, which has sensitivities that are generally fovorable - all coming out as small fractions. In contrast, Sallen-Key has less favorable sensitivities in general, with the Q sensitivities having terms that increase with Q. [Note that this is basically due to the D = 3-K equation, where K can approach 3, with the difference being very sensitive.] Negative gain VCVS has sensitivities that are in general better than M.F.I.G., being somewhat smaller fractions, and containing additional terms, if any, that go as 1/Q or as $1/Q^2$ (which is a minor adjustment). Based on these findings, negative gain VCVS would seem the best, and Sallen-Key the worse, with M.F.I.G. in the middle. [The state-variable approach has sensitivities that are comparable with M.F.I.G. (see problems at end of chapter).] A final judgement, however, must also consider the active sensitivity, and the design and tuning ease.

In addition to getting general overall ideas about the relative sensitivities of different configurations, we can use sensitivity calculations to see how a performance parameter can be trimmed by adjusting different passive components. This, and a number of other matters discussed above, will be illustrated in the example below:

EXAMPLE 7-1 Find the passive sensitivities of the M.F.I.G. bandpass and consider a scheme for adjusting the frequency and Q independently, if possible.

The M.F.I.G. bandpass was first discussed in Section 2-4. However, the analysis given there is not entirely suitable for sensitivity calculations, since we had set two capacitors equal initially. Accordingly, this is a case where we have to go back for a more general case. Fig. 7-2 shows the M.F.I.G. bandpass with the capacitors separately identified. Equations (2-15), (2-16), and (2-17) now become:

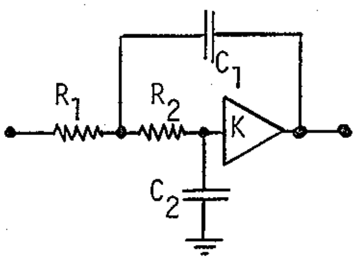
$$T'(s) = -sC_1R_2 \tag{7-14}$$

$$\frac{V_{in}-V'}{R_1} = \frac{V'-V_{out}}{1/sC_2} + \frac{V'-0}{1/sC_1} \tag{7-15}$$

$$T(s) = \frac{-s/C_2R_1}{s^2 + s\dfrac{C_1+C_2}{C_1C_2R_2} + \dfrac{1}{C_1C_2R_1R_2}} \tag{7-16}$$

and the design equations (2-22) and (2-24) become:

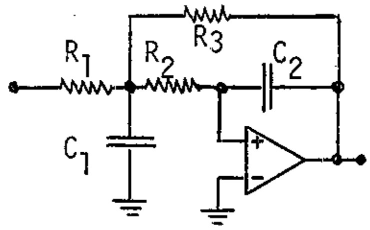
Sallen-Key Low-Pass



Nominal Point:

$R_1 = R_2 \quad C_1 = C_2$

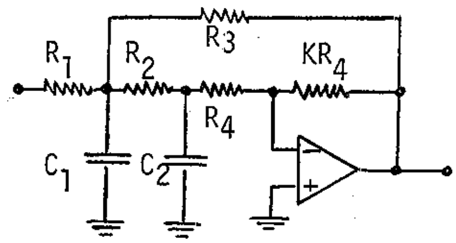
M.F.I.G. Low-Pass



Nominal Point:

$R_1 = R_2 = R_3$

(-) Gain VCVS Low-Pass



Nominal Point:

$R_1 = R_2 = R_3 = R_4 \quad C_1 = C_2$

$$S^Q_{R_1} = -S^Q_{R_2} = -1/2 + Q$$

$$S^Q_{C_1} = -S^Q_{C_2} = -1/2 + 2Q$$

$$S^Q_K = 3Q - 1$$

$$S^{\omega_o}_{R_1,R_2,C_1,C_2} = -1/2$$

$$S^{\omega_o}_K = 0$$

$$S^Q_{R_1} = 1/3$$

$$S^Q_{R_2} = S^Q_{R_3} = -1/6$$

$$S^Q_{C_1} = -S^Q_{C_2} = 1/2$$

$$S^{\omega_o}_{R_1} = 0$$

$$S^{\omega_o}_{R_2,R_3,C_1,C_2} = -1/2$$

$$S^Q_{R_1} = \frac{1}{5} - 1/25Q^2$$

$$S^Q_{R_2} = -\frac{1}{10} + 1/25Q^2$$

$$S^Q_{R_3} = -\frac{3}{10} + 3/50Q^2$$

$$S^Q_{R_4} = \frac{1}{5} - 3/50Q^2$$

$$S^Q_{C_1} = -S^Q_{C_2} = 1/10$$

$$S^Q_K = \frac{1}{2} - 1/10Q^2$$

$$S^{\omega_o}_{R_1} = -1/25Q$$

$$S^{\omega_o}_{R_4} = -3/50Q$$

$$S^{\omega_o}_{R_2} = -\frac{1}{2} + 1/25Q$$

$$S^{\omega_o}_{C_1} = -S^{\omega_o}_{C_2} = -1/2$$

$$S^{\omega_o}_{R_3} = -\frac{1}{2} + 3/50Q^2$$

$$S^{\omega_o}_K = \frac{1}{2} - 1/10Q^2$$

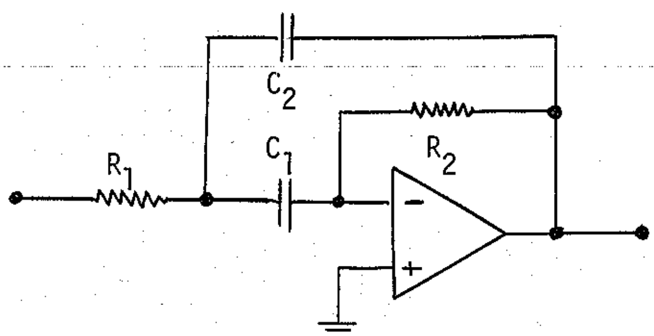


Fig. 7-2

M.F.I.G. Bandpass showing different notation for two capacitors.

$$\omega_0 = 1/\sqrt{C_1 C_2 R_1 R_2} \qquad (7\text{-}17)$$

$$D = \sqrt{\frac{R_1}{R_2}}\left[\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}}\right] \qquad (7\text{-}18)$$

From equation (7-17) and (7-18), we can calculate some typical sensitivities (or just write them down by inspection, as discussed above), such as:

$$S^{\omega_0}_{R_1,R_2,C_1,C_2} = -1/2 \qquad (7\text{-}19)$$

$$S^D_{R_1} = -S^D_{R_2} = 1/2 \qquad (7\text{-}20)$$

This leaves us to calculate the sensitivity of D to C_1 and to C_2:

$$S^D_{C_1} = \frac{C_1}{D}\sqrt{\frac{R_1}{R_2}}\left[\,C_2^{-1/2}(\tfrac{1}{2})C_1^{-1/2} + C_2^{1/2}(-\tfrac{1}{2})C_1^{-3/2}\right] \qquad (7\text{-}21)$$

Some simplification of equation (7-21) is possible, but not necessary. [Note that $S^D_{C_2}$ is the same as $S^D_{C_1}$ as can be seen by symmetry.] We can just evaluate equation (7-21) at the nominal point, $C_1=C_2$, in which case:

$$S^D_{C_1} = S^D_{C_2} = 0 \qquad (7\text{-}22)$$

This interesting result does not mean that D does not depend on C_1 or on C_2, as clearly from equation (7-18), it does. Rather it means that about the nominal case of $C_1=C_2$, a small change of either C_1 or C_2 will have only a very small effect. This is a result of the fact that D depends on the sum of two numbers, both of which are very close to 1, and which are reciprocals. In fact, we can show directly, using equation (7-18), that a 50% change in either C_1 or C_2 causes only a 2% change in D, which is a truly impressive insensitivity. Of course, this 50% change does not leave the filter unchanged. From equation (7-19), the center frequency would change by 25%. [Contrast this zero sensitivity with the two cases of zero sensitivity in Fig. 7-1. In these cases, the sensitivity was zero because the parameter simply did not depend on the component in question - ever. In equation (7-21), the sensitivity is only zero at the nominally equal point, but is still small for a normal spread of nominally equal capacitor values.]

The result of our study suggests that we can in fact adjust the frequency of the bandpass, independent of the Q, simply by making adjustments to one or both of the capacitors. In fact, equation (7-19) tells us that we need to adjust C_1 or C_2 by 1/2 the percentage by which the frequency is in error, and in a direction opposite to that frequency error.

7-3 REAL OPERATIONAL AMPLIFIERS:

The usual ideal op-amp assumptions include the idea that the gain is infinite at all frequencies. The single most important design consideration that arises from this assumption is that the differential input is forced to zero through the operation of negative feedback, or in the absence of working negative feedback, that the output is pinned at one of the supply rails. In practical terms, the gain is of course not infinite, nor is it independent of frequency. In fact, the so-called "open loop" gain curve of a typical op-amp is as shown in Fig. 7-3. There are a number of important points about this curve that should be understood.

First, the curve is exactly that of a first-order low-pass filter, which is probably the one filter that we understand better than any other. Note that the 3db "cutoff" occurs at a very low frequency of about 10 Hz. On the other hand, A_o is very large - 10^5 to 10^6 or so. Why is the frequency response so bad? The answer is that most op-amps are designed with built-in "unity gain compensation." This means that the poor roll-off of Fig. 7-3 is entirely intentional. It is built-in to assure stability under closed-loop conditions, as will be discussed more below. By "open loop" we mean that there is no feedback involved, and that the test is being made by applying very very small signals to the input or inputs, in a manner as suggested in Fig. 7-4. [In fact, the test is very difficult to make in this way, and a number of tricks must be employed to get the curve. These will be covered in a problem at the end of the chapter. None the less, the idea of Fig. 7-4 is the correct idea in theory.] In this view, the open-loop gain curve is exactly a frequency response curve, which is no surprise, since we have noted above that the op-amp is a first-order low-pass filter. One final point is that the open loop gain curve is the way the op-amp always works, whether or not it is in fact being used in a closed loop configuration. This is logical if we consider that the op-amp merely produces an output voltage in response to input voltages. The op-amp has no way of "knowing" that it is in a closed loop, and that its inputs are seeing voltages that are at least in part, produced by its own output. Looked at as a three-terminal device, it is always behaving according to the curve of Fig. 7-3. Of course, any configuration using the op-amp, which involves additional components in general, does not have the same response curve.

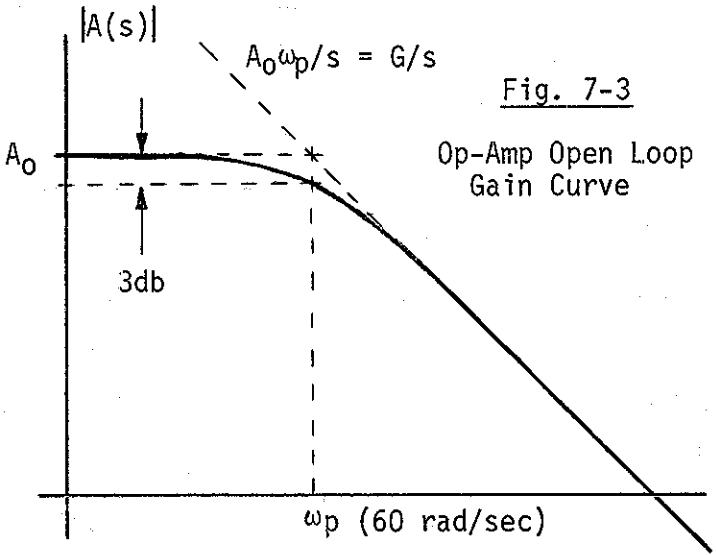
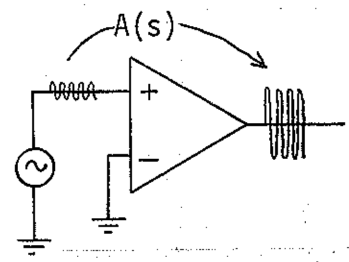


Fig. 7-3

Op-Amp Open Loop Gain Curve

Fig. 7-4 Open-Loop Gain Test (in theory)



$V_{out} = A(s)[V_+ - V_-]$

The need for compensation in the first place can be understood in terms of our needing to employ op-amps in feedback structures. In most of the cases studied here, the feedback is supposed to be negative, and is established as being negative by using the (-) input of the op-amp. However, the monolithic op-amp is fabricated to be very small, and stray capacitances within the chip can be significant. In such a case, additional phase shifts can accumulate, and these get larger and larger at higher and higher frequencies. Eventually a frequency can be found where the phase shift across the chip itself reaches 180°. At this frequency, the feedback which was assumed to be negative (and which is still negative at lower frequencies) becomes positive. If the gain around the feedback loop is equal to or greater than 1, a condition of oscillation is expected. It is the purpose of compensation to shape the op-amp's frequency response so that the gain will always be less than 1 in such a case.

Our general experience with amplifiers and similiar circuits may lead us to associate oscillations with very high gain amplifiers, and this is correct. It may come as a surprise therefore to learn that the case where there is the most "danger" or oscillation is the unity-gain voltage follower configuration of the op-amp, and that in fact, configurations requiring higher gains may be much more stable. To understand this it should be realized that it is the amount of feedback that matters. If there is a 180° phase shifted signal at the op-amp output, this is not a problem until it is fed back, and until it reaches the (-) input with a net gain of 1 or greater. In this view, the voltage follower is 100% negative feedback. There is no attenuation in the feedback loop, and the full "trouble-making" signal reaches the (-) input. Op-amp amplifier circuits requiring gain will have attenuators in the feedback loop (this is how the configuration gets the gain), and the "trouble-making" signal is attenuated. Accordingly it can be larger at the op-amp output and still not reach the (-) input with enough amplitude to cause oscillation.

Thus unity gain with its 100% feedback is the worse possible case. This means that op-amp designers and users have several compensation options. Each of these involves giving the op-amp an intentional roll-off so that the gain is reduced at high frequencies. If we knew that an op-amp was only going to be used for a gain of 10 or more, we would require less compensating roll-off, and could expect benefits of more bandwidth and faster slew rate. In such a case, a "custom compensated" op-amp would make sense. With custom compensation, the user looks at the gain situation in various configurations and supplies only as much compensation as is needed. This is usually done by connecting a single capacitor between two pins on the op-amp IC package. If a unity gain follower is being employed, the maximum amount of compensation (largest capacitor - usually about 30 picofarads) is needed. For higher gains, the capacitor can be decreased.

A second compensation option is to have the compensating capacitor built-in. This means that the user need not bother with that extra capacitor, and in fact can't get access to it at all. Accordingly, in nearly all cases, enough compensation is built in that the op-amp is stable at unity gain. This means that it is over-compensated at higher gains. This is the trade-off, and it is almost always resolved by using an internally compensated op-amp. In part this can be understood because the better op-amps available today, even with their internal compensation, have much larger bandwidths than even the best of the custom compensated op-amps of a few years ago could offer. Thus we are almost always dealing with internally compensated op-amps, which is what we have described in Fig. 7-3.

The so-called "industry standard" of op-amps for many years has been the type "741." This was the first of the internally compensated op-amps, and it had a 1 MHz gain-bandwidth product (see below). People still use 741's in designs at times. However, the newer "BiFET" type op-amps offer gain-bandwidth products of about 5 MHz and have much much lower input bias currents (under 50 picoamps). Thus, in many cases, the BiFETS types, of which the LM351 is typical, have become a new industry standard. It is fair to warn however that it is sometimes wise not to go for this extra bandwidth and speed unless it is needed, since the faster op-amps have a greater tendency for high frequency instabilities. Some ideas as to what measure of gain-bandwidth product is needed will appear in our active sensitivity studies below.

The single-pole roll-off compensation curve needs to be fully characterized in order to use it. The points of interest in Fig. 7-3, assumed to be a log-log plot, are the DC gain (A_O), and the pole frequency ω_p. The curve of Fig. 7-3 is a single-pole low-pass, and can be written as the equation:

$$A(s) = \frac{A_O\omega_p}{s+\omega_p} \qquad (7\text{-}23)$$

In general, $f_p = \omega_p/2\pi$ is around 10 Hz, so for frequencies much greater than 10 Hz, the ω_p can be neglected in the denominator relative to s, and we have $A(s) \to A_O\omega_p/s$. From equation (7-23) it is clear that:

$$|A(s)| = [A(j\omega)\cdot A(-j\omega)]^{\frac{1}{2}} = A_0\omega_p\left[\frac{1}{\omega^2 + \omega_p^2}\right]^{\frac{1}{2}} \tag{7-24}$$

which takes on the value $A_0\omega_p/\sqrt{2}$, 3db down from A_0, when $\omega = \omega_p$. If we look at the limiting form for large frequency, $A(s) \to A_0\omega_p/s$, when $\omega = \omega_p$ here, $|A(s)| \to A_0$, and we can understand the intersection of the straight line $A_0\omega_p/s$ curve with the DC gain A_0 at ω_p (see Fig. 7-3). Setting $A_0\omega_p = G$, where G is called the Gain-Bandwidth Product of the op-amp, we can end up with a convenient equation good for medium frequencies and above (relative to 10 Hz).

$$A(s) = G/s \tag{7-25}$$

Now, the value of knowing A(s) is that we can write an equation relating the input voltages of the op-amp to the output voltage as:

$$V_{out} = (V_+ - V_-)A(s) = (V_+ - V_-)\,\frac{G}{s} \tag{7-26}$$

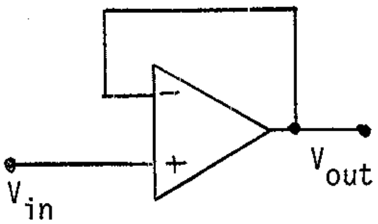
where V_+ and V_- are the voltages at the (+) and (-) inputs of the op-amp respectively. Thus when we want to examine the first approximation to a real op-amp instead of an ideal one, we use equation (7-26), and if we need to work at low frequencies, we use the better approximation of equation (7-23) for A(s) in equation (7-26). The most important thing to note in going from an ideal op-amp to a non-ideal one is that now the differential input voltage is <u>not</u> assumed to be zero, but instead has some value which is related to the output voltage according to equation (7-26). In all cases, once we work out the new result, if we let G approach infinity, we should get back our ideal op-amp result, and this is a useful check.

Note that the gain-bandwidth product G relates to the 45° downward slope of the open-loop curve, and can be related to the first-order low-pass and integrator curves of Fig. 6-3. In fact, when we use the "G/s model of the op-amp," we are specifically treating the op-amp <u>as an integrator</u>. The G/s roll-off curve is thus a 6db/octave roll-off. When the frequency increases by a factor x, the response decreases by 1/x. Consequently, any point on this curve is such that if we multiply the gain by the frequency, we get a constant value of G. Since we can interpret this frequency point as a bandwidth (since it is the 3db point), we can understand how G is called a gain-bandwidth product, and that bandwidth is inversely proportional to the gain. Note that G is assumed to be in radians-per-second. When it comes to plugging in numbers, which usually are quoted as in MHz, the conversion factor of 2π should not be forgotten.

7-4 AMPLIFIER CONFIGURATIONS WITH REAL OP-AMPS:

As a first application of our real op-amp equation, equation (7-26), we will look at our usual op-amp amplifier configurations. We are interested in these, first of all because we will learn about what we can expect from real op-amp amplifiers, such as we often need in audio and instrumentation. Secondly, many of our active filters used these amplifiers, and when we study these active filters, we will be able to transfer the amplifier results without rederivation.

Above we described the real op-amp in terms of an open-loop gain which depended on frequency, as A(s). Note that A(s) <u>is</u> a transfer function every bit as much as any T(s) we have written. When we now look at amplifiers, we will be obtaining gains which will be functions of frequency, and we can write them as K(s), and these are also transfer functions. [In fact, the original gains K were transfer functions, but were constants not depending on frequency, so we left off the (s).] These amplifier gain transfer functions K(s) will only have the frequency dependence that comes from the op-amp, and will thus also be first-order low-pass filters. The differences between them will be due to different cutoff frequencies and different dc gains.

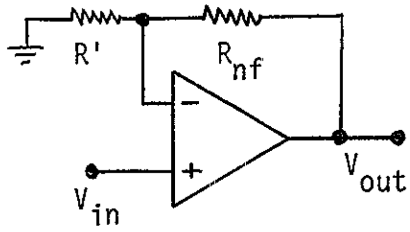


Follower: $K_1(s)$

Ideal: $K_1 = 1$

Real: $K_1(s) = G/(s+G)$

Fig. 7-5a

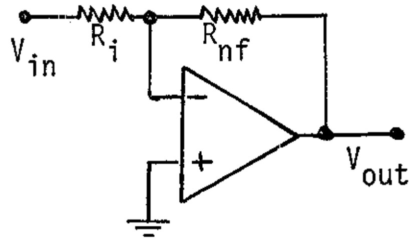


Non-Inverter: $K(s)$

Ideal: $K = 1 + R_{nf}/R'$

Real: $K(s) = G/(s + G/K)$

Fig. 7-5b



Inverter: $K_i(s)$

Ideal: $K_i = -R_{nf}/R_i$

Real: $K_i(s) = \dfrac{GK_i/(1-K_i)}{s + G/(1-K_i)}$

Fig. 7-5c

Fig. 7-5 shows the three amplifier configurations studied in Chapter 1 for the ideal op-amp case. Here we will re-analyze them for the G/s op-amp model, and will write the final results in terms of G, s, and the gain in the ideal case. We will begin with the follower of Fig. 7-5a. Using equation (7-26) we have:

$$V_{out} = \frac{G}{s}(V_+ - V_-) = \frac{G}{s}(V_{in} - V_{out}) \qquad (7\text{-}27)$$

which is solved for the follower gain, which we will call $K_1(s)$ as:

$$K_1(s) = V_{out}/V_{in} = \frac{G}{s + G} \qquad (7\text{-}28)$$

We note that this means that the follower done with a real op-amp has a pole at -G, which can be of very little consequence for frequencies that are very small relative to G. We can take the limit as s goes to 0 for the dc gain of 1. Also, note that as G goes to infinity, we get the ideal case back of $K_1(s) = 1$. Accordingly we have the follower as a first-order low-pass with dc gain of 1 and 3db cutoff at G.

The non-inverter of Fig. 7-5b can be solved by first finding V_- by the voltage divider back from V_{out}:

$$V_- = V_{out}\frac{R'}{R' + R_{nf}} = V_{out}/K \qquad (7\text{-}29)$$

where K is the gain in the ideal op-amp case. Then using equation (7-26) we have:

$$V_{out} = \frac{G}{s}(V_+ - V_-) = \frac{G}{s}(V_{in} - V_{out}/K) \qquad (7\text{-}30)$$

which can be solved for K(s) as:

$$K(s) = V_{out}/V_{in} = \frac{G}{s + G/K} \qquad (7\text{-}31)$$

This is a first-order low-pass filter with dc gain K and with a pole (and corresponding 3db cutoff) at G/K. This means that the pole is now moving in on us. The larger the gain K we try to get, the lower the cutoff. Of course, equation (7-28) for the follower is a special case of equation (7-31), for K=1.

The inverting amplifier case is handled by first finding V_- as:

$$V_- = \frac{V_{in}R_{nf} + V_{out}R_i}{R_{nf} + R_i} = V_{in}\frac{K_i}{K_i - 1} + V_{out}\frac{1}{1 - K_i} \qquad (7\text{-}32)$$

where K_i is the gain in the ideal case ($= -R_{nf}/R_i$). Since the (+) input of the op-amp is grounded, equation (7-26) gives:

$$V_{out} = -\frac{G}{s} V_- = -\frac{G}{s}\left[V_{in}\frac{K_i}{K_i - 1} + V_{out}\frac{1}{1 - K_i}\right] \tag{7-33}$$

which can be solved for $K_i(s)$ as:

$$K_i(s) = V_{out}/V_{in} = \frac{GK_i}{s(1-K_i) + G} \tag{7-34}$$

This is a first-order low-pass with dc gain of K_i and with a 3db cutoff at $G/(1-K_i)$. Keeping in mind that $K_i = -R_{nf}/R_i$ is a negative number, we see that the pole again moves in with a higher gain.

It is interesting to compare the unity-gain voltage follower [equation (7-28)] with the unity-gain inverter [equation (7-34) with $K_i=-1$]. In both these cases, the "circuit gain" is of magnitude 1. That is, neither circuit provides any amplification. The follower has a pole at -G, and a corresponding 3db cutoff at G. On the other hand, the inverter has a pole at -G/2, and a corresponding 3db cutoff at G/2, so there is only half the bandwidth with the inverter.

A unifying concept that will help us understand the position of the pole and the resulting cutoff, is the notion of "noise gain." The "noise gain" is the reciprocal of the feedback factor from the output of an op-amp back to the (-) input. The position of the pole will always be -G divided by the noise gain. We will denote the noise gain by G_N.* We can see how this concept is useful by considering the op-amp network shown in general form in Fig. 7-6. Here we are concerned with the voltages that actually appear on the (-) and (+) inputs of the op-amp. The voltage V_1 appears on V_+ and this may be the scaled sum of any number of sources. The voltage V_- on the (-) input is the sum of a feedback term from the output, and possibly a scaled sum of other sources, denoted by V_2. The path multiplier β tells us how much of V_{out} actually gets back to V_-, and it is mainly β that we are concerned with here. Applying equation (7-26) we have:

$$V_{out} = \frac{G}{s}(V_+ - V_-) = \frac{G}{s}(V_1 - \beta V_{out} - V_2) \tag{7-35}$$

which can be solved as:

$$\frac{V_{out}}{V_1 - V_2} = \frac{G}{s + G\beta} \tag{7-36}$$

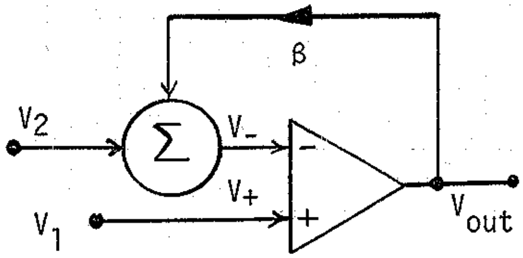


Fig. 7-6 Feedback factor β determines Noise Gain $G_N=1/\beta$

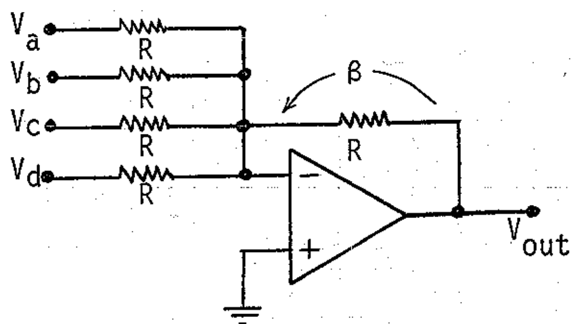


Fig. 7-7 Four input summer is 1:1 for any input/output relationship, but only has bandwidth of G/5 since G_N=5

* It is important to distinguish the gain bandwidth product of the op-amp, G, the noise gain G_N, and the "normalized gain-bandwidth product" g_n (which will appear in the next section).

This shows that regardless of the way external voltages arrive at V_+ and V_-, there will always be a pole at $s = -\beta G$. This pole depends only on the feedback factor. Since the noise gain G_N is the reciprocal of this feedback factor, the pole is always at:

$$s_p = -G/G_N \qquad (7\text{-}37)$$

The determination of the feedback factor β is a matter of looking at V_{out} as the only voltage source, and applying the superposition idea. Thus, looking at the amplifiers of Fig. 7-5, we have β=1 for the follower, $\beta = R'/(R'+R_{nf})$ for the non-inverter, and $\beta = R_i/(R_i+R_{nf})$ for the inverter. For the follower and the non-inverter, only β is responsible for V_-, so $V_2=0$, and $V_+ = V_1 = V_{in}$. Using these facts, equation (7-36) can lead to equations (7-28) and (7-31). For the inverter, $V_+=V_1=0$, and $V_2 = V_{in}R_{nf}/(R_i+R_{nf})$, which leads equation (7-36) to equation (7-34). [See problems at end of chapter.]

One additional example of noise gain will prove useful, and this is the four-input inverting summer of Fig. 7-7. Here in the ideal case the output is just the inverted sum of the input voltages, so the circuit gain is just of magnitude 1 for any input to the output. On the other hand, looking back from Vout, the four inputs are considered grounded, resulting in a net resistance of R/4 in the lower leg of the voltage divider. Thus β = 1/5, and G_N=5. The pole is at -G/5, and the 3db bandwidth is G/5. Thus while we are mixing signals at 1:1 ratios, the bandwidth is only 1/5 of what it would be for a follower, or 2/5 of what it would be for a simple one input inverter. This sort of consideration can be very important in audio circuits, for example.

It is important to realize that the discussion above permits us to understand the bandwidth limitations of amplifiers based on op-amps. Equally important for our purposes here, we know how to handle these amplifiers when they appear inside active filters of interest, and we have also looked at the concept of noise gain.

7-5 ACTIVE SENSITIVITY OF SECOND-ORDER FILTERS:

Here we will be applying the real op-amp model of equation (7-26) to second-order active filters. This will involve at least some degree of re-analysis of the configuration, and will result in a third-order network. When useful, we will distinguish 2nd and 3rd order transfer functions by $T_2(s)$ and $T_3(s)$. In becoming third-order, these transfer functions will now have a third pole that is real, in addition to what is usually a complex conjugate pole pair. In general, this third pole is far enough away on the negative real axis that it is not, in itself, very important (in fact, it would enhance the asymptotic roll-off by 6db/octave). However, the nominal second-order poles may be sufficiently perturbed from their ideal positions that the resulting response becomes unsatisfactory. Fig. 7-8 indicates an overview of the type of analysis that we will be doing, and it should be carefully studied.

We are familiar with the idea of analyzing second order networks, and going back and forth between the transfer function and the pole/zero diagram using only the quadratic equation. In the case of the real op-amp, we will have a third-order denominator, and this requires factoring by computer program. We will then generally be unconcerned with the real pole, but will rather turn our attention to the complex pole pair, of which we need look at only one of the two. Thus a typical active sensitivity diagram will be the detailed region as indicated in the lower right of Fig. 7-8, which will plot the pole (one of the pair) as a function of nominal Q (second-order Q) and of various values of g_n. When g_n is infinite (G is infinite - which is the ideal op-amp case), the poles are on the unit circle in their second-order positions. For finite values of g_n, the poles move. We are concerned with the motion, and with its direction. If the pole moved inward along a radius

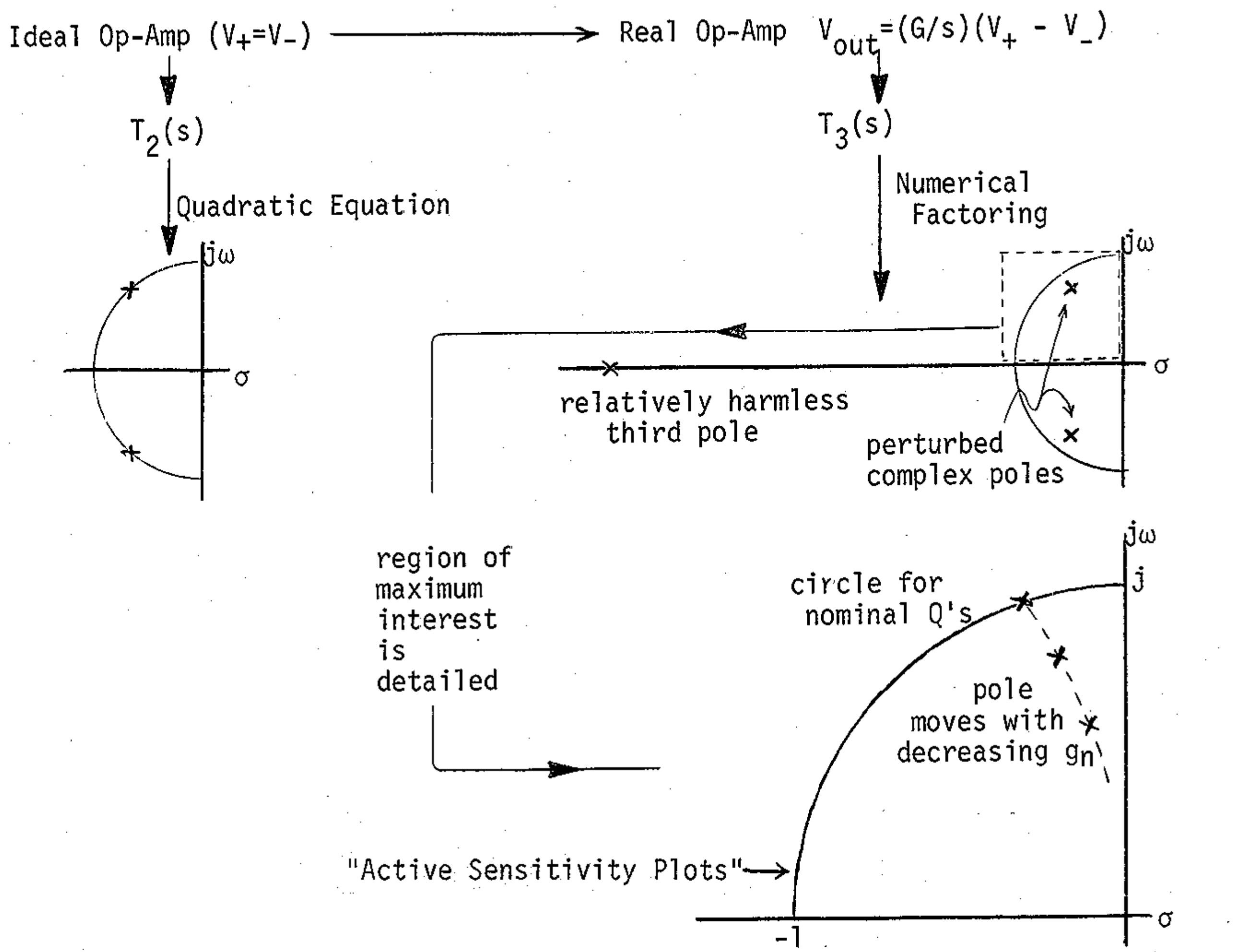


Fig. 7-8 Active sensitivity of 2nd-order filters is done by using the real op-amp model, giving a 3rd-order $T_3(s)$. In the ideal case, we had $T_2(s)$ which yielded to the quadratic equation. $T_3(s)$ must be solved numerically, and shows a third real pole (relatively harmless) plus some perturbation of the complex pole pair, which may be significant. Since this is of the most interest, and because only one quadrant is unique, this region is detailed as shown at the lower right. When g_n is infinite, the pole is on the circle at its second-order position. For finite values of g_n, the poles move, in general inward, and a bit (or a lot) toward the jω-axis.

toward s=0, the pole frequency would change, but its Q (or damping) would remain the same. If it moves off of this radial line (as it actually always does to some degree), then the Q changes, increasing if the curve is toward the jω-axis. In some cases, the poles will not move inward significantly, but will head almost directly toward the jω-axis, which is obviously a bad situation.

As a first example, consider the case of our well-studied Sallen-Key low-pass which has transfer function (Section 2-3):

$$T_2(s) = \frac{K/R^2C^2}{s^2 + (3-K)(s/RC) + 1/R^2C^2} \tag{7-38}$$

We could begin the analysis by going back to equation (7-26) for the real op-amp, but we have already carried the non-inverting amplifier used in the Sallen-Key through to the real op-amp case [equation (7-31)]. Thus we can just substitute K(s) for K in equation (7-38), which gives:

$$T_3(s) = \frac{\frac{GK}{G + sK}(1/R^2C^2)}{s^2 + \left[3 - \frac{GK}{G+sK}\right]\frac{s}{RC} + 1/R^2C^2} \quad (7\text{-}39)$$

For convenience, we will set $\omega_0 = 1/\sqrt{RC} = 1$. This will mean that all frequencies are now relative to ω_0, and this includes the frequency G. Accordingly we define a "normalized gain-bandwidth product" g_n as:

$$g_n = G/\omega_0 \quad (7\text{-}40)$$

With this normalization, and some simplification, equation (7-39) becomes:

$$T_3(s) = \frac{g_n}{s^3 + s^2(g_n/K + 3) + s(3g_n/K - g_n + 1) + g_n/K} \quad (7\text{-}41)$$

This allows us to solve for the actual poles, of which there are three. The factoring must be done numerically with a "root finder" program. Table 7-1 lists the pole positions for several values of D=1/Q=(3-K) and of g_n.

Table 7-1

Damping D	g_n	Complex Poles	Real Pole
1.414	1,000,000	-0.707 ±0.707j	-630518
1.414	1000	-0.705 ±0.707j	-632
1.414	100	-0.689 ±0.707j	-64.7
1.414	10	-0.558 ±0.676j	-8.19
1.414	5	-0.465 ±0.623j	-5.22
1.414	2	-0.339 ±0.487j	-3.58
1.414	1	-0.274 ±0.360j	-3.08
0.1	1,000,000	-0.050 ±0.999j	-344830
0.1	1000	-0.050 ±0.995j	-348
0.1	100	-0.047 ±0.959j	-37.4
0.1	10	-0.063 ±0.736j	-6.32
0.1	5	-0.087 ±0.609j	-4.55
0.1	2	-0.126 ±0.430j	-3.44
0.1	1	-0.151 ±0.301j	-3.04

In the table, we have examples for $Q=1/\sqrt{2}$, or D = 1/Q = 1.414, which is a Butterworth Q, and for Q=10 as a second case. We see that when g_n is large enough, the complex pole pair is near to its nominal second-order position, while the added real pole is quite far away. When g_n is relatively small, the poles degrade in the sense that they move away from their nominal positions, and the real pole moves in closer to s=0.

We need to be clear on what is meant by g_n. In general, g_n as given by equation (7-40) does not change as a result of a change in G, since G is a fixed parameter of any op-amp type, and only changes if we change type. Rather g_n changes by having ω_0 change, which means that we try to use the op-amp at different frequencies. As long as ω_0 is very small relative to G, g_n is large and the poles and performance are near nominal. As we try to design for higher and higher frequencies ω_0, g_n effectively gets smaller, and the poles are less and less nominal.

For example, let's say we have a 1 MHz gain-bandwidth product op-amp (such as the famous type 741), and that we are trying to design a filter for f_0 = 10,000 Hz, and for a Q of 10. This gives us:

$$g_n = G/\omega_o = G(Hz)/f_o = 1{,}000{,}000/10{,}000 = 100 \qquad (7\text{-}42)$$

From Table 7-1, we see that the poles for this case are listed as -0.047 ±0.959j, with the real pole at -37.4. By a simple comparison with the g_n = 1,000,000 case, which should be very close to ideal, we see that there is a relatively slight movement of the poles in this case. The data listed, like all frequencies in this normalized case, are relative to ω_o. We could multiply these pole positions by $\omega_o = 2\pi f_o$ to get the poles in rad/sec., and then divide by 2π to get the poles in Hz. It is simpler to just reinterpret the unit circle which contains the nominal poles as being at f_o. In this case, we avoid multiplication and subsequent divisions by 2π. With f_o = 10,000, the poles are at -470 ± 9,590j Hz, which is a radius of about 9,602 Hz and a damping of (-2)(-470)/9602 = 0.0979, which is a Q of 10.21. Thus we would expect the frequency to be low by about 4% and the Q high by about 2% for this particular op-amp and nominal design. As we might have expected, designing at 10,000 Hz with a 1 MHz op-amp works out fairly well. It is easy to see from the data, however, that for lower effective g_n values, the result could differ significantly from nominal.

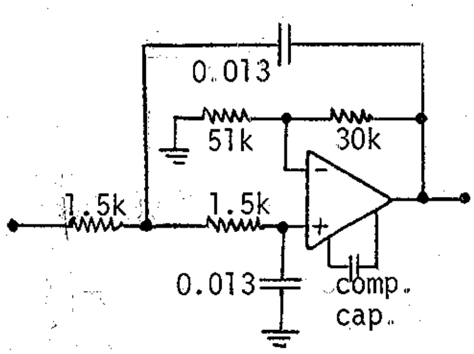
We can make an experimental test of this theory. Because it is inconvenient to work at frequencies approaching 1 MHz, we can effectively "slow down" an op-amp for our purposes for testing. This we will do by choosing a custom compensated op-amp (the type 748, which is essentially a type 741 with the internal compensation removed). In the experimental test, a measured gain-bandwidth product of 1.6 MHz was obtained with 27 pf capacitance of compensation, while 2000 pf of compensation resulted in only 23 kHz for the gain-bandwidth product. For a test, a second-order Butterworth with an 8 kHz cutoff was chosen (R=1.5k, C=0.013 microfarad, R'=51k, and R_{nf} = 30k). With the 27pf compensation, the g_n was 1.6 MHz/8 kHz = 200, while 2000 pf compensation gave g_n = 23 kHz/8 kHz = 2.9. Fig. 7-9 shows the near-nominal 2nd-order Butterworth that occurs when g_n = 200. In contrast, when g_n = 2.9, the solution for the poles of equation (7-41) gives complex poles at -0.386 ±0.548j, and a real pole at -4.04. The theoretical frequency response is obtained by multiplying these poles by 8,000, and by calculating according to equation (1-35) as usual. The solid line for g_n = 2.9 gives this theoretical curve, while the open circles are overplotted experimental points. The agreement is excellent. Note that all three of the poles were used for the calculation, and we begin to see evidence of the third-order roll-off. Incidentally, we note that the result is not a bad filter at all - taken on its own merits - it just is not what we were designing for.

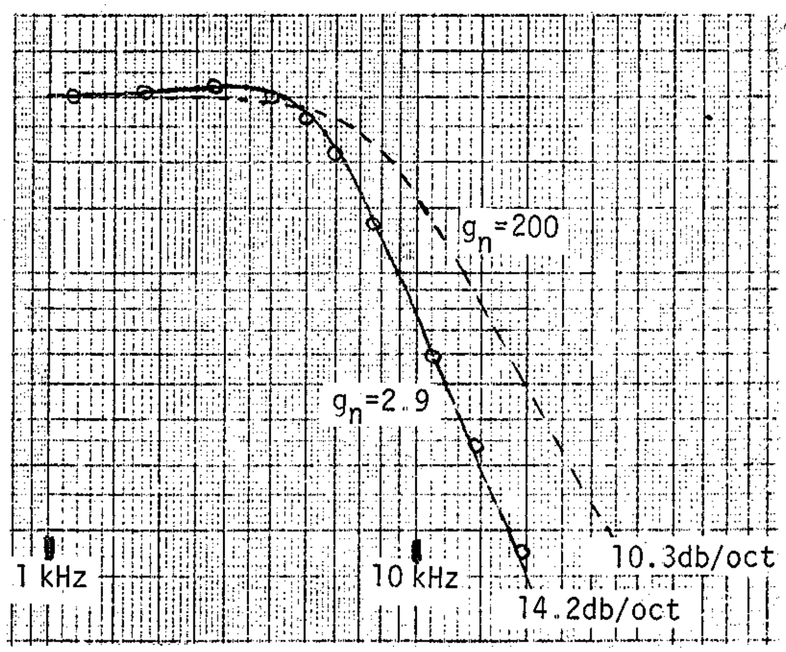
Fig. 7-9 Experimental Test.

2nd-order Butterworth low-pass, 8 kHz cutoff.

g_n=200 (27 pf compensation)

g_n=2.9 (2000 pf compensation)





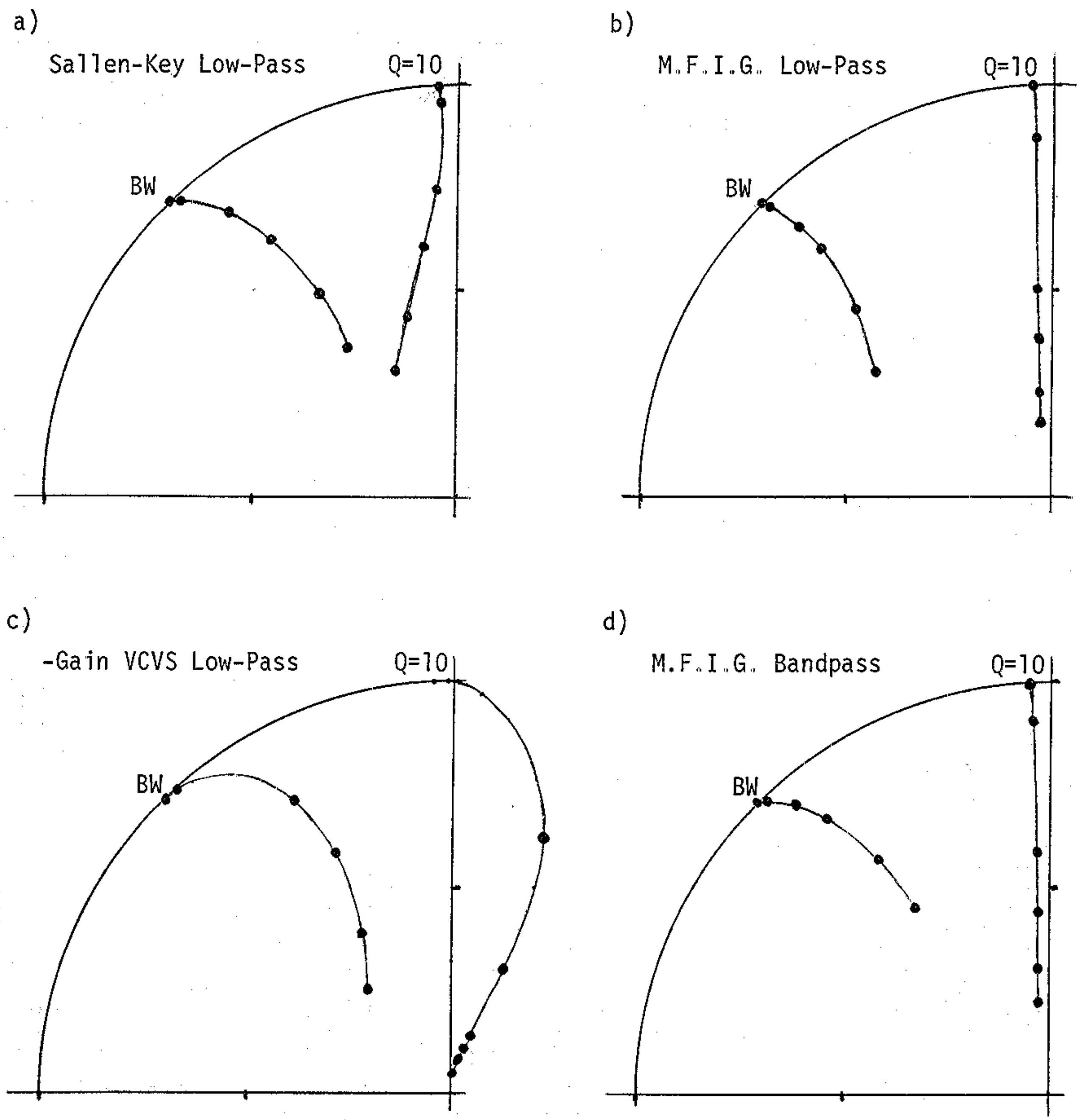


Fig. 7-10 Active sensitivity charts for three single-op-amp low-pass configurations, and for the M.F.I.G. bandpass configuration. Each shows pole positions for two different values of Q: $Q = 1/\sqrt{2}$ (Butterworth) and Q=10. Each pole position curve runs from g_n = infinity to g_n=1, with the g_n=infinity positions being on the unit circle, and with heavy dots for g_n = 1000, 100, 10, 5, 2, and 1 curving inward toward the center of the circle. The nominal cases for the configurations are as follows:

a) Sallen-Key Low-Pass: $R_1=R_2=R$, $C_1=C_2=C$

b) M.F.I.G. Low-Pass: $R_1=R_2=R_3=R$

c) -Gain VCVS Low-Pass: $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

d) M.F.I.G. Bandpass: $C_1=C_2=C$

Fig. 7-10a shows the data of Table 7-1 plotted out in a single quadrant, completing the analysis and presentation scheme proposed in Fig. 7-8. (The other three plots of Fig. 7-10 will be discussed as they come up later.) Of course, we could add many more curves to these diagrams, since each point on the unit curcle is a possible value for a nominal-Q case, and would have its own pole-position curve curving inward from it. Some text and journal papers will often show many curves for many values of nominal Q, and with "cross-gridding" to connect like values of g_n. In such a case, it is possible to make some general estimates of actual performance. However, a full analysis will usually require more than just the gridworks these diagrams offer. Individual calculations, solving the third-order denominators with a computer "root-finder" are possible and very useful. Thus here we find it most useful to do a comparative study of different types of configurations. Accordingly, we have done all our diagrams for the same two nominal values of Q, and for the same six values of g_n plotted as heavy dots (even when more of the curve is shown as a solid line).

The analysis of the Sallen-Key low-pass with a real op-amp has been convenient since we were able to directly employ a previous result - the non-inverting amplifier with a real op-amp. In some other cases, we must use the real op-amp equation (7-26) directly from the start. Such a case is the M.F.I.G. low-pass (Section 5-1, Fig. 7-1, etc.) which is considered here for the real op-amp case in Fig. 7-11.

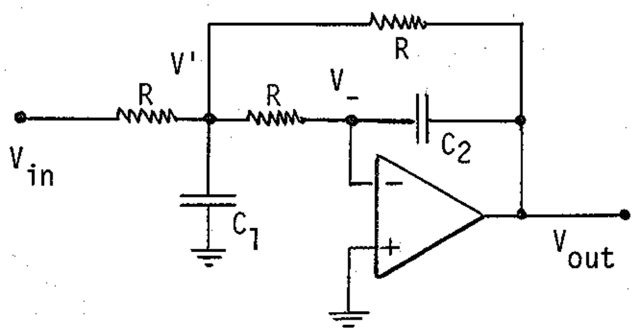


Fig. 7-11 M.F.I.G. Low-Pass considered here for its active sensitivity

In the ideal case, we had a transfer function and design equations for the "equal R" case as:

$$T_2(s) = \frac{1/R^2C_1C_2}{s^2 + (3\sqrt{C_2/C_1})(1/R\sqrt{C_1C_2})s + 1/R^2C_1C_2} \qquad (7\text{-}43)$$

$$\omega_o = 1/R\sqrt{C_1C_2} \qquad (7\text{-}44)$$

$$D = 3\sqrt{C_2/C_1} \qquad (7\text{-}45)$$

For the case where the (+) input is grounded, equation (7-26) leads to:

$$V_{out} = (G/s)(V_+ - V_-) = -(G/s)V_- \qquad (7\text{-}46)$$

or:

$$V_- = -(s/G)V_{out} \qquad (7\text{-}47)$$

Here we obtain V_- as the voltage node between two impedances as:

$$V_- = \frac{V'/sC_2 + V_{out}R}{R + 1/sC_2} \qquad (7\text{-}48)$$

but do not set this to 0, as we would in the ideal op-amp case [instead we have equation (7-47)]. Finally we sum currents at the V' node as:

$$\frac{V_{in} - V'}{R} = \frac{V' - V_{out}}{R} + \frac{V' - V_-}{R} + \frac{V'}{1/sC_1} \tag{7-49}$$

Equation (7-49) is solved using equations (7-48) and (7-47). However, before we write down the expression for the transfer function, we will set $\omega_0 = 1$, which has two effects. First, when we see $R\sqrt{C_1C_2}$ in our algebra, we can replace it with 1. Secondly, G is now in units of $\omega_0=1$, and when it occurs, we replace it with g_n. It is left to the reader as an exercise to show that this gives:

$$T_3(s) = \frac{V_{out}}{V_{in}} = \frac{-g_n}{s^3 + s^2(3C_2R + C_1R + g_n) + s(3C_2Rg_n+2) + g_n} \tag{7-50}$$

In fact, we have not yet achieved quite the form we want, since equation (7-50) is still a function of the R's and C's, and not just of g_n and Q (or of D).* We need to replace the C's and R's by use of equations (7-45) and (7-44). It is convenient to see how this works by writing C_2R as:

$$C_2R = \sqrt{C_2}\, R\sqrt{C_2}\, \frac{\sqrt{C_1}}{\sqrt{C_1}} = \sqrt{\frac{C_2}{C_1}} = D/3 \tag{7-51}$$

and

$$C_1R = \sqrt{C_1}\, R\sqrt{C_1}\, \frac{\sqrt{C_2}}{\sqrt{C_2}} = \sqrt{\frac{C_1}{C_2}} = 3/D \tag{7-52}$$

which allows us to write equation (7-50) as:

$$T_3(s) = \frac{-g_n}{s^3 + s^2(D + 3/D + g_n) + s(Dg_n + 2) + g_n} \tag{7-53}$$

As with the Sallen-Key, we can now solve for the three poles of T(s) numerically and plot them as seen in Fig. 7-10b. Again we show nominal Q cases for Q=0.707 and for Q=10. As a quick evaluation, we might say that M.F.I.G. looks a bit better than Sallen-Key, mainly because the Q=10 case seems to loose frequency without curving much away from a radially outward line. Thus it tends to maintain its Q better. This will be discussed more later after we look at some additional cases.

Another case of interest is the negative-gain VCVS low-pass (Section 5-1, Fig. 7-1, etc.), which was seen to have excellent passive sensitivity properties. In the nominal case of $R_1=R_2=R_3=R_4 = R$, and $C_1=C_2 = C$, we had the second-order transfer function:

$$T_2(s) = \frac{-K/R^2C^2}{s^2 + 5s/RC + (5+K)/R^2C^2} \tag{7-54}$$

where:

$$\omega_o = \sqrt{5+K}\,/\,RC = \sqrt{5-K_i}\,/\,RC \tag{7-55}$$

and:

$$D = 5/\sqrt{5+K} = 5/\sqrt{5-K_i} \tag{7-56}$$

* Compare this with the corresponding step, equation (7-41) for the Sallen-Key. We were successful there in substituting just for $\omega_0=1$ because the damping depended only on K (which is 3-D) and not on the R's and C's. This can be compared with Sallen-Key using a unity-gain amplifier rather than a gain of K (see problems at end of chapter), where we need to make further simplifications, of the type we must do at this point for the M.F.I.G.

where: $K_i = -R_{nf}/R_i = -K$ (K is positive, K_i is negative) (7-57)

It might be supposed that this case can be handled in a manner similar to the Sallen-Key case by employing a previously studied amplifier stage [the inverting amplifier of equation (7-34) in this case]. However, this case provides a couple of interesting "wrinkles" to complicate the problem. First, substituting $-K_i(s)$ from equation (7-34) for K in equation (7-54) does properly relate the two corresponding voltages in the network, and does give a good approximation to the actual transfer function. The problem is that equation (7-54) comes from an ideal op-amp analysis, which assumed that the (-) input of the op-amp was grounded, while here it is actually at a voltage $-(s/G)V_{out}$, which simply using equation (7-34) does not change (see problems at end of chapter). The second problem relates to the normalization to $\omega_0 = 1$, which according to equation (7-55), gives us:

$$\sqrt{5 - K_i} = RC \qquad (7\text{-}58)$$

so we do not just get to set RC=1 here. Analysis of the network gives (see problems at end of the chapter).

$$T_3(s) = \frac{(1-Q^2)g_n/AQ^2}{s^3 + s^2[20Q - \frac{3}{Q} + g_n]/A + s[2 - \frac{1}{5Q^2} + \frac{g_n}{Q}]/A + g_n/A} \qquad (7\text{-}59)$$

$$A = 25Q^2 - 4 \qquad (7\text{-}60)$$

where equations (7-55) and (7-56) have also been used.

As in the previous examples, the denominator of equation (7-59) can be factored numerically, and poles as seen in Fig. 7-10c are the result. In contrast to the Sallen-Key and the M.F.I.G., these result are much less satisfactory. Note that the poles not only move more, but move fairly rapidly into the right half-plane. For example, a Q of 10 is unstable even when g_n is as large as 1000. *

We could continue with numerous other second-order sections. The second-order M.F.I.G. bandpass (Fig. 7-2 with C_1=C_2=C, etc.) comes out as:

$$T_3(s) = \frac{-2sQg_n}{s^3 + s^2[2Q + 1/Q + g_n] + s[g_n/Q + 1] + g_n} \qquad (7\text{-}61)$$

by an analysis that is very similar to that for the M.F.I.G. low-pass above. The M.F.I.G. bandpass is represented by Fig. 7-10d. A first evaluation would suggest that the M.F.I.G. bandpass is similar in active sensitivity to the M.F.I.G. low-pass.

Before we review the overall situation with regard to these second-order configurations, it will be useful to look at two more configurations. The unity-gain Sallen-Key of Fig. 5-1 can be examined in the case where R_1=R_2=R, in which case the third-order transfer function is obtained using equation (7-28) along with equations (2-11) and (5-4) (see problems at end of chapter).

$$T_3(s) = \frac{g_n}{s^3 + s^2[g_n + 2/D + D] + s[1 + g_nD] + g_n} \qquad (7\text{-}62)$$

This has poles as shown in Fig. 7-13a, which is noticable different from the case where K is greater than 1 (Fig. 7-10a). Incidentally, the active sensitivity plot

*In fact, the active sensitivity is so bad that the extra real pole is also a problem, and is itself within the circle in many cases.

for the Sallen-Key high-pass (Fig. 3-20b), which is an equal-R, equal-C case, is identical to the equal-R, equal-C low-pass case (Fig. 7-10a). We may note a slight improvement when we compare the unity gain case to the equal-R, equal-C cases.

Fig. 7-12 shows a M.F.I.G. high-pass configuration, the analysis of which is very similar to that for the M.F.I.G. low-pass and bandpass cases, resulting in the third-order transfer function (equal-C case) of:

$$T_3(s) = \frac{-s^2 g_n/2}{s^3 + s^2(g_n/2 + 3/2D + D/2) + s(1/2 + Dg_n/2) + g_n/2} \qquad (7\text{-}63)$$

where:

$$D = 3\sqrt{R_1/R_2} \qquad (7\text{-}64)$$

The pole position plot for this case is seen in Fig. 7-13b.

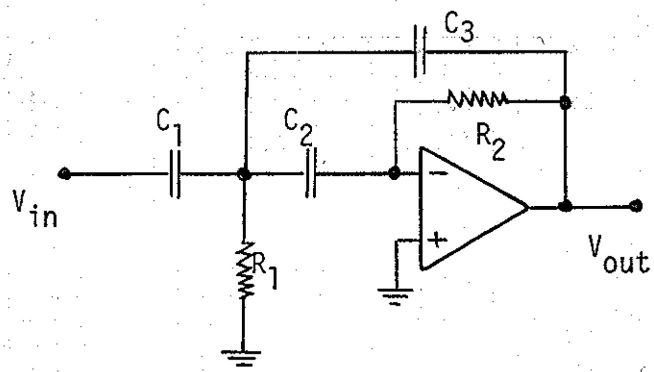


Fig. 7-12

M.F.I.G. High-Pass

Nominal Case: $C_1=C_2=C_3=C$

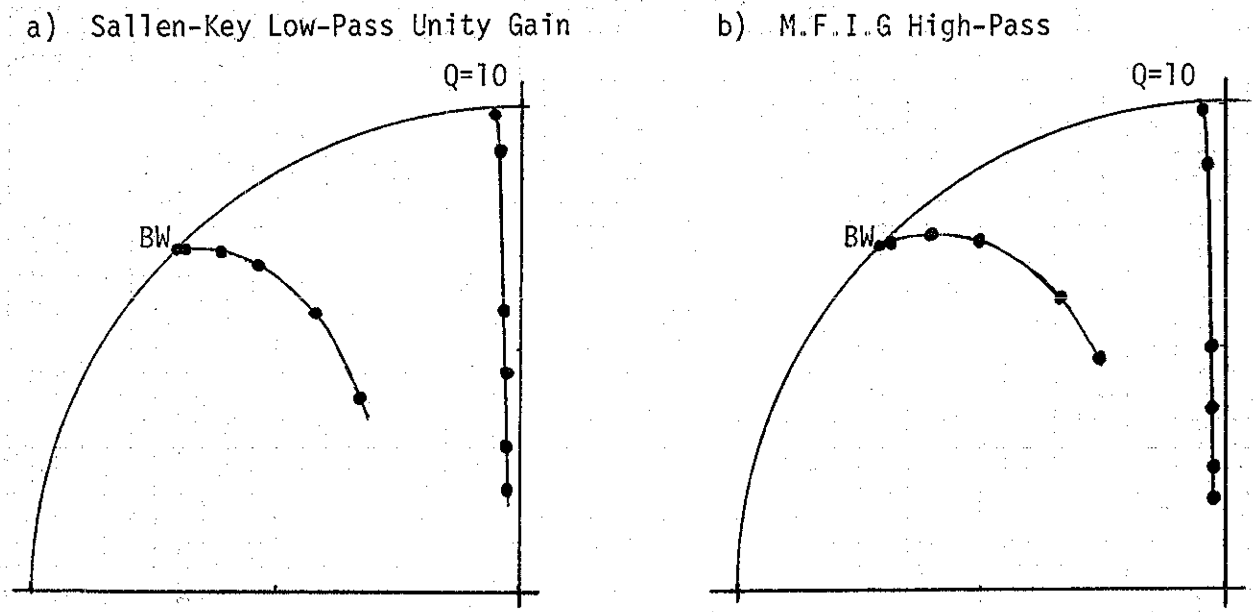


Fig. 7-13 Active sensitivity charts for two additional examples: Sallen-Key low-pass with unity gain, and M.F.I.G. high-pass. The Sallen-Key low-pass is for the nominal case of $R_1=R_2 = R$, and the M.F.I.G. nominal case is $C_1=C_2=C_3 = C$. The values of Q are BW (Butterworth or Q=0.7071) and Q=10, and the values of g_n are (spiraling inward): 1000, 100, 10, 5, 2, and 1.

We now have enough data on active sensitivity to make a few generalizations. First, it is clear that the performance tends to be relatively constant for a particular style of configuration. We have noted that Sallen-Key low-pass and high-pass filters share the identical active sensitivity plot. We have also seen a good deal of similarity between MFIG low-pass, bandpass, and high-pass: Fig. 7-10b, Fig. 7-10d, and Fig. 7-13b. If we were to look at other negative-gain VCVS structures, we would find them all similarly poor. Consequently, we might say in general that MFIG looks best, Sallen-Key is not as good, and negative-gain VCVS is rather poor when it comes to active sensitivity. This is essentially correct. If we look carefully, however, at the case where a unity-gain amplifier was used in a Sallen-Key approach (Fig. 7-13a), we note its basic similarity to the MFIG graphs. This might lead us to suspect that there is something a bit more fundamental going on here.

What this more fundamental thing is is noise gain. Configurations with lower noise gain have better active sensitivity. As we saw at the end of Section 7-4, noise gain was related to a certain feedback factor, and the op-amps effective bandwidth was reduced by a factor equal to this noise gain. The poor active sensitivity performance of negative-gain VCVS structures can be related to the large negative gains required (-10 to -100 or more), with correspondingly large noise gains and associated small bandwidths. Somewhat better are the Sallen-Key positive gain VCVS structures, which have more modest gain requirements (+1 to +3). Best of all are the MFIG structures and the unity-gain Sallen-Key configurations which have a noise gain of 1. *

In a sense, the VCVS designs (positive or negative) are being "cheated", since we are assuming that we are using unity-gain compensated op-amps [equation (7-26)]. This means that the op-amps are overcompensated in most cases. The situation would be somewhat improved, for example, if we were using a negative-gain VCVS circuit and were able to custom compensate it for the particular gain being used. This would be done by reducing the value of the compensating capacitor. In such a case, values of G approaching 10 times the unity-gain bandwidth of the internally compensated version are possible. **

While this (overcompensation) point of view adds some perspective to the active sensitivity problem, the graphs we have looked at do properly discribe the practical situation with regard to the use of ordinary (internal compensation for unity gain) op-amps. This is, as mentioned above, because the overall performance of the best of the internally compensated op-amps available today is probably superior to that which we could get with custom compensation of older types. ***

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* For the MFIG types, we are considering that there is no "excess feedback." Certainly there is negative feedback - that's how our op-amp circuits work. But there is only feedback that is needed to make the configuration work in the first place - no extra that is just to stabilize the op-amp used.

** In theory, as the attenuation from the output to the (-) input is increased, the noise gain is increased, and the potential oscillation-causing signal is decreased. Accordingly we should be able to reduce the compensation (move the 6db/ocatve roll-off upward in frequency) proportional to the noise gain, and thereby increase bandwidth. This does work in that way, but only for improvements of five to ten at most. This is because the 6db/octave compensation is not being imposed on an ideal op-amp [as equation (7-26) might lead one to suppose], but on a high gain differential amplifier that is capable of exceeding the compensation limits, but which is certainly not unlimited in bandwidth. Thus relaxing our imposed compensation only pushes us up closer to the next physical limitations that await.

*** In light of footnote just above, we can understand this design improvement as not so much one of improving the amplifier stages, but of reducing spurious →

We can use active sensitivity plots and data tables in several ways. The first and most obvious way is to try to choose a configuration with low active sensitivity in the first place. The second way would be to examine the active sensitivity data and try to choose regions were the performance comes out close enough to nominal so that no corrections would be necessary. The third way to use this approach would be to see how much a given design situation degrades, and to try to compensate for this by "overdesigning" the filter in the first palce. The idea is that since we know that the performance is going to degrade a bit, we design it a bit beyond what we need with the expectation that it will fall back closer to the nominal specifications. At times we can make useful estimates of the degree to which the filter should be over-designed.

As an example, let's consider the design of a second-order low-pass section with f_0 = 30,000 Hz, Q=2, using Sallen-Key low-pass. Assume that we are using a 1 MHz gain-bandwidth product op-amp, so g_n = 1,000,000/30,000 = 33.33. We can put these numbers into equation (7-41), and the denominator is solved for the following poles:

$$s_{p1,p2} = -0.2147 \pm 0.8901j \quad (7\text{-}65)$$

$$s_{p3} = -15.90 \quad (7\text{-}66)$$

These poles are relative to the pole frequency f_0, so we can multiply them by 30,000 to see where they are in our actual example:

$$s_{p1,p2} = -6441 \pm 26{,}703j \text{ Hz} \quad (7\text{-}67)$$

$$s_{p3} = -477{,}000 \quad (7\text{-}68)$$

These we can compare with the nominal positions for second-order f_0=30,000 and Q=2 which would be:

$$s_{p1,p2} = -7500 \pm 29{,}047j \text{ Hz} \quad (7\text{-}69)$$

The actual complex poles [equation (7-67)] have a radius of 27,469 Hz and a damping of D = (2·6441)/27,469, or Q = 1/D of 2.132. At the same time, we feel that the real pole at -477,000 Hz [equation (7-68)] can be ignored. Thus we find that the frequency of the section came out about 8% low, while the Q came out about 7% high. Whether or not this is acceptable depends on the application.

In cases where this is not acceptable, we need to do an overdesign. To a degree, this is a trial-and-error process, although we can make some useful guesses about which direction to move the original specifications. One method would be to try designing by moving the f_0 and Q specs in the direction opposite to which they fell, and by the same percentages. Since we designed for f_0 of 30,000 and ended up with f_0 = 27,469, 8% low, we will design now for f_0 8% high, or 32,764. Since we were after a Q of 2, and got instead 2.132, 7% high, we will design now for a Q that is 7% low, or 1.876. We repeat the design process. Note that here g_n becomes 1,000,000/32,764 or 30.52, and is not 33.33 as in the original case. We put g_n = 30.52 and D = 0.533 into equation (7-41), and get poles which correspond to equations (7-65) and (7-66) as:

$$s_{p1,p2} = -0.2267 \pm 0.8820j \quad (7\text{-}70)$$

$$s_{p3} = -14.92 \quad (7\text{-}71)$$

phase shifts so that the compensation curve can be moved upward. That is, a 4.5 MHz unity gain compensated op-amp of today could not be improved to 45 MHz simply by making it custom compensated, even though this order of improvement was available for a 1 MHz op-amp.

These poles are multiplied by f_0 for this example, which is 32,764, and the result is:

$$s_{p1,p2} = -7428 \pm 28{,}897j \text{ Hz} \tag{7-72}$$

$$s_{p3} = -488{,}839 \text{ Hz} \tag{7-73}$$

which are at a radius of 29,837 (-0.5% short of 30,000) and at a Q of 2.01 (+0.5% over 2.00). This would probably be close enough for most applications.

Alternative to adjusting the specifications for f_0 and for Q, we might try to adjust the real and imaginary parts of the poles. For our example, we started with desired poles at -7,500 ± 29,047j Hz, and they moved to -6,441 ± 26,703j Hz. We could try an overdesign by percentage fall-back, as we did for f_0 and Q. Or we could just add the amount of the fall-back to the original design. For a second example, we will try this latter approach. The overdesign is thus represented by poles at -(7500 + 7500-6441) ± (29,047 + 29,047-26,703)j Hz = -8559 ± 31,391j Hz. These overdesigned poles have a radius of 32,537 Hz and a Q of 1.901. The gn for this case is 1,000,000/32,537 = 30.73, and D = 1/Q = 0.5260. These values can be put in equation (7-41) and give poles at:

$$s_{p1,p2} = -0.2239 \pm 0.8829j \tag{7-74}$$

$$s_{p3} = -15.00 \tag{7-75}$$

When we multiply these by the pole radius of 32,537, we get:

$$s_{p1,p2} = -7285 \pm 28{,}727j \text{ Hz} \tag{7-76}$$

$$s_{p3} = -488{,}055 \text{ Hz} \tag{7-77}$$

which have a radius of 29,636 Hz (1% low) and a Q of 2.034 (2% high).

We see that in either case, we get a significant improvement, and one which could become even better in an iterative procedure. Here in fact, the overdesign of the frequency and the Q worked a bit better than the overdesign of the poles. However, no hard and fast rules can be offered as to what works best for any particular case. It is clear that overdesign can work, and that there are several procedures for approaching it. It is probably also obvious that since we need a computer to solve the third-order equations, that we may well want to program the entire overdesign procedure.

By far the most important of the non-ideal properties of the op-amp as far as active filter design is concerned is the G/s behavior of A(s). At times, we need to look at other limitations, of which the finite slew rate of the output stage can cause some interesting complications. Because of a limited current drive, it is possible for the input and the rest of the op-amp to move faster than the output can. In such a case, the output may "slew" for certain periods of time - going as fast as it can but remaining "behind" where it should be. This could occur for example when a very fast step appears at the input, in which case, the output "ramps" up to the new level as fast as it can, but not instantaneously. Slew rate limiting can also occur even with sinusoidal inputs, if the amplitude and the frequency are both high enough so that the dv/dt required exceeds the dv/dt available, at least for a portion of the waveform. In such a case, the waveform becomes more triangular in shape. The situation constitutes a failure of our negative feedback ideas, and can also be interpreted as a non-linearity at the op-amp output. This can be seen since the output should be a sinusoidal, but is instead a more triangular shape of the same frequency, and thereby being a waveform with a fundamental and at least some harmonic distortion, as would occur if the sinusoidal were driven through a non-linear circuit. This can lead to the phenomenon of "jump resonance."

It is easy to determine the frequency at which slew rate limiting and the beginning of a non-linear behavior will occur (see problems at end of chapter). However, what is usually seen first is an unexpected "jump" in the measurement of the frequency response curve. Typically, in a bandpass case, as the frequency is increased up the lower slope before the peak, there will be a sudden jump upward in the response. This jump may be small enough that it seems ignorable at first. However, if we then measure the curve again with decreasing frequency, we find that the corresponding downward jump does not occur, but that at a slightly lower frequency, a somewhat larger jump downward occurs, and this one we can't ignore (see Fig. 7-14)

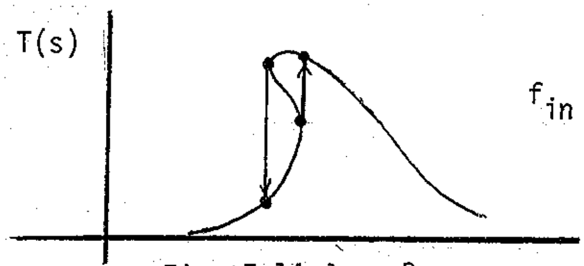


Fig. 7-14 Jump Resonance

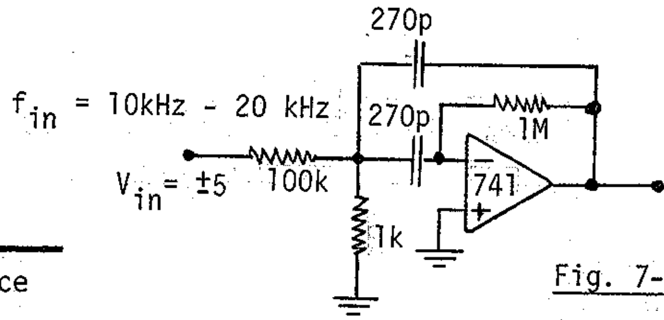


Fig. 7-15

We may also observe that if we cut back the input amplitude, that the problem either gets much smaller or goes away. Fig. 7-15 shows a bandpass circuit along with voltage levels that can be used to demonstrate jump resonance.

The phenomenon of jump resonance is far more common in mechanical systems than it is in electrical ones, owing to the fact that extended ranges of linearity are the norm in electrical systems, but not in mechanical ones. Accordingly, more information on jump resonance will be found in texts on machanical vibrations, and only an outline of what is occuring will be given here. Because of the non-linearity, the actual frequency response curve bends over and becomes double valued, as is suggested in Fig. 7-14. We can therefore understand the smaller upward jump as being from the lower part of the curve to the upper part. The larger downward jump occurs because the response is on the upper part of the curve, and effectively, falls off the end. In fact, if one watches the waveform just before this jump, it is easily possible to see the triangular-like stretching of the sinusoidal, and the "snapping" of the waveform to a "shorter length" seems very natural. We can understand that jump resonance will not occur if the input amplitude is reduced, since the output requirement will be below the slew-rate limit. A similar jump responance can be observed in some voltage-controlled filters (Chapter 8) due to the non-linearity of the input stage of the control elements: the operational transconductance amplifiers (OTA's). The OTA input becomes significantly non-linear unless the signal is attenuated to ±10 mV or less.

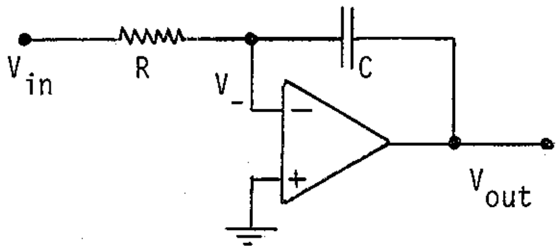
There are sometimes other effects that can be found with real op-amps. While we ideally have no output impedance, it is actually on the order of 50 to 75 ohms, although this is almost always "hidden" by negative feedback effects. A problem can come up however when a capacitive load is attached to the output, thus forming an RC low-pass in the feedback loop. The excess phase can destabilize the unity-gain compensation faster than the additional loop roll-off can prevent it. One such case is common with faster op-amps such as the LF351 BiFET type with its 4 MHz gain-bandwidth product. A scope cable in excess of about 5 feet length or similar capacitive load can cause this op-amp to go into a small amplitude but high-frequency oscillation of several MHz. This is often relatively harmless, particularly in the case where it is caused by the scope cable, in which case it is not there except during the times we are actually looking at it. For a number of reasons, it is sometimes wise to use a slower op-amp such as a 741 type with 1 MHz bandwidth unless the higher speed is actually needed. A 741 type will not normally oscillate with a scope cable capacitive load.

7-6 SENSITIVITY OF STATE-VARIABLE FILTERS:

As we have seen in Chapter 6, the state-variable (or other "Biquad") approach offered a useful alternative to the single-op-amp configurations used for second-order sections. In this sense, our study here is similar to that of Section 7-5, in that our goal is a usable second-order section. However, there are a couple of differences as well. First, since there are three (or more) op-amps involved in the state-variable approach, we expect that the real op-amp model will have fifth-order behavior instead of just 3rd-order as we have encountered so far. Secondly, we will find that the passive sensitivity is quite acceptable (comparable to M.F.I.G.), but that the active sensitivity is very bad - at least as bad as negative-gain VCVS. What must be kept in mind therefore is that we are designing with bad elements (integrators and summers) which we know we can eventually fix up (Section 7-7).

Our first step in the analysis of state-variable filters with real op-amps is to develop the transfer function of a real integrator which we can then put into the particular state-variable configuration of interest. We will look at the popular inverting integrator structure, as in Fig. 7-16.

Fig. 7-16 Inverting integrator $T_i(s)$ with a real op-amp



We can identify the voltage V- as a voltage node between two impedances connected to two voltages sources, and write an expression for V_- as:

$$V_- = \frac{V_{in}(1/sC) + V_{out}R}{1/sC + R} = \frac{V_{in} + V_{out}sCR}{1 + sCR} \tag{7-78}$$

With the ideal op-amp we would set $V_- = V_+ = 0$ and arrive at the ideal integrator as $T(s) = -1/sCR$. Here however we have the real op-amp and will use the G/s model of equation (7-26), which for V_+ grounded gives us:

$$V_{out} = \frac{G}{s}(-V_-) = -\frac{G}{s}\left[\frac{V_{in} + V_{out}sCR}{1 + sCR}\right] \tag{7-79}$$

which can be solved for the real integrator transfer function, which we will call $T_i(s)$ as:

$$T_i(s) = \frac{-1}{sCR[1 + s/G + 1/RCG]} \tag{7-80}$$

This result we can use whenever this particular integrator appears. Note that there are two poles here, one at s=0 as expected, and the second at approximately -G.

Next we must choose a particular version of the state-variable configuration. We will choose the three op-amp Version 2 seen in Fig. 6-6d. This circuit is repeated in Fig. 7-17 here, where the integrator $T_i(s)$ of Fig. 7-16 and equation (7-80) is shown as a block for convenience. This will automatically take care of the active sensitivity of the integrator itself, and we are left with the question of how to handle the summing op-amp. This we will do by applying equation (7-26). As in the ideal op-amp case, it is true that:

$$V_- = (V_H + V_L + V_{in})/3 \tag{7-81}$$

and:

$$V_+ = V_B R'/(R'+R_Q) = V_B/3Q \tag{7-82}$$

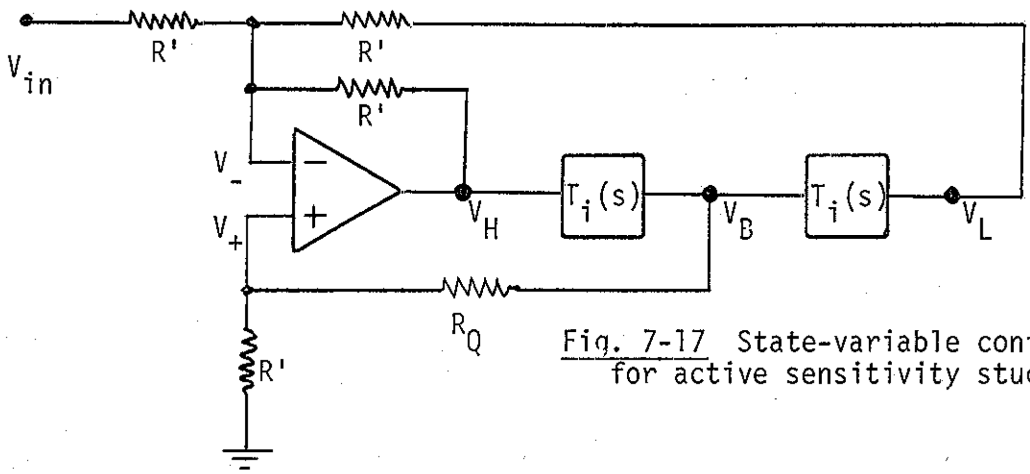


Fig. 7-17 State-variable configuration for active sensitivity study.

where $Q = (R'+R_Q)/3R'$ is the Q in the ideal case. Putting these values for V+ and V_ into equation (7-26) we arrive at:

$$T_H(s) = V_H/V_{in} = \frac{-1}{3s/G - T_i(s)/Q + T_i^2(s) + 1} \qquad (7\text{-}83)$$

Equation (7-83) is the correct answer, but we need to put it into a more useful form in order to work with it. We will be setting $\omega_0 = 1/RC = 1$, so that frequencies are normalized to ω_0 and G will become g_n, as we have done before. Using equation (7-80) and working out the algebra, we can convert equation (7-83) to:

$$T_H(s) = \frac{-g_n D_i^2(s)}{as^5 + bs^4 + cs^3 + ds^2 + es + f} \qquad (7\text{-}84)$$

where $D_i(s)$ is the denominator of $T_i(s)$, and the other coefficients are given by:

$$a = 3/g_n^2 \qquad (7\text{-}85a)$$

$$b = 7/g_n + 6/g_n^2 \qquad (7\text{-}85b)$$

$$c = 5 + 8/g_n + 3/g_n^2 \qquad (7\text{-}85c)$$

$$d = 1/Q + 2 + g_n + 1/g_n \qquad (7\text{-}85d)$$

$$e = (g_n + 1)/Q \qquad (7\text{-}85e)$$

$$f = g_n \qquad (7\text{-}85f)$$

Equation (7-84) is the real op-amp equation for this particular state-variable configuration. It is fifth-order, as it should be, with two orders being due to the ideal case, and three additional poles being contributed by the op-amps. Note from the equations for the coefficients [equations (7-85a) through (7-85f)] that the coefficients for the 5th, 4th, and 3rd powers of s are "weak" if g_n is large. Thus as g_n goes to infinity, we get the ideal op-amp case back, as we should. We can compare equation (7-84) with earlier results for other configurations, such as equations (7-41), (7-53), etc. Here the difference is that we get fifth rather than third-order, and that for convenience, the coefficients are listed separately.

We will be interested in where the five poles of equation (7-84) occur. In particular, where are the three extra poles, and how much do the two closest poles move from their nominal positions. We can plug in some numbers and solve equation (7-84) numerically. Such calculations show that the extra poles occur with two at approximately $-g_n$, while the third occurs at approximately $-g_n/3$. In the unnormalized

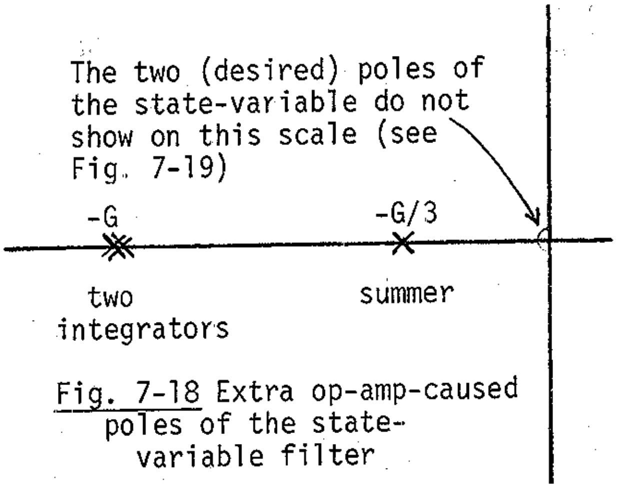


Fig. 7-18 Extra op-amp-caused poles of the state-variable filter

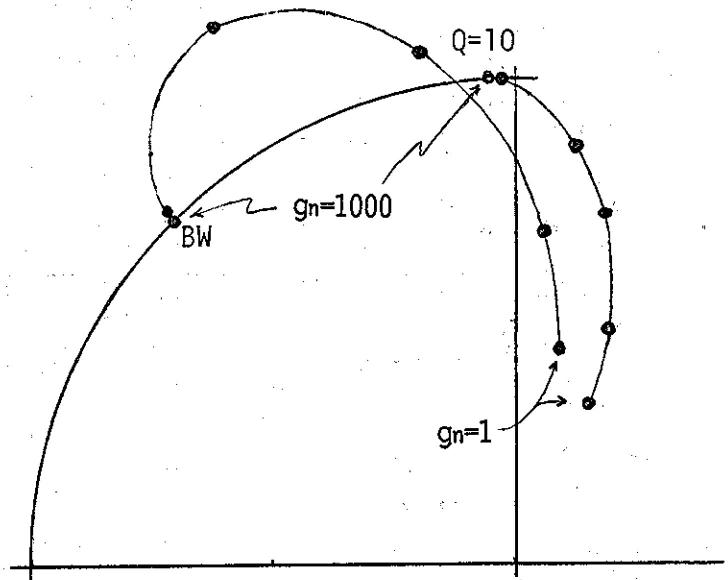


Fig. 7-19 Active sensitivity chart for the state-variable of Fig. 7-17. Heavy dots are for g_n=1000,100,10,5,2, and 1

case, these are of course at -G and at -G/3 (Fig. 7-18). Thus we can reasonably associate the two poles at -G with the poles of the real op-amp integrators that are used in the state-variable structure [see equation (7-80)]. The pole at -G/3 can be seen to be due to the summer, as can be understood as the summer having a noise gain G_N of 3, and a corresponding pole at $-G/G_N$ [equation (7-37)]. Further, we have already seen that this particular summer was working at a non-inverting gain of 3, a result that led to finding the Q in the ideal case [see equation 6-23)].

It is important and interesting to understand the positioning of the three extra poles, but our major concern is the perturbations of the more dominant pole pair at the design frequencies. Accordingly we look at the active sensitivity curve for these poles that corresponds to those for other configurations seen in Fig. 7-10 and Fig. 7-13. It is clear that these are as bad or worse than any that we have seen so far. We can understand this mainly in terms of the added number of op-amps. We would thus suspect that the state-variable would only be useful for low frequencies unless we do something to compensate the network. Fortunately we can do this where it becomes necessary (next section).

We have commented above that the passive sensitivity of the state-variable filter is reasonably good. Because there are so many different state-variable and "biquad" configurations, we will mainly be looking at generalities here. However, the configuration of Fig. 6-6c will be used for an example, and is repeated with all the passive components separately identified, in Fig. 7-20. The first thing to keep in mind was that the pole frequency was basically identified with the inverse of the product of the time constants of the integrator. In fact, in all cases, we saw ω_0 = 1/RC. What we will see here is that this will mean that the sensitivities of ω_0 to the passive components in the integrators will be -1/2, as expected. However, there are now other components that matter: R_3 and R_4 in the upper loop, which in effect, in changing the gain in this loop, affect the overall time constants of the integrators. The second thing to keep in mind is that the Q of the state-variable was given by the inverse of the gain from the bandpass output back to the highpass output - the gain in the lower loop. This gain is nominally -1/Q, and since:

$$S^{Q}_{1/Q} = -1 \qquad (7\text{-}86)$$

we expect sensitivities to be close to -1, and smaller than -1 if a given component does not directly affect this gain.

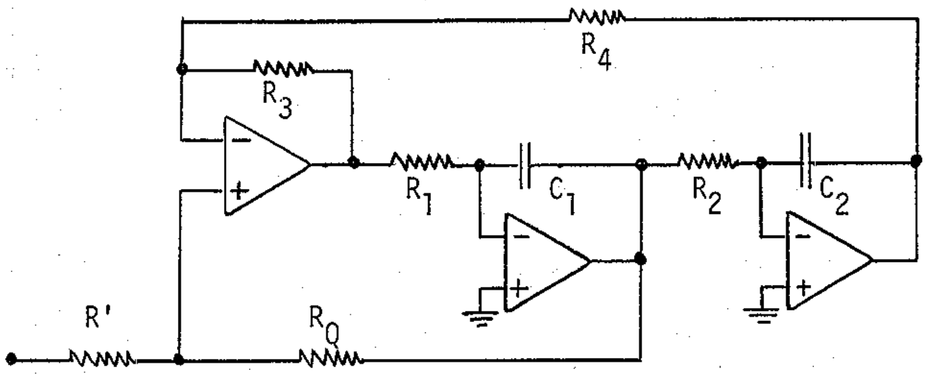


Fig. 7-20

State-variable used for finding passive sensitivity

The denominator of the transfer function of Fig. 7-20 is given by:

$$D(s) = s^2 + s \frac{1}{R_1C_1} \frac{1+R_3/R_4}{1+R_Q/R'} + \frac{R_3/R_4}{R_1R_2C_1C_2} \qquad (7\text{-}87)$$

from which we see that:

$$S^{\omega_0}_{R_1,R_2,R_4,C_1,C_2} = -1/2 = -S^{\omega_0}_{R_3} \qquad (7\text{-}88)$$

From equation (7-87), we can get the Q as:

$$Q = \frac{1+R_Q/R'}{1+R_3/R_4}\left[\frac{R_3R_1C_1}{R_4R_2C_2}\right]^{1/2} \qquad (7\text{-}89)$$

From equation (7-89) we can get, for example:

$$S^Q_{R_Q} = \frac{2Q-1}{2Q} \qquad (7\text{-}90)$$

As an example, if Q is nominally 10, then it must have been set by:

$$Q = 10 = \frac{R' + R_Q}{2R'} \qquad (7\text{-}91)$$

according to equation (6-18), or $R_Q = 19R'$. If now R_Q is in error by +5%, then Q = 10.475, which is 4.75% high. According to equation (7-90), for a Q of 10 the sensitivity is 0.95, so a 5% change is R_Q should give a 4.75% change in Q, as we see. Thus we see a percentage change that is the same as the percentage change in the gain of the loop from V_B to V_H, as we suggested.

The dependence of Q on R_Q and R' is well understood, but from equation (7-89), Q also has sensitivities of 1/2 to R_1 and to C_1, and sensitivities of -1/2 to R_2 and to C_2. Q is also sensitive to R_3 and to R_4, but much less so than one might first expect. Even though a change of R_3 or R_4 causes a significant change of the gain across the summer, this is not reflected directly in changes of Q, since there are compensating effects. In fact, it takes nearly a 2:1 change of R_3 or R_4 just to cause a 5% change in Q. [Of course, the frequency would change by 50% while this was going on.] The Q is always reduced by an unequal ratio of R_3 to R_4. (See problem at end of chapter).

In total, the passive sensitivity of the state-variable approach is acceptable and comparable with the M.F.I.G. structures. Accordingly we next need to see how the poor active sensitivity can be fixed up by compensating integrators and summers.

7-7 COMPENSATION OF LINEAR CIRCUIT BLOCKS FOR THE EFFECTS OF REAL OP-AMPS

We have seen that "overdesign" was one approach to handling the problem of active sensitivity. Another approach, which works where a filter is actually made up of component blocks, is to improve the blocks individually. This can be done in the case of state-variable and signal-flow-graph "ladder" filters. These are composed of integrators and summing amplifiers. If we can improve these blocks, in effect before we use them, we should end up with a better filter. We will be looking at both passive and active ways of compensating these devices. It will be seen that a general method of locating poles and zeros for improved response can be employed.

We can begin with the most fundamental of our building blocks - the inverting integrator. We have looked at the inverting integrator using a real op-amp and found its transfer function [equation (7-80)]. This had the desired pole at s=0, but also a second pole at -G - 1/RC, which is at approximately -G in most cases. Passive compensation of the integrator is achieved by cancelling the extra pole. There are two ways of doing this, as seen in Fig. 7-21. In Fig. 7-21a, a small capacitor C' is placed in parallel with R. The voltage at the (-) input of the op-amp is thus:

$$V_- = \frac{V_{in}(1/sC) + V_{out}\frac{R}{1+sC'R}}{1/sC + R/(1+sC'R)} \qquad (7\text{-}92)$$

Using equation (7-26), with V+ grounded, also gives us that $V_- = -V_{out}(s/G)$. Thus we can solve for the transfer function with C' added as:

$$T(s) = \frac{-(1 + sC'R)}{s^2 \frac{R}{G}(C+C') + \frac{s}{G} + sCR} \qquad (7\text{-}93)$$

When we separate out the pole at s=0, what is left is:

$$T(s) = \frac{-(1 + sC'R)}{sCR\left[\frac{s(C+C')}{CG} + \frac{1}{CRG} + 1\right]} \qquad (7\text{-}94)$$

which has a zero at -1/C'R and a second pole at:

$$s_{p2} = \frac{-(RCG+1)}{(C+C')R} \qquad (7\text{-}95)$$

If we set the extra pole to the zero position, we arrive at, simply:

$$C' = 1/RG \qquad (7\text{-}96)$$

Analysis of the second structure, Fig. 7-21b gives a similar result:

$$R' = 1/CG \qquad (7\text{-}97)$$

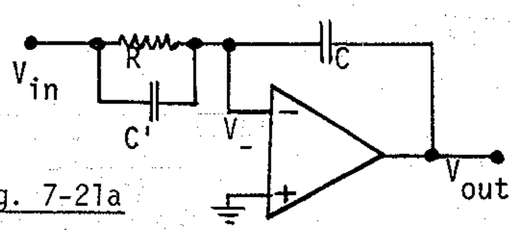


Fig. 7-21a

passive comp. of inverting integrator using shunt C'

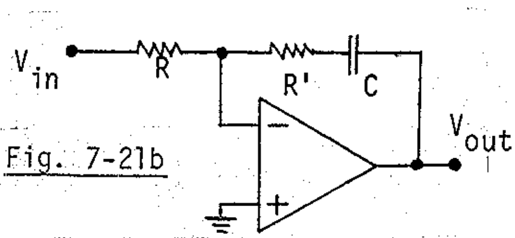


Fig. 7-21b

passive comp. of inverting integrator using series R'

This passive compensation technique seems very effective, since it seems possible to completely cancel the unwanted pole. However, it must be remembered that there are tolerances here to consider. We do not know exactly what value of G any given op-amp will have, and the capacitor (or resistor) we use to compensate it also has some tolerance associated with it. Nonetheless, even getting the extra zero relatively close to the unwanted pole can mean a substantial improvement.

There is also an active way of compensating the non-inverting integrator. This is done by putting a second op-amp configured as a follower in the feedback loop, as shown in Fig. 7-22. In this case it will be seen that we do not get a zero that cancels the extra pole. Rather we get a special array of one zero and two poles (in addition to the one at s=0) which has a favorable phase response. This type of array will prove the key to a good number of additional compensation schemes that we will look at later.

The analysis of Fig. 7-22 begins by recognizing that the op-amp follower in the feedback is known to us, and we can employ equation (7-28) to give:

$$V' = V_{out}\frac{G}{s+G} \qquad (7\text{-}98)$$

Now, applying equation (7-26) to the lower op-amp of Fig. 7-22 we have:

$$V_- = \frac{V_{in}(1/sC) + V'R}{R + 1/sC} = -\frac{s}{G}V_{out} \qquad (7\text{-}99)$$

which in turn gives a transfer function:

$$T(s) = \frac{-G(s+G)}{sCR[\,s^2 + s(G + 1/RC) + G(G+1/RC)]} \qquad (7\text{-}100)$$

Here we see that the added pole at -G which was put in the feedback loop has resulted in a zero at -G, something we might suppose will be very helpful since the inverting integrator has a zero there. Yet a curious thing happens in that the pole that was at -G has in effect already moved out, pairing with the feedback amplifier's pole. In fact, for the case where G is much greater than 1/RC, we can find the poles at:

$$s_{p1} = 0 \qquad s_{p2}, s_{p3} \approx -G/2 \pm jG\sqrt{3}/2 \qquad (7\text{-}101)$$

The pole/zero plot for T(s) of equation (7-100) is shown in Fig. 7-23. We need to consider if this is in any way better than just having a pole at -G, and if so, how much better, and why.

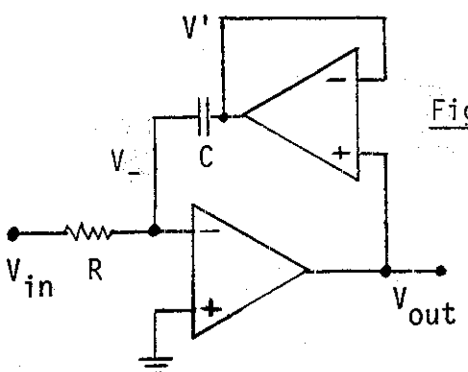


Fig. 7-22

Active Compensation of Inverting Integrator Using Follower in Feedback Loop

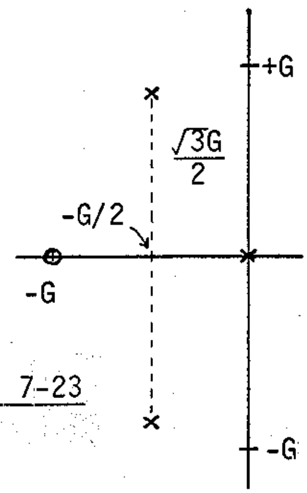


Fig. 7-23

In this case of an integrator, we expect a 1/f roll-off and a 90° phase shift. The pole at -G in the uncompensated case, or the pole/zero array of Fig. 7-23 in the actively compensated case do not in general have a major effect on the 1/f behavior of the integrator. However, they have an immediate effect on the phase. This can be seen by considering the pole at -G as an example. As we move from s=0 and begin up the jω-axis, the distance to the pole gets longer, but only very gradually at first, since we are moving perpendicular to the line to the pole. On the other hand, the phase begins to run immediately, being proportional to the distance along the jω-axis. Accordingly it is phase that is of concern to us first. In fact, it is what we call "excess phase" that is of concern: phase in excess of the 90° we do want and expect. It is this excess phase that is most responsible for the non-ideal behavior of circuits using the inverting integrator without compensation.

The excess phase of the uncompensated inverting integrator is determined by the extra pole at -G, and is given by (where frequencies are in units of G):

$$\phi = -\mathrm{Tan}^{-1}\,\omega \qquad (7\text{-}102)$$

In comparison, the excess phase due to the two poles and single zero of Fig. 7-23 is given as:

$$\phi = \mathrm{Tan}^{-1}\omega - \left[-\mathrm{Tan}^{-1}\frac{\frac{\sqrt{3}}{2}-\omega}{0.5}\right] - \mathrm{Tan}^{-1}\frac{\frac{\sqrt{3}}{2}+\omega}{0.5} \qquad (7\text{-}104)$$

Fig. 7-24 and Fig. 7-25 give plots of these phases. As a means of comparison, we also show a case of passive compensation where we assume that there is a 10% error between the actual G and the placement of the cancelling zero. [Of course, if the passive compensation is done perfectly, there is no excess phase at all.]

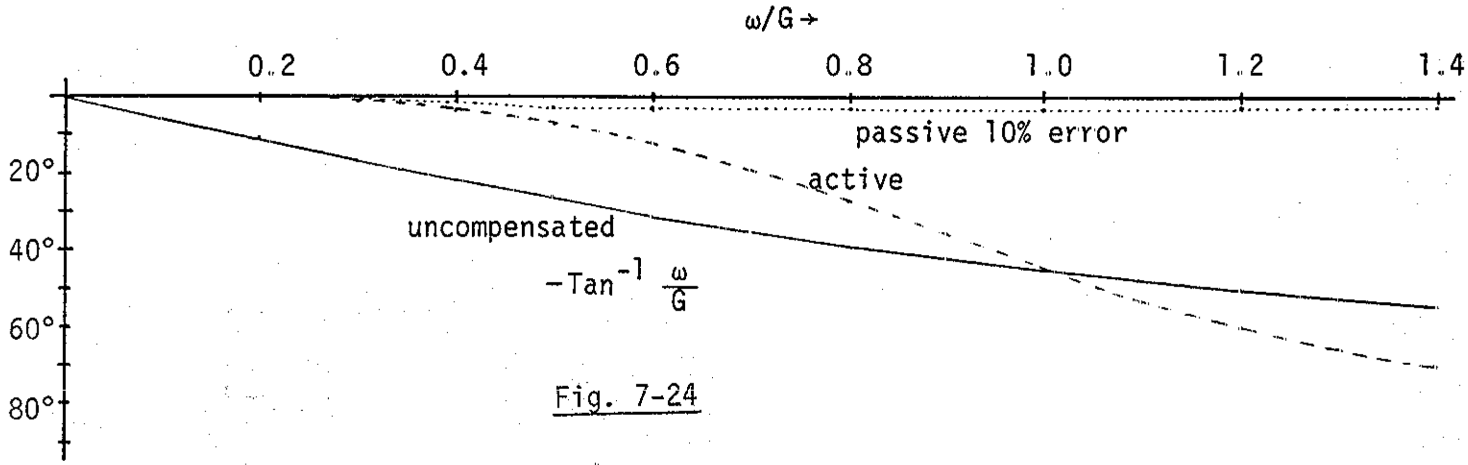


Fig. 7-24

In fact, from Fig. 7-24, it is clear that both the passive and the active compensations do work, as compared to the uncompensated case. Further, it seems from Fig. 7-24 that passive compensation is superior to active. However, we need to take a closer look at the low-frequency region, and this is shown in Fig. 7-25. From this we see that the active compensation is superior in that the slope of the active compensated case is zero at zero frequency, while that of the passive compensated case is non-zero. In fact, the active compensated case has only a small fraction of a degree of phase shift even at frequencies of 0.1G, which is equivalent to designing at 100,000 Hz with a 1 MHz op-amp - something we would be a bit hesitant to do without some assurance of compensation, since there would be a phase error or about 5° without any compensation.

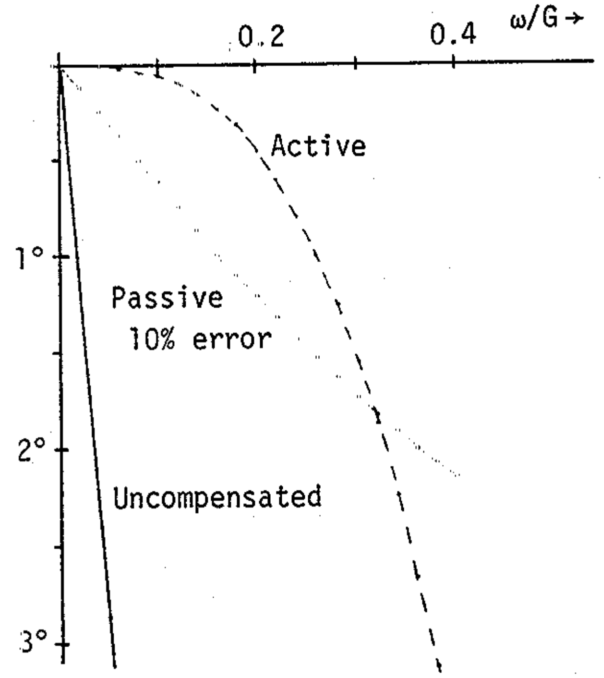


Fig. 7-25

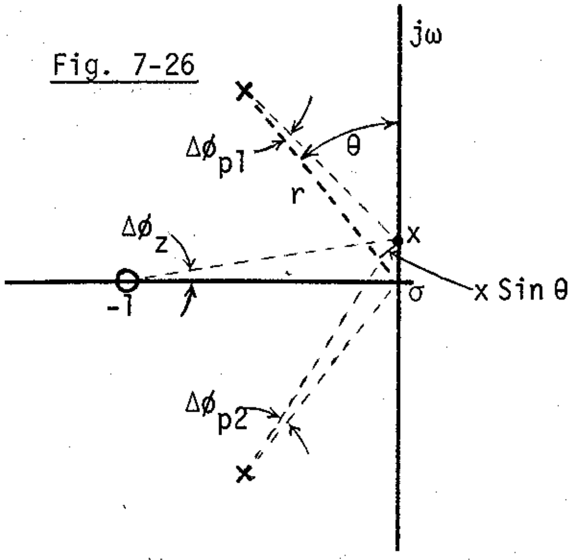


Fig. 7-26

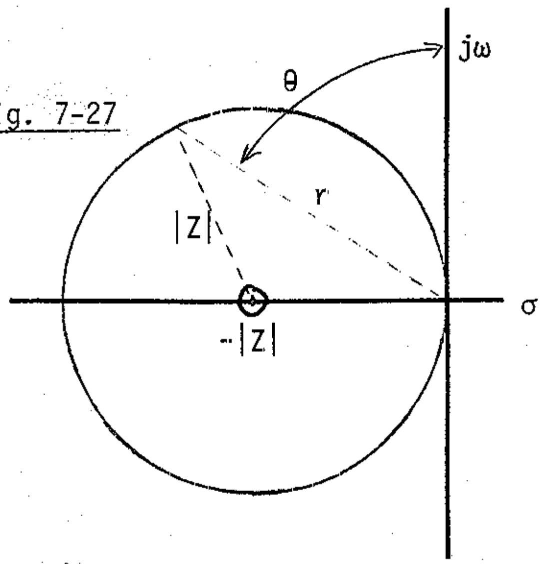


Fig. 7-27

Consider the general problem of replacing an unwanted pole with a zero and a pair of poles. Of course, if we could place a zero right on top of a pole, and the pole stands still, we just get rid of it, and that's great. This does seem to happen if we are successful in passive compensation of the integrator. But we can ask ourselves if it is possible to cancel out an op-amp-caused pole from an amplifier, the way we did with the inverting integrator using passive compensation. If we could, that would be great, as we would be getting an ideal op-amp back from a real one. That's not going to happen - we don't cancel the pole. Instead we are able to get an "array" of a zero and two poles - very similar to what we got in the active compensation of the inverting integrator - which has favorable phase properties. We saw in the active compensation of the inverting integrator that poles at $-1/2 \pm (\sqrt{3}/2)j$, relative to a zero at -1, had a favorable phase response (Fig. 7-25). Here we are looking for a general solution of which this is a special case. We will find the result useful for establishing and checking passive and active attempts to compensate amplifiers and other linear blocks.

Fig. 7-26 shows the case where a zero appears at -1, and a pair of poles appears at angle θ and radius r as shown. At DC, the phase adds up to zero, and we would like it to remain zero for a small frequency displacement x along the jω-axis. Thus we will be looking for the phase changes across the poles to cancel the phase change across the zero. We are assuming that the frequency change x is small, so the angles are all small, and can be approximated in radians by the arc length divided by the radius. The arc seen at the upper of the two poles is, by simple geometry, $x \sin\theta$, so the phase change is:

$$\Delta\phi_{p1} = x \sin\theta / r \qquad (7\text{-}105)$$

The phase change at the zero is it's arc divided by the radius (which is 1) so:

$$\Delta\phi_{z} = x/1 = x \qquad (7\text{-}106)$$

and the phase change at the lower pole is the same as for the upper one. The phase shift off the poles should be negative, relative to positive phase shifts off the zero, so for a net of zero phase change for small frequencies:

$$\Delta\phi_{total} = \Delta\phi_{z} - \Delta\phi_{p1} - \Delta\phi_{p2} = x - 2x \sin\theta/r = 0 \qquad (7\text{-}107)$$

which relates r and θ as:

$$r = 2 \sin\theta \qquad (7\text{-}108)$$

This is easily generalized for the case where the zero is at a distance $-|Z|$ as:

$$r = 2|Z| \sin\theta \qquad (7\text{-}109)$$

Thus equation (7-109) provides a simple check of a proposed phase compensation technique, or a means of achieving one if a free parameter allows for the adjustment of r or θ. The equation is seen also to represent a circle of radius $|Z|$ about the zero. Thus the poles should be placed on a circle, about the zero, such that the circle also passes through s=0. Note that equation (7-107) meets these conditions. Fig. 7-27 shows the "circle of solutions" that pertains to favorable phase. What this means is that the phase change, from DC to small frequencies, will start out with zero slope. Thus a region of near-zero phase change will also exist for sufficiently small frequencies.

Since there is a solution for any angle θ [equation (7-109)], we need to see which of these solutions are the best. It is not always the case that a proposed compensation method will have a pole/zero placement that satisfies equation (7-109). In other cases, the pole/zero placement may satisfy equation (7-109), but we are then stuck with that particular solution [as in equation (7-101) for example]. In still other cases we may be able to manipulate a free parameter to satisfy equation (7-109), and then perhaps even choose a more favorable soultion to (7-109). Here we will look at the possible solutions to see which is best, even though in many cases we do not have the freedom to choose among these. We will see that the intuitive notion that the further back these poles are, the better, applies to this consideration.

In evaluating the cases shown here, it should be remembered that the results are for poles placed relative to a zero at -1. While this zero might actually occur at -G, or at -G/4, or some such position relative to the G of the op-amp, we are scaling everything to a zero at -1. Thus we need to be aware not just of having poles as far back as possible, relative to the zero, but also having the zero far back to begin with. Thus for example, a solution with a zero at -G/10 and two poles at -G/20 may be best, given the necessity of a zero at -G/10. However, another solution with a zero at -G and poles at $-G \pm jG$ would be better, even though the poles, relative to the zero, are not as far back as in the first case.

Figures 7-28 and 7-29, along with corresponding Tables 7-1 and 7-2, show the advantage of solutions of equation (7-109) that have their poles as far back (largest negative real part, relative to zero) as possible. These figures show the phase and

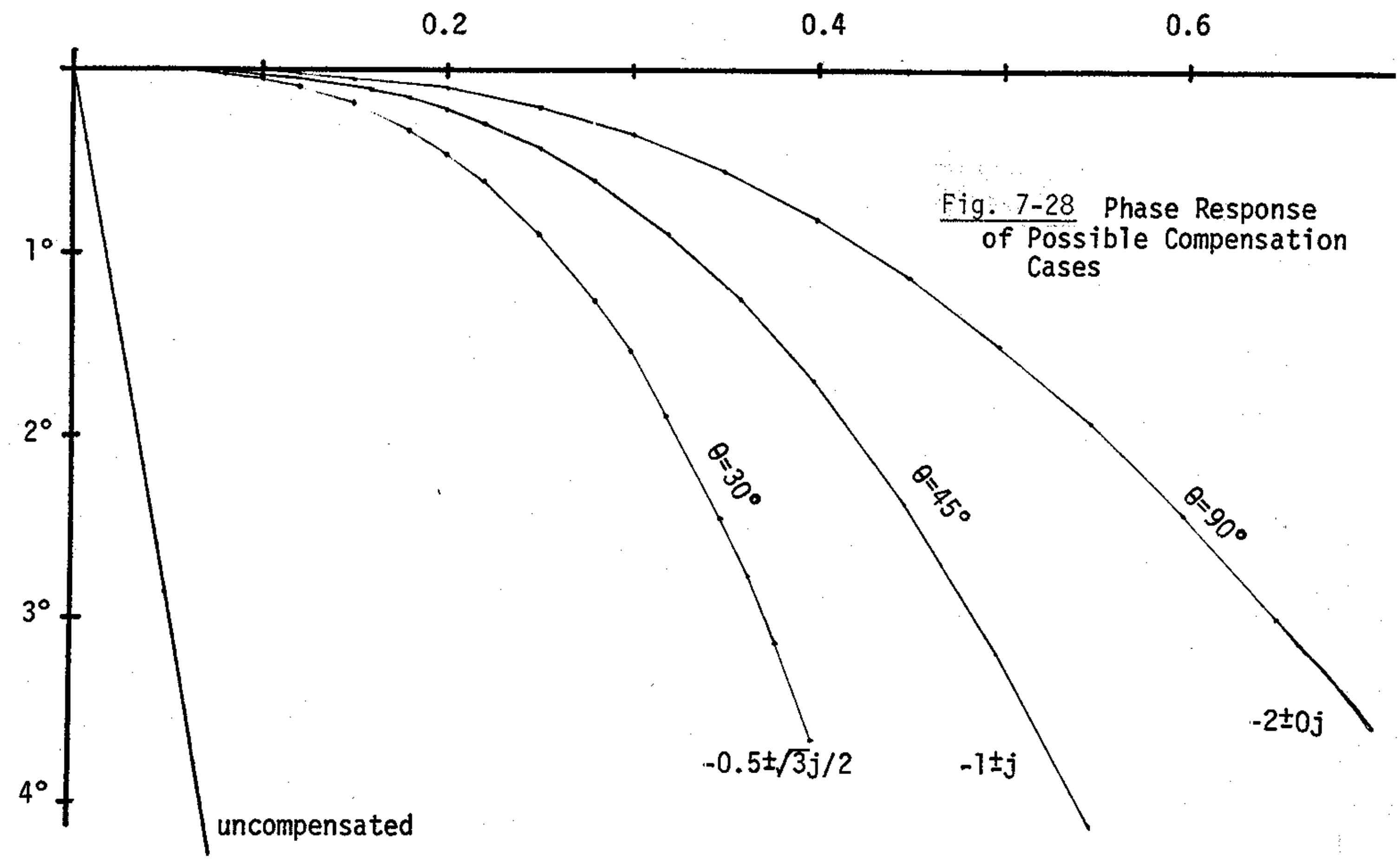


Fig. 7-28 Phase Response of Possible Compensation Cases

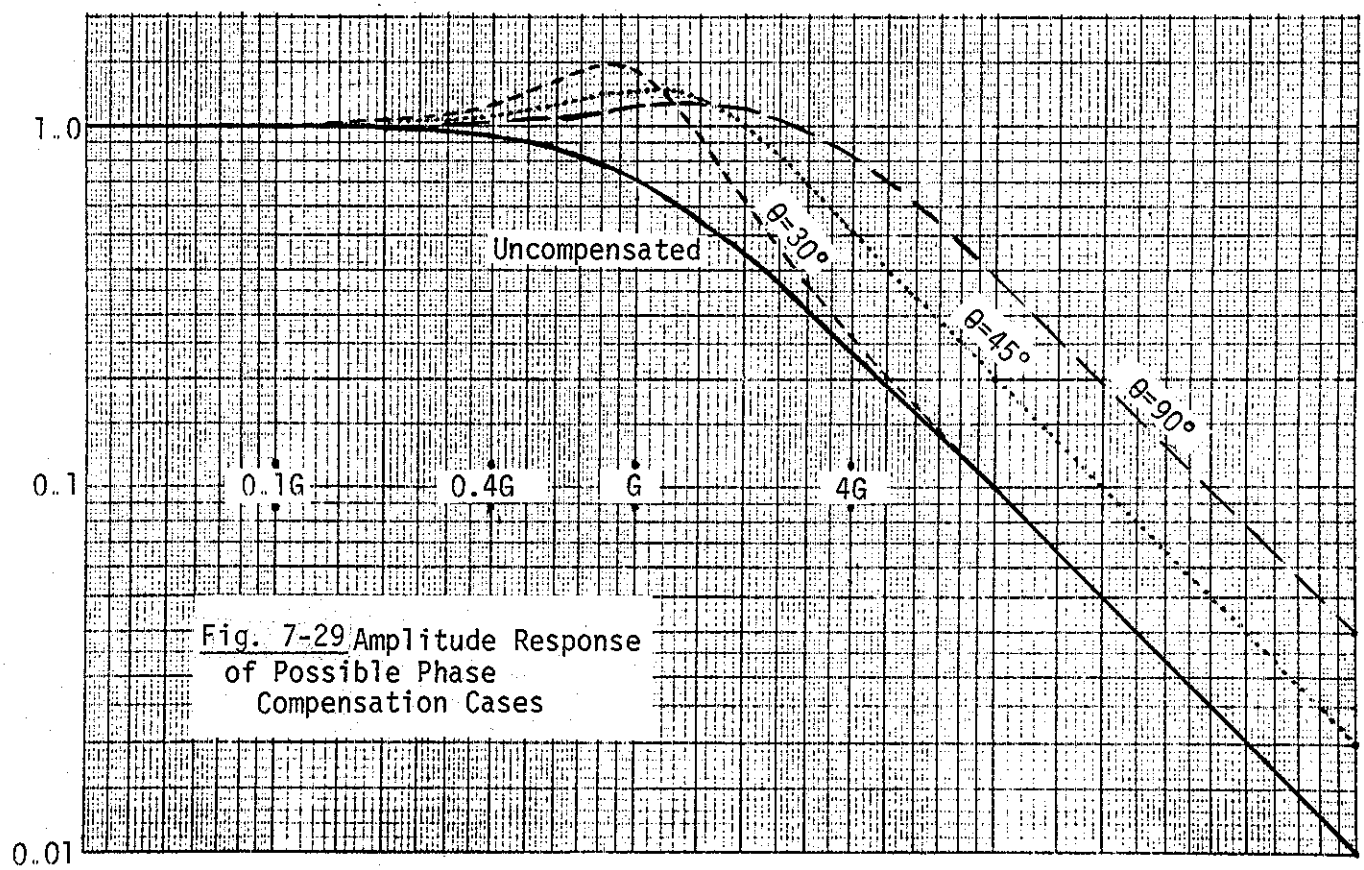


Fig. 7-29 Amplitude Response of Possible Phase Compensation Cases

Table 7-1: PHASE RESPONSE

Poles	-1	$-0.5\pm\sqrt{3}j/2$ (θ=30°)	-1±j (θ=45°)	-2±0j (θ=90°)
Zero	none	-1	-1	-1
Freq for 0.1° phase excess	0.00174	0.121	0.152	0.193
Freq for 1° phase excess	0.0174	0.26	0.33	0.43

Table 7-2: Amplitude Response

Poles	-1	$-0.5\pm\sqrt{3}j/2$ (θ=30°)	-1±j (θ=45°)	-2±0j (θ=90°)
Zeros	none	-1	-1	-1
Error at 0.01	0.99995 -0.005%	1.0001 0.01%	1.0000 0%	1.0000 0%
Error at 0.05	0.99875 -0.125%	1.0025 0.25%	1.0012 0.12%	1.0006 0.06%
Error at 0.1	0.99504 -0.5%	1.0100 1.0%	1.0050 0.5%	1.0025 0.25%
Error at 0.2	0.98058 -1.98%	1.0399 4%	1.0196 1.96%	1.0097 0.97%

amplitude responses for four cases. The first case is the uncompensated case, and the other three are compensated cases corresponding to angles θ of 30°, 45°, and 90°. In addition to showing the relative merits of different values of θ, a performance evaluation for any one of the cases is possible.

In general it is seen that the replacement of the pole with a zero and the pole pair greatly improves the phase response, and has relatively small effect on the amplitude response. In some cases, the amplitude response is a bit worse, while in others it is improved, although relatively speaking the effect on the amplitude response is minor, and is secondary to the vast improvement in phase response. The two tables are intended to supplement the information given on the graphs, giving some details that can not be seen on the graphs. It should be kept in mind that all frequencies given are relative to the position of the zero. This zero is usually placed as a significant fraction of G, so frequencies as small as 0.1 still represent fairly large operating frequencies for the linear circuit blocks using the op-amps.

Fig.7-28 thus shows the vast improvement of the use of the zero and pole pairing, giving phases of less than 0.1° at frequencies exceeding 0.1, relative to a phase of nearly 6° at 0.1 for the uncompensated. Table 7-1 further reflects these results. In Table 7-1 the case of no zero and a pole at -1 is the uncompensated case, while the other three cases, from left to right, correspond to angles of 30°, 45°, and 90°. The case of 90° places two poles at -2, and cancels the phase by virtue of two poles at twice the distance, relative to the zero.

Fig.7-29 shows that the effect of the use of the zero and pole pair is to cause a peaking in the amplitude response at higher frequencies. Note that if we were only concerned with amplitude response, that generally we find an increase in available bandwidth, if we can tolerate the peaking. For more modest frequencies, Table 7-2 shows that the error in the amplitude respose is a bit worse than the uncompensated case when the poles are to the right of the zero position, is equal to the uncompensated case when the poles are above and below the zero, and gets better as the poles move to the left of the zero. Thus the most favorable cases of phase correction (approaching the best case of poles at -2±0j) correspond to cases where there is also an improvement in amplitude response. In Table 7-2, two values are indicated in each box. The upper value is the actual response value, relative to an ideal value of 1, while the lower value represents the corresponding error in %. Note that for frequencies up to 0.1, the error is at most 1%.

Having now looked at both passive and active compensations of an inverting integrator, and then developed a general theory for a pole/zero array with a minimum amount or excess phase, we can turn our attention to amplifiers and summers, which are also blocks of our filters, in addition to being of inherent interest of their own. We will look at passive and active compensation techniques on the amplifiers. However, we have suggested earlier that we do not expect the extra op-amp-caused pole to just get cancelled, as this would give us back ideal op-amp performance. We will take this same approach, however, and see what happens.

Fig. 7-30 shows the inverting amplifier structure. It is clear that the voltage at the (-) input is given by:

$$V_- = \frac{V_{in}aR + V_{out}R}{aR + R} = \frac{aV_{in} + V_{out}}{a+1} \qquad (7\text{-}110)$$

In the ideal case, $V_- = V_+ = 0$, and we have $T(s) = -a$, an inverting amplifier with gain of a. In the case of the real op-amp, V_- is set equal to $-(s/G)V_{out}$ and we arrive at:

$$T(s) = \frac{-aG}{s(a+1) + G} \qquad (7\text{-}111)$$

which shows a pole at [compare same result from equation (7-34)]:

$$s_p = -G/(a+1) \qquad (7\text{-}112)$$

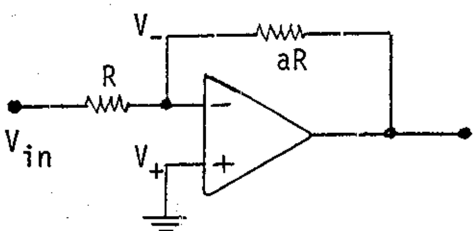


Fig. 7-30 Inverting Amplifier with Gain of a Using No Compensation

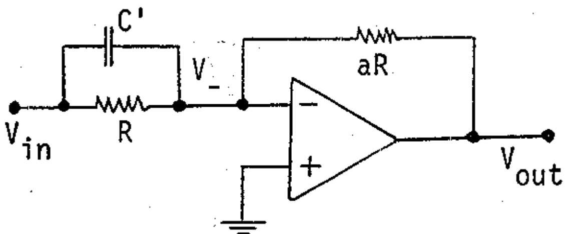


Fig. 7-31 Inverting Amplifier with Gain of a Using Passive Compensation with Capacitor C'.

First we attempt to look at passive compensation of the inverter through the use of a small capacitor C' in parallel with the input resistor R, as shown in Fig. 7-31. This has a voltage V- given by:

$$V_- = \frac{V_{in}aR + V_{out}\frac{R}{1+sC'R}}{aR + \frac{R}{1+sC'R}} = -\frac{s}{G}V_{out} \qquad (7\text{-}113)$$

From this the transfer function is obtained as:

$$T(s) = \frac{-\frac{G}{C'R}[1 + sC'R]}{s^2 + \frac{(1+a)}{aC'R}s + \frac{G}{aC'R}} \qquad (7\text{-}114)$$

Which has a zero at:

$$s_z = -1/RC' \qquad (7\text{-}115)$$

and poles at:

$$s_{p1}, s_{p2} = \frac{-(1+a)}{2a}\frac{1}{RC'} \pm \frac{j}{2a}\frac{1}{RC'}\sqrt{4GRC'a - (1+a)^2} \qquad (7\text{-}116)$$

The basic ploy here is to choose a value of C' such that the poles of equation (7-116), relative to the zero of equation (7-115), meet the general solution of equation (7-109). While an analytic match probably would not be too difficult, here an empirical approach was used, based on the intuitive idea that the zero should be matched to the original pole. We thus set equations (7-112) and (7-115) equal, giving:

$$-1/RC' = -G/(a+1) \qquad (7\text{-}117)$$

Making this substitution into equation (7-116) gives us:

$$s_{p1}, s_{p2} = -G/2a \pm (jG/2a)\sqrt{\frac{3a-1}{a+1}} \qquad (7\text{-}118)$$

Calculation of these poles shows that they do in fact correspond to one of the solutions of equation (7-109). Some of the solutions are plotted in Fig. 7-32 for different values of a. Note that the a=1 case corresponds to the simple inverter.

It should be clear that the use of passive compensation of the inverting amplifier stage is both useful and simple to implement. It is useful in that the phase response can be greatly improved, and that the magnitude response may be significantly improved, or at least not greatly harmed. For example, the case of a unity gain inverter in Fig. 7-32 corresponds exactly to the -1±j case of Figures 7-28 and 7-29 examined earlier. It is simple in that the determination of C' is just a matter of placing a zero in the position where the pole was, and the desirable placement of a zero and two poles results automatically. The only needed design equation is thus:

$$C' = (a+1)/RG \qquad (7\text{-}119)$$

Having successfully found a passive compensation technique for the inverting amplifier, we might try an active compensation technique that worked in the case of the inverting integrator - the follower in the feedback loop, as seen in Fig. 7-33. This does not in fact work, and we must use a different technique that will lead to a new compensation principle, which is seen in Fig. 7-34, and will be discussed in a moment. First, we analyze Fig. 7-33, and get its transfer function as:

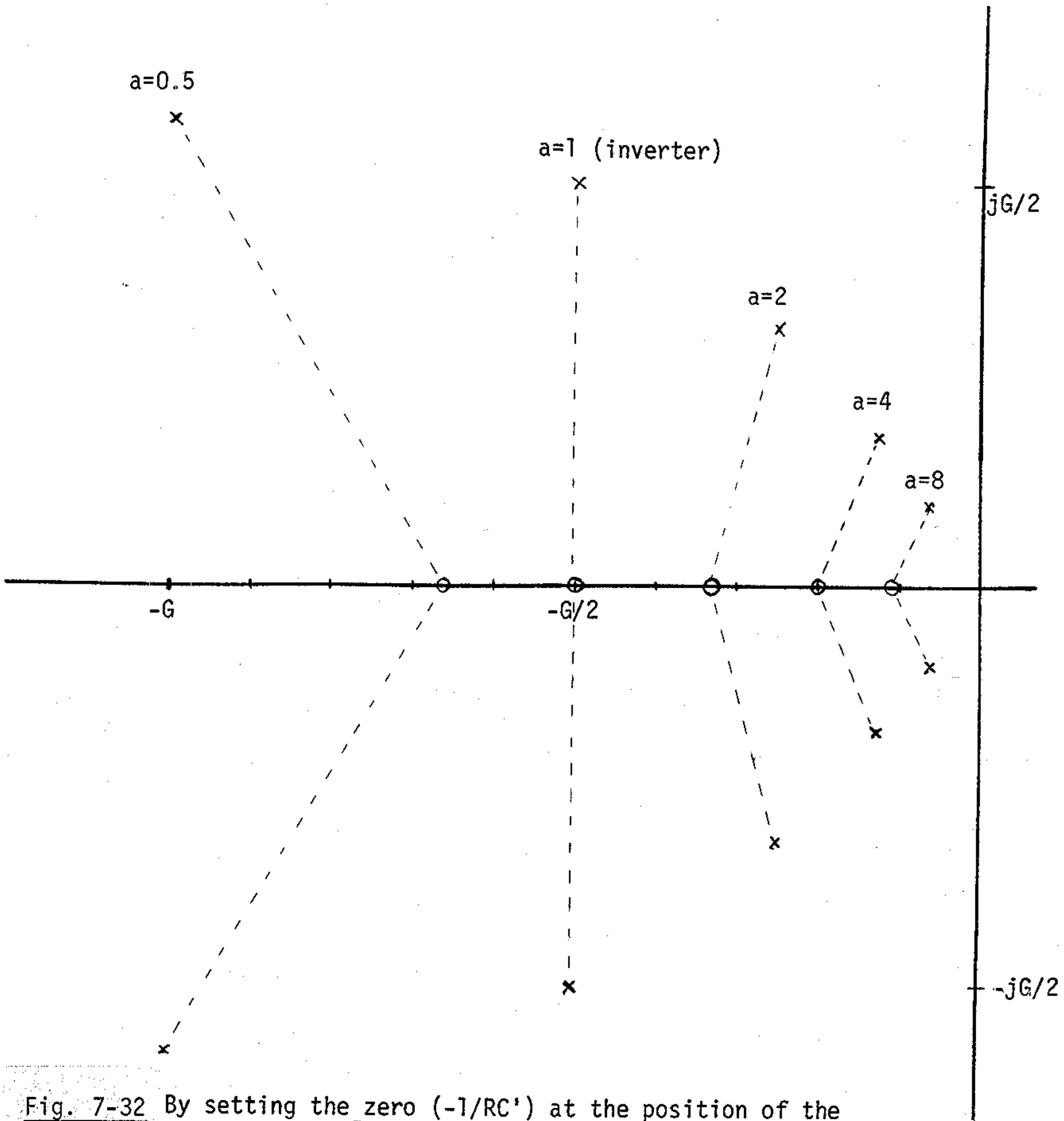


Fig. 7-32 By setting the zero (-1/RC') at the position of the original pole [-G/(1+a)] a positioning of a zero and two poles with favorable phase is achieved.

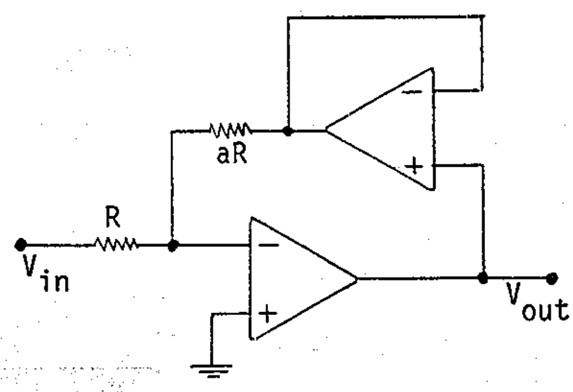


Fig. 7-33 An Attempt at Active Compensation of the Inverting Amplifier (unsuccessful)

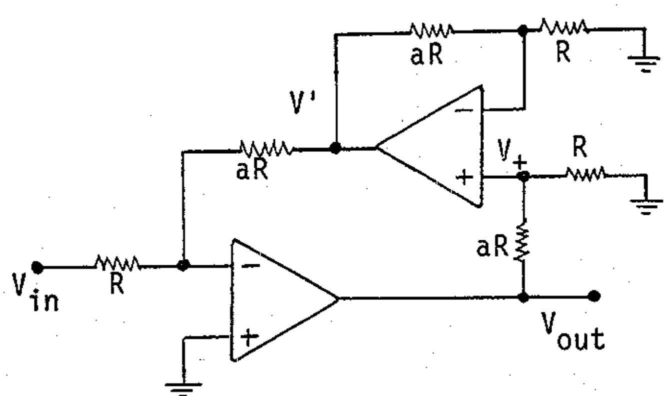


Fig. 7-34 Successful Active Compensation of of Inverting Amplifier through Use of Gain of (1+a) in the feedback loop

$$T(s) = \frac{-Ga(s+G)/(a+1)}{s^2 + sG + G^2/(a+1)} \quad (7\text{-}120)$$

This has a zero at s=-G, and poles at:

$$s_{p1}, s_{p2} = -G/2 \pm (G/2)\sqrt{(a-3)/(a+1)} \quad (7\text{-}121)$$

This has a favorable phase solution only for a=0 (the same solution as the inverting integrator) which is not useful since it represents zero gain. For useful gains where a is greater than 0, the pole pair always lies inside the required circle rather than on it. For the unity gain inverter (a=1) the poles are at -G/2 ± jG/2 which is inside the circle of radius G centered at -G.

A successful method of achieving proper active compensation is shown in Fig. 7-34. What has been done here is to put an op-amp in the feedback loop, such that the overall configuration around the op-amp has unity gain, but so that the amplifier is working at a noise gain that is the same as that of the original op-amp. This is a principle that we will find to work in general. This principle of placing a pole in the feedback loop which is the same as the pole of the original configuration, is in itself a case of placing a zero on top of the original poles, as is the usual case with passive compensation. The result is the placement of a zero at the original pole postion, and the proper placement of a pole pair for favorable phase response.

In this particular case of Fig 7-34, applying equation (1) to the non-inverting amplifier at the top would give a transfer function from the (+) input to the output V' of:

$$V'/V_+ = (1+a)G/[s(1+a) + G] \quad (7\text{-}122)$$

and the voltage V_+ is in turn determined from V_{out} by the voltage divider which provides a loss of 1/(1+a) so the net gain from V_{out} to V' is ideally 1 and for the real op-amp:

$$V'/V_{out} = G/[s(1+a) + G] \quad (7\text{-}123)$$

Thus for the lower op-amp we have:

$$V_- = \frac{V_{in}aR + V_{out}GR/[s(1+a)+G]}{R + aR} = -\frac{s}{G}V_{out} \quad (7\text{-}124)$$

Which is solved to give the transfer function of Fig. 7-34 as:

$$T(s) = \frac{\frac{-aG}{(1+a)^2}[s(1+a) + G]}{s^2 + Gs/(1+a) + G^2/(1+a)^2} \quad (7\text{-}125)$$

This equation has a zero at:

$$s_z = G/(1+a) \quad (7\text{-}126)$$

and poles at:

$$s_{p1}, s_{p2} = -\frac{G}{2(1+a)} \pm \frac{jG\sqrt{3}}{2(1+a)} \quad (7\text{-}127)$$

Here equations (7-126) and (7-127) represent one of the successful solutions of equation (7-109), for the θ = 30° case in fact.

We are interested in improving amplifiers for their own sake, especially as this may lead to improved autio circuits and the like. However, in filter design, we are equally concerned or more concerned with summers. Accordingly we will now look at passive and active compensation of inverting summers. Fig. 7-35 shows a passively compensated inverting summer. The transfer function is given by:

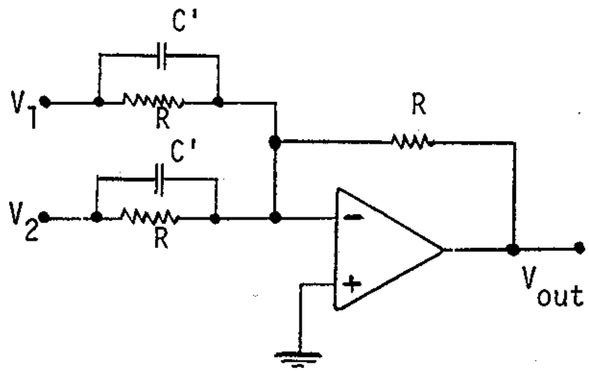


Fig. 7-35 Passive Compensation of the Inverting Summer

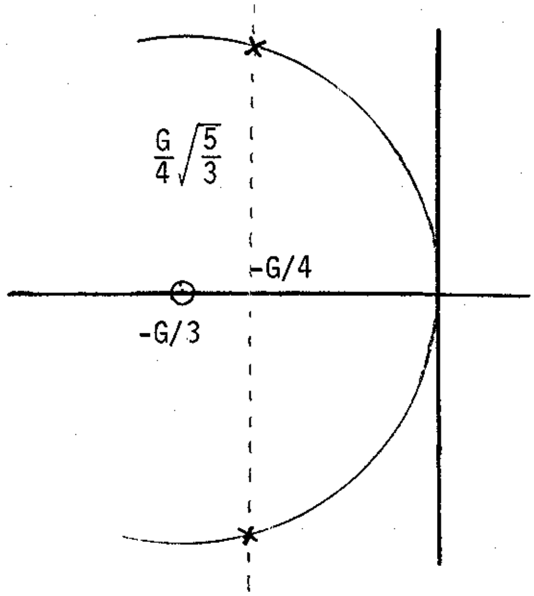


Fig. 7-36 Pole/Zero Plot for Fig. 7-35 with 1/RC' = G/3

$$T(s) = \frac{V_{out}}{V_1 + V_2} = \frac{(-G/2C'R)(1 + sC'R)}{s^2 + \frac{3s}{2C'R} + \frac{G}{2C'R}} \qquad (7\text{-}128)$$

Equation (7-128) has a zero at -1/RC' and poles at:

$$s_{p1}, s_{p2} = \frac{-3}{4RC'} \pm \frac{1}{2}\sqrt{\frac{9}{4}\frac{1}{(RC')^2} - 2G\frac{1}{RC'}} \qquad (7\text{-}129)$$

The procedure here will be to choose C' so that the zero falls on the position where the original poles was. Without the capacitors C', the network has a pole at -G/3. Setting 1/RC' = G/3 we find that the poles fall at:

$$s_{p1}, s_{p2} = -G/4 \pm (jG/4)\sqrt{5/3} \qquad (7\text{-}130)$$

This pole zero positioning is shown in Fig. 7-36, and it can be seen that the poles do in fact fall on a circle passing through s=0 centered on the zero. Thus it is one of the favorable phase cases, and the compensation is successful.

We can extend these ideas to cases where there are more than two inputs, where the circuit amplifies as well as sums, and where the inputs are not equally summed (so that the input R resistors are not equal). In such a case, it is necessary to find the fraction of the output voltage that is fed back to the (+) input when all the input voltages are grounded (and no C' capacitors). This fraction is the reciprocal of the noise gain G_N. The pole of the op-amp would thus be at $-G/G_N$. [Above the noise gain was 3, and in the original one input inverter, it was 1+a (Fig.7-31)]. Finally it would be necessary to adjust the individual C' to the particular R so that 1/RC' = G/G_N.

Fig. 7-37 is a fairly simple extension of the inverting amplifier of Fig. 7-34. The upper op-amp has an overall gain of 1 from V_{out} to V' when the op-amp is ideal. In the real case, the combination of the attenuator on the (+) input of the upper op-amp, along with the non-inverting gain gives us (using the noise gain idea):

$$\frac{V'}{V_{out}} = \frac{G}{s(1+3a) + G} \qquad (7\text{-}131)$$

The voltage V_- for the lower op-amp is then given as:

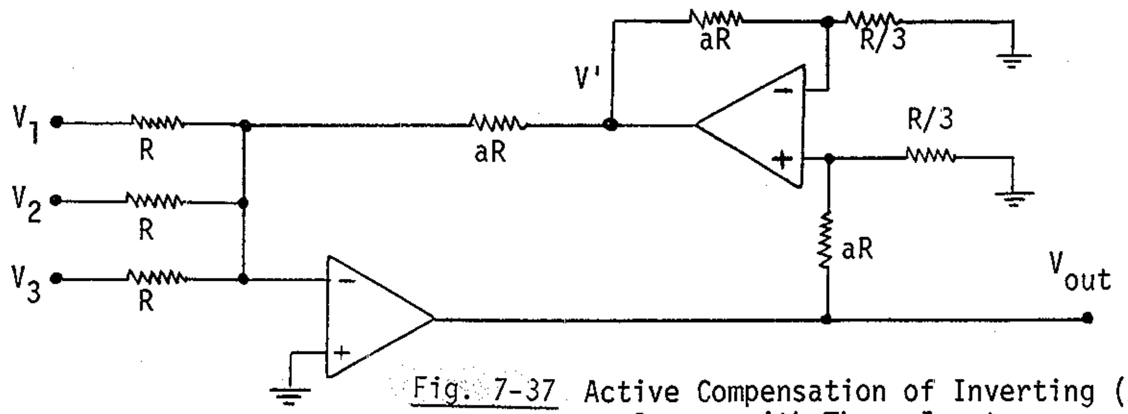


Fig. 7-37 Active Compensation of Inverting (Amplifying) Summer with Three Inputs

$$V_- = \frac{a(V_1+V_2+V_3) + GV_{out}/[s(1+3a)+G]}{1+3a} = -\frac{s}{G}V_{out} \qquad (7\text{-}132)$$

and from this:

$$T(s) = \frac{V_{out}}{V_1+V_2+V_3} = \frac{\frac{-Ga}{(1+3a)^2}[s(1+3a)+G]}{s^2 + Gs/(1+3a) + G^2/(1+3a)^2} \qquad (7\text{-}133)$$

This has poles and zeros at:

$$s_z = -G/(1+3a) \qquad (7\text{-}134)$$

$$s_{p1}, s_{p2} = \frac{-G}{2(1+3a)} \pm \frac{jG\sqrt{3}}{2(1+3a)} \qquad (7\text{-}135)$$

These are clearly the $\theta = 30°$ solution of equation (7-109). Notice how the solution carries the number of inputs (3) through so that it appears always in conjunction with 3a, so it is fairly obvious how to modify things if we have a different number of inputs.

To handle the more general case, we have to keep in mind that the feedback op-amp has two identical attenuators so that its gain from V_{out} to V' is 1. This attenuation is the same as that from V' to the (-) input of the lower op-amp. The pole/zero pattern has a zero at G times this attenuation factor, and the poles are the $\theta = 30°$ solution to equation (7-109). Notice that while all have the same $\theta = 30°$ solution, not all these are equally good. When the gain goes up and/or the number of inputs goes up, the solution circle gets smaller, with the poles and zero moving in. Thus there is the usual gain-bandwidth product penalty to pay, and nothing is free here.

We can also look at non-inverting structures, and consider passive and active compensation for these. Fig. 7-38 and Fig. 7-39 show a non-inverting amplifier and a three-input non-inverting summer, with passive compensation applied. Without compensation, these structures have a pole at -G/(1+a). The transfer function of Fig. 7-38 is found to be:

$$T(s) = \frac{\frac{G}{C'R}\left[\frac{(1+a)}{a} + sC'R\right]}{s^2 + \frac{(1+a)}{aC'R}s + \frac{G}{aC'R}} \qquad (7\text{-}136)$$

Equation (7-136) is identical to equation (7-114) for the inverting amplifier except for the term in brackets in the numerator which is here (1+a)/a instead of just 1 [and equation (7-114) has a - sign of course]. Both have the same poles. The zero is at:

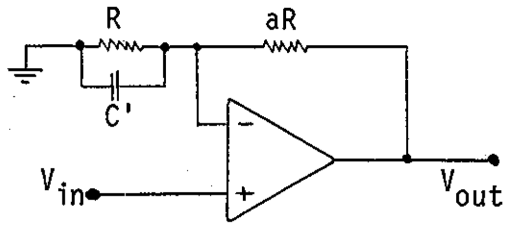


Fig. 7-38 Passively Compensated Non-Inverting Amplifier (Ideal case omits C')

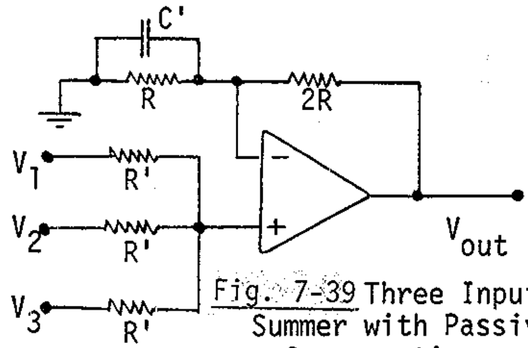


Fig. 7-39 Three Input Summer with Passive Compensation

$$s_z = \frac{-(1+a)}{a}\frac{1}{RC'} \tag{7-137}$$

If we follow our usual procedure now, we had an original pole at -G/(1+a) and setting this to the zero of equation (7-137) gives us:

$$C' = (a+1)^2/aRG \tag{7-138}$$

This choice gives us poles of equation (7-136) at:

$$s_{p1}, s_{p2} = \frac{-G}{2(1+a)} \pm \frac{jG\sqrt{3}}{2(1+a)} \tag{7-139}$$

and since $s_z = -G/(1+a)$, we recognize this as one of the favorable phase solutions, the familiar $\theta = 30°$ solution to equation (7-109). Notice that unlike equation (7-116), here we get a solution where θ is independent of a. The equation for the compensating capacitor C' is simple to use in both cases.

Fig. 7-39 shows a simple way in which a passively compensated non-inverter can be made into a passively compensated non-inverting summer. Here shown for a case of three inputs, the inputs are averaged by the passive network of three R' resistors, and then amplified by a gain of 3. The capacitor C' is set according to equation (7-138) with a equal to 2, as should be clear. It should be clear how this can be extended for more inputs, and for a summer with gain. Unequal sums can be handled as long as the passive averaging at the (+) input is properly solved.

Fig. 7-40 shows how the active compensation of the non-inverting amplifier can be accomplished. As in previous cases, the amplifier is put in the feedback loop with noise gain equal to that in the original case. In this particular case, this is particularly evident since we can even share the feedback attenuator between the two amplifiers. The attenuator on the right side of Fig. 7-40 serves to give the feedback op-amp an overall gain of 1. Normally this upper op-amp would have its own feedback attenuator (as in Fig. 7-34), and this could be done. However, the non-inverting amplifier itself has its gain equal to the noise gain, so in putting

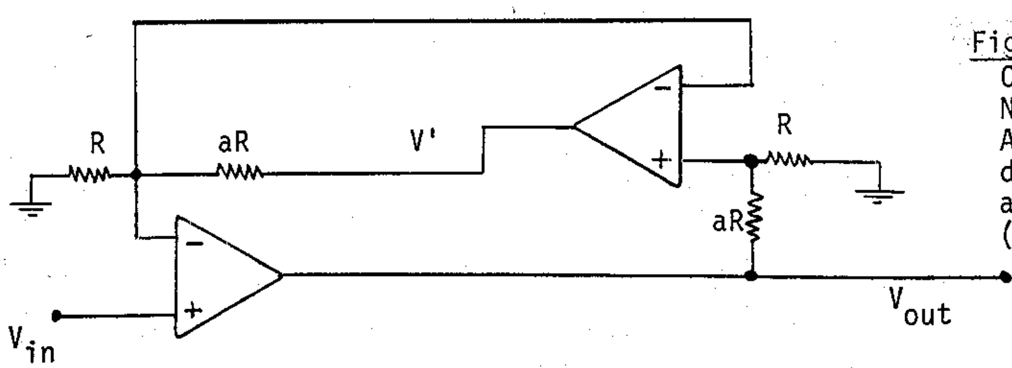


Fig. 7-40 Active Compensation of the Non-Inverting Amplifier (note the double use of same attenuator on both (-) inputs)

an amplifier in the feedback with gain equal to the noise gain, we are using the same attenuator. Of course it makes sense to save parts, and here there is no chance of a mismatch of passive components, and Fig. 7-40 is a particularly attractive case.

Fig. 7-40 is analyzed by realizing that V' is obtained from V_{out} exactly as in equation (7-123). and then using equation (7-26) we have:

$$V_{out} = \frac{G}{s}\left[V_{in} - \frac{1}{1+a} \frac{GV_{out}}{s(1+a) + G} \right] \qquad (7\text{-}140)$$

and from this the transfer function is found as:

$$T(s) = \frac{\frac{G}{1+a}\left[s(1+a) + G \right]}{s^2 + \frac{Gs}{1+a} + \frac{G^2}{(1+a)^2}} \qquad (7\text{-}141)$$

This is identical to equation (7-125) except for the (-) sign and a factor of a/(1+a) which appears in equation (7-125). We can understand these different factors if we take the limiting forms of equations (7-125) and (7-141). As G goes to infinity, equation (7-125) goes to -a as it should, while equation (7-141) goes to (1+a) as it should. Since both equations have the same poles and zeros, equation (7-141) is obviously also one of the favorable phase cases.

Fig. 7-41 shows how active compensation of a non-inverting summer can be achieved. Here a three input case is shown, but the results can be generalized in the way we have been doing. Note that here we are summing on a (+) input which becomes a summing node by virtue of the fact that an inverter is put in the feedback loop. This may look strange at first. All that remains is to achieve the same noise gain in both amplifiers, and this is the purpose of the R/2 resistor shown, which is the same as giving the upper op-amp the same number of input resistors (3) that the lower one has, thus equalizing the attenuation factors (and thereby the noise gains).

We can consider the R/2 resistor as being two R resistors to ground, and thus the (-) input of the upper op-amp is a resistor average of four voltages, one of which is V', a second of which is V_{out}, and the other two or which are zero. Thus for the upper op-amp V_- is $(V_{out}+V')/4$, and using equation (7-26) we have:

$$V' = \frac{-GV_{out}}{(G + 4s)} \qquad (7\text{-}142)$$

Likewise the (+) input of the lower op-amp is the average of four voltages:

$$V_+ = (V_1+V_2+V_3+V')/4 \qquad (7\text{-}143)$$

and using equation (7-26) we get the transfer function of Fig. 7-41 as:

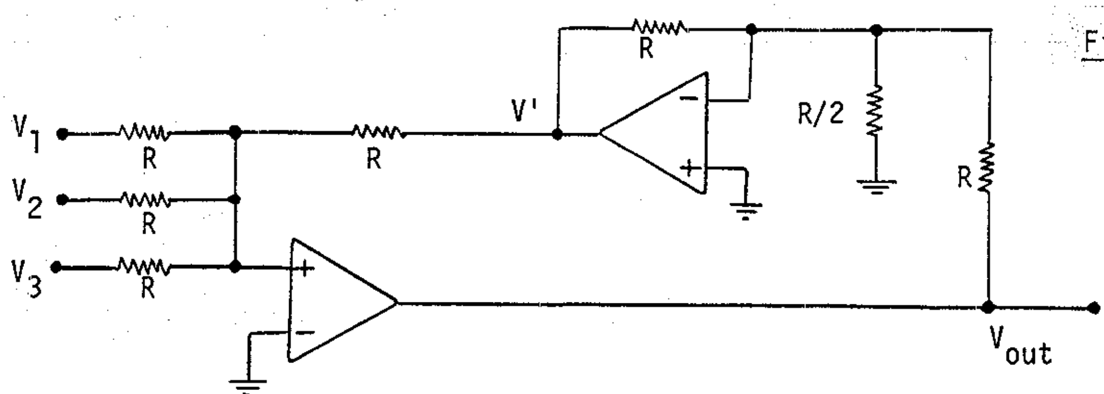


Fig. 7-41 Active Compensation of the Non-Inverting Summer.

$$T(s) = \frac{(G/16)(G + 4s)}{s^2 + Gs/4 + G^2/16} \qquad (7\text{-}144)$$

which has a zero at -G/4 and poles:

$$s_{p1}, s_{p2} = -G/8 \pm jG\sqrt{3}/8 \qquad (7\text{-}145)$$

and this is one of our familiar θ = 30° solutions to equation (7-109), and thus one of our favorable phase cases.

In some cases, we may be successful in achieving phase compensation, but end up in inverted form, or need an inversion that is not present. In such a case, we consider adding on an op-amp inverting stage. This however costs us in that the inverter itself brings in an extra unwanted pole (at G/2 in the case of a unity-gain inverter). Thus we would have to think about adding phase compensation to this extra inverter. However, in many cases, there is an easier and better way. This nice method can be used to achieve a phase-free inversion as long as the op-amps are well matched in G.

Fig. 7-42 shows the usual way in which an op-amp inverter is added on. Here we are considering the special (but not at all unusual) case where the stage to be inverted involves a grounded (+) input (virtual ground). Normally we would just ground the (+) terminal of the added inverter, as shown. However, in the ideal case we can just as well connect this (+) input back to the virtual ground of the previous stage, as seen in Fig. 7-43. There would be no real reason for doing this in an ideal situation. However, in the case of real op-amps, something quite unexpected happens.

We can see, using equation (7-26) as usual, that:

$$V_2 = -(G/s)V_1 \qquad (7\text{-}146)$$

and likewise, for the added op-amp:

$$V_4 = (G/s)[V_1 - V_3] = (G/s)[V_1 - \frac{V_2 + V_4}{2}] \qquad (7\text{-}147)$$

putting equation (7-146) into equation (7-147) we obtain:

$$V_4 = \frac{G}{s}\left[-\frac{s}{G}V_2 - \frac{V_2}{2} - \frac{V_4}{2}\right] \qquad (7\text{-}148)$$

or:

$$V_4/V_2 = -[1 + (G/2s)]/[1 + (G/2s)] = -1 \qquad (7\text{-}149)$$

This is a pure inversion, dispite the fact that an op-amp is involved. We can think of it as using the first op-amp to pre-compensate the second. The idea is so useful that we can consider using it in most new designs, and even possibly to modify old circuits.

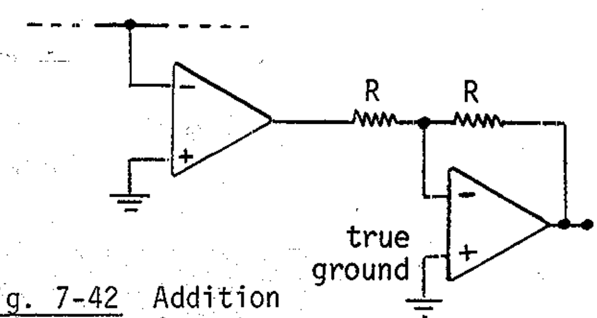


Fig. 7-42 Addition of op-amp inverter.

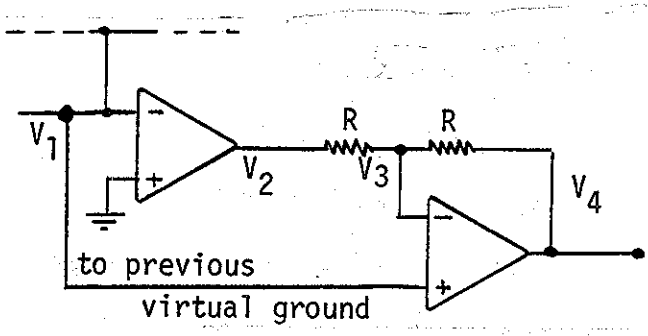


Fig. 7-43 Connection for phase-free inversion.

(ACTIVE SENSITIVITY IN A COMPENSATED STATE-VARIABLE FILTER)

----continued from page 2

that is far away and relatively harmless, while the two "desired" poles are close to (or not so close to!) our desired second-order pole positions. Whether or not the poles are sufficiently nominal depends primarily on the pole radius (essentially, the frequency at which the filter section is to operate) relative to the gain-bandwidth product G of a particular op-amp. Filters designed for lower frequencies can be expected to be near nominal.

In cases where the actual poles are too much displaced from their nominal positions, we can do what we call "overdesign." In this procedure, perhaps we find that with a real op-amp, the pole radii drop to 95% the desired value while the Q rises to 108% the desired value. Intuitively we might redesign the network, now with the frequency 5% higher, and the Q 8% lower than the original specifications, supposing that the real op-amp will still cause the frequency to fall by about 5% and the Q to rise by about 8%. If necessary, additional iterations (trial and error) can be used to get as close as necessary to the nominal case. In general, much or all of this iteration is simply a matter of computer calculations. It is somewhat analogous to building a bridge, and finding that it sags under its own weight, so you rebuild it, bulging it up slightly, so that it sags back to exactly where you intended it.

In the second approach, favored in cases with more op-amps, and for configurations composed of simple "building blocks" such as integrators and summers, we displace the usual active sensitivity problem by stating that, in cases where it is a problem, we will replace our simple (ideal) op-amp realizations of these building blocks with improved ("compensated") blocks. This we do using extra resources, passive, or active compensation tricks (a few "trimming" passive components, or an extra op-amp). In the bridge analogy, we invest more resources in stronger beams so that sag is not expected.

It would seem that there is a clear advantage to the first (overdesign) approach, as aside from some engineering time, it costs us less in production. This is true. But there are cases where it is not practical. Readers of this newsletter are familiar with voltage-controlled filters as used in electronic music synthesizers, which have variable characteristic frequencies (pole radii) and Q (pole angle). Clearly overdesign is only well-suited to filters with fixed parameters. In such a case, we have in mind a particular frequency and Q, and we calculate the necessary (non-nominal) values of resistors and capacitors, and solder them in place. With a voltage-controlled filter, both frequency and Q may change rapidly, continuously, and over a wide range. We HAVE TO fix the building blocks of the filter in this case.

For reasons of algebraic entanglement, we have tended to neglect recalculation once compensated building blocks are designed and employed in place of the "textbook" blocks. Yet the supposition that the method is sound is suggested first by its reasonable nature, and is borne out in experiments. Indeed, any of us who has built a voltage-controlled filter with the compensating shunt capacitors across the transconductor input stage has done one form of this experimental verification. Over the years, we have seen numerous other successful experiments. Yet, why not do the calculations for at least one example?

AN IDEAL STATE-VARIABLE FILTER

Our starting point is the three op-amp version of the state-variable filter first presented in Chapter 6 of Analog Signal Processing (last issue), and which is used as the active-sensitivity example in Chapter 7 (this issue). The circuit is repeated in Fig. 1 below:

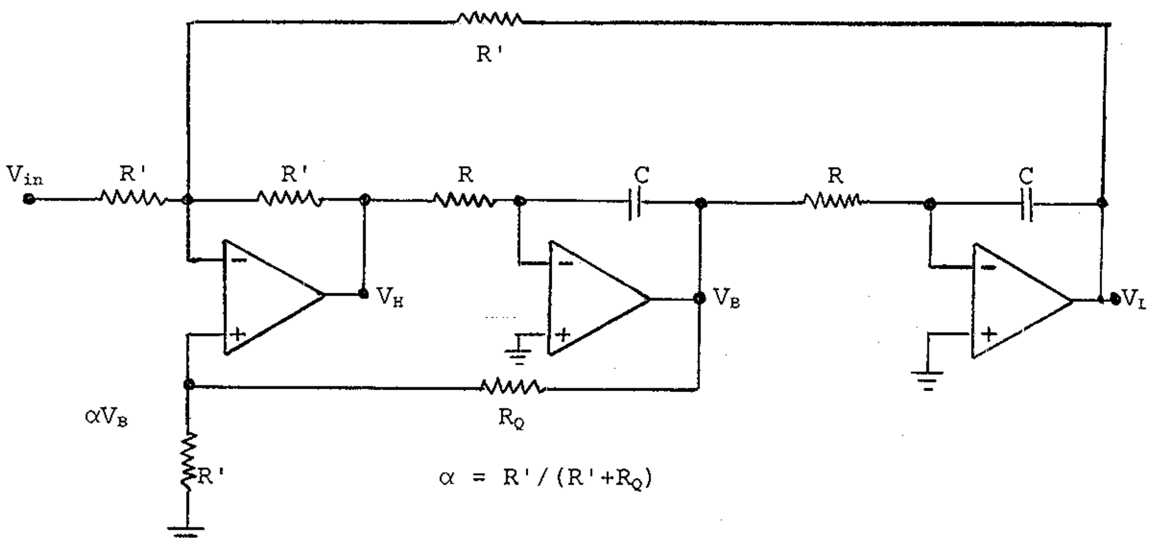


Fig. 1 Uncompensated State-Variable Filter

In the ideal op-amp case, $V_+=V_-$ for all three op-amps and the high-pass transfer functions is purely second order:

$$T_H(s) = V_H(s)/V_{in}(s) = -s^2 / (s^2 + 3\alpha s/RC + 1/R^2C^2) \quad (2)$$

so the nominal poles are at:

$$s_{p1,p2} = -\omega_0/2Q \pm j\omega_0(\sqrt{4Q^2-1})/2Q \quad (3)$$

where:

$$\omega_0=1/RC \quad (4)$$

is the pole radius and the Q is related to the attenuation factor α as:

$$Q = 1/3\alpha \quad (5)$$

THE NON-IDEAL STATE-VARIABLE FILTER

The case of the non-ideal (real op-amp) version is covered in Section 7-6 of Chapter 7 (this issue) with the result (Fig. 7-19 of Chapter 7) showing relatively poor active sensitivity. Note that at this point, we could easily employ the overdesign approach, just as we have for one op-amp versions of second-order networks. Again, we would be restricted to fixed parameter cases, but otherwise, there would be no additional problems of much significance.

Instead at this point we have "bailed out" stating that we would fix up the summer and the integrators later, and this we did in detail and in several ways in Section 7-7. But we did not actually put the pieces together.

PUTTING IN THE COMPENSATED BLOCKS

Where are we? Well, ideal op-amps gave perfect results. Then, using the real op-amp model, we saw excessive pole migration. What we want to do now is bring in compensated blocks, replacing Fig. 1 with Fig. 2. Here we are employing the active compensation methods, so note that we now have six op-amps total.

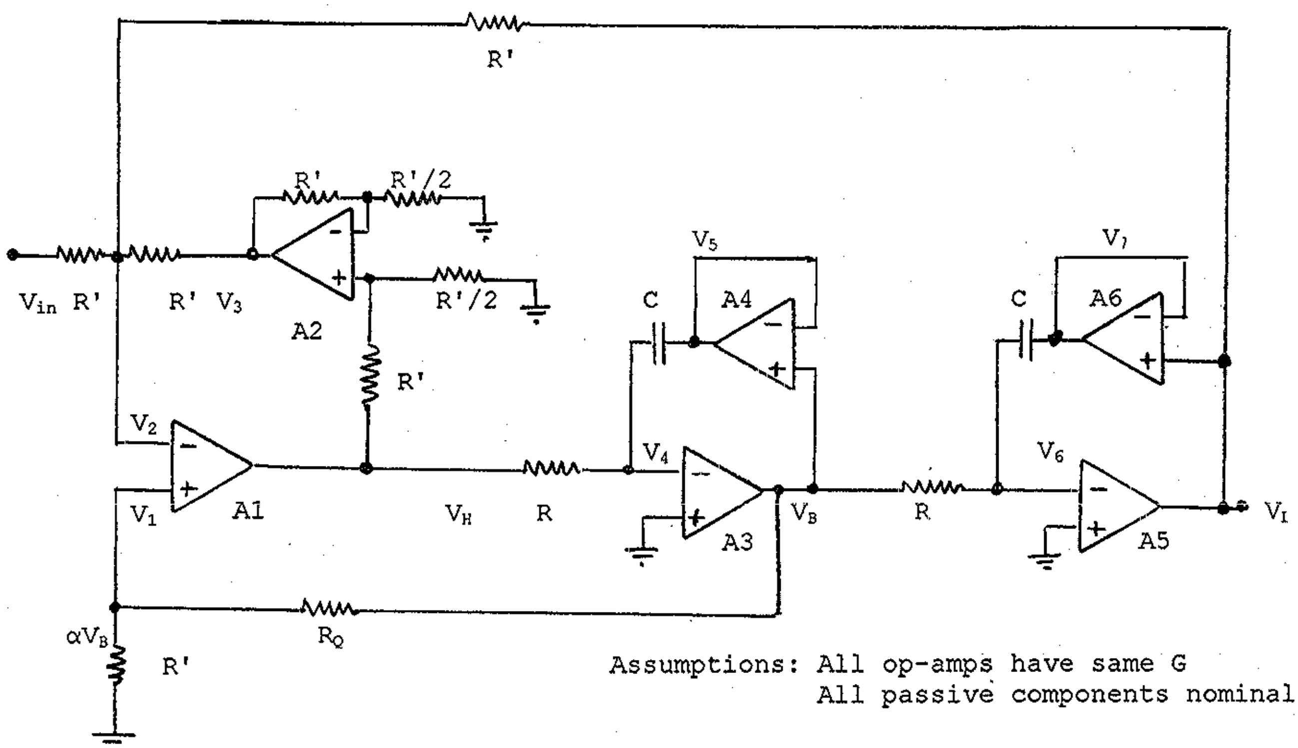


Fig. 2 State-Variable Filter with Compensated Blocks

We now have what is going to be an eighth-order network (6 poles from the op-amps and two intentional capacitors). What we hope to find is two near-nominal poles with the remaining poles far away and/or masked in part by zeros. Note that we must consider all the op-amps to be real (otherwise, we get perfect results). This is where the algebra gets messy, even though portions of the mess have been worked out already in Chapter 7.

THE SUMMER PORTION

The active compensated summer consists of op-amp A1 with A2, configured for the same noise gain (3), in the feedback loop. Note that A2 is configured for a gain of 3 and is preceded by an attenuator of 1/3, so it is ideally unity from V_H to V_3. However, as a real op-amp, we apply equation (1) to A2 to get:

$$V_3 = (G/s)\ (V_H/3 - V_3/3) \qquad \text{6a)}$$

or

$$V_3 = [\ G/(G+3s)\ ]\ V_H \qquad \text{(6b)}$$

which has a pole at s=-G/3. This is what we want. Now applying equation (1) to A1, and using equation (6b) we arrive at:

$$V_H = (G/s)\ [V_1 - (V_{in} + V_I + V_3)/3\ ]$$

$$= G\alpha V_B/s - GV_{in}/3s - GV_I/3s - G^2V_H/[3s(G+3s)] \qquad (7)$$

This equation (7) is the key to solving our problem. Once we relate V_B and V_I to V_H by analyzing our compensated integrators, we can transform equation (7) into the form $V_H(s)/V_{in}(s)$ and find the poles and zeros.

THE ACTIVE COMPENSATED INTEGRATOR

Applying equation (1) to A4 we get:

$$V_5 = G/(s+G)\ V_B \qquad (8)$$

which is a real voltage follower with a pole at s=-G. Once again, using equation (1), this time on A3 we get:

$$V_B = -\ (G/s)\ (V_H/sC + V_5R)/(1/sC + R) \qquad (9)$$

or, using equation (8)

$$V_B(s)/V_H(s) = -G(s+G)\ /\ sRC[s^2 + s(G+1/RC) + G(G+1/RC)] \qquad (10)$$

This is just the compensated integrator result, equation (7-100) of Chapter 7. Of course equation (10) also relates $V_I(s)/V_B(s)$, and $V_L(s)/V_H(s)$ is the square of equation (10). This completes the pieces.

THE NEAR-NOMINAL POLES

Rewriting equation (7) using equation (10) as well gives us:

$$V_H(s)/V_{in}(s) = -[Gs^2R^2C^2(s^2+sB+GB)^2(G+3s)] / [2s\, s^2R^2C^2(s^2 + sB + GB)^2(G+3s) + 3G^2\alpha(s+G)sRC(s^2 + sB + GB)(G+2s) + G^3(s+G)^2(G+3s) + G^2s^2R^2C^2(s^2 + sB + GB)^2] \qquad (11)$$

where we have substituted B = G+1/RC. There is a temptation to regard G as being much bigger than 1/RC, but this we must not assume. Further, we might have set RC=1 "without loss of generality" but this we resisted, as doing so would have destroyed dimensional checking which can be extremely useful in complicated algebraic manipulations. (All the terms of equation (11) have dimensions of frequency to the 6th power.)

Equation (11) is the answer we need, although we do need to do more algebra in order to put the denominator in a from that can be factored to the poles. When we do this, the denominator becomes:

$$d_8s^8 + d_7s^7 + d_6s^6 + d_5s^5 + d_4s^4 + d_3s^3 + d_2s^2 + d_1s + d_0 \qquad (12)$$

where:

$$d_8 = 9\ R^2C^2$$

$$d_7 = 3\ G\ R^2C^2 + 18\ R^2C^2B$$

$$d_6 = 24\ R^2C^2GB + 9\ R^2C^2B^2 + G^2\ R^2C^2$$

$$d_5 = 6\ R^2C^2G^2B + 21\ R^2C^2GB^2 + 9\ G^2\alpha RC + 2BG^2R^2C^2$$

$$d_4 = 15\ R^2C^2G^2B^2 + 12G^3\alpha RC + 9G^2\alpha BRC + 2G^3BR^2C^2 + G^2B^2R^2C^2 \qquad (13)$$

$$d_3 = 3R^2C^2G^3B^2 + 3G^4\alpha RC + 21G^3\alpha BRC + 3G^3 + 2G^3B^2R^2C^2$$

$$d_2 = 15G^4\alpha BRC + 7G^4 + G^4B^2R^2C^2$$

$$d_1 = 3G^5\alpha RCB + 5G^5$$

$$d_0 = G^6$$

RESULTS

The general results show that the method of using compensated blocks works. Yet there are nonetheless some curious findings that are perhaps not too difficult to explain. For our study, we have chosen to expand on the results in

Section 7-6 of Chapter 7. Accordingly the results of equations (7-84) and (7-85) will provide the comparison case. In Section 7-6, we saw results for values of g_n of 1000,100,10,5,2, and 1, and for a Butterworth Q (0.7071) and for a Q of 10. As we might expect, the pole positions become unacceptable for g_n=100 for a Q of 10, and for g_n=10 for the Butterworth Q (see Fig. 7-19). What happens when we use the compensated blocks? Do the poles stand still?

Fig. 3 shows a graph similar to Fig. 7-19, except here we superimpose the compensated case, and show the desired poles for Butterworth and values of g_n of 1,000,000, 1000, 500, 100, 50, 20, 10, 5, 2, and 1. The compensated case does not show poles that do not move significantly. The uncompensated case spirals out and around becoming unstable. The compensated case tends inward. If we look carefully (the zoom of Fig. 4), the displacement in the compensated case is roughly only half that of the uncompensated case. Also, we have noted that motion of the poles inward more or less along a radius is a less serious mater than an arc, as the Q changes in the case of an arc. However, for the most part, we don't see a dramatic improvement, and this is largely a matter of recognizing that a Butterworth Q of 0.7071 is not too taxing on the op-amps. Table 1 lists the pole positions for this case.

Somewhat more revealing is the case of Q=10. Running this case for the uncompensated case (shown here as Fig. 5) shows the same result as Fig. 7-19 of Chapter 7. In fact, for this case, the poles become unstable somewhere between g_n=50 and g_n=20. Also from Fig. 7-19 we saw that the pole motion is more or less direct into the right half plane - as bad as you can get. In fact, Fig. 5 is kind of messy, with some nasty-looking compensated case poles. Who ordered them! Of course, these are for g_n=2 and g_n=1, likely pushing our op-amps beyond what we could hope for anyway.

Let's look at Fig. 6 which is a zoom of Fig. 5. Once again, it is clear that the uncompensated case just heads directly for trouble. Finally we see a clear and dramatic advantage of the compensated case. The poles move inward toward the origin and not to the right. At g_n=20, where the uncompensated case is already hopelessly unstable, the compensated case shows 97% of the desired frequency, and little if any change of Q. This is what we were looking for. The method works. Table 2 shows a listing of the actual pole positions.

But what about those horrible cases for g_n=1 and g_n=2? Well, what has apparently happened is that when the compensated case gets around to deteriorate, it deteriorates rather suddenly. To extend the bridge analogy, we are building with better beams, which deflect much less, but when they do break, the bend suddenly and dramatically. Some notion of why this is can be seen by looking at Fig. 7-25 of Chapter 7, which shows the phase response of the compensated integrator. It starts out flat, but when it bends (at about g_n=3), it bends. But to reiterate, this is interesting but secondary, because we really have no business trying such low g_n's anyway.

Fig. 3 Uncompensated case poles (+) and compensated case poles (x) for Butterworth and for value of g_n as indicated

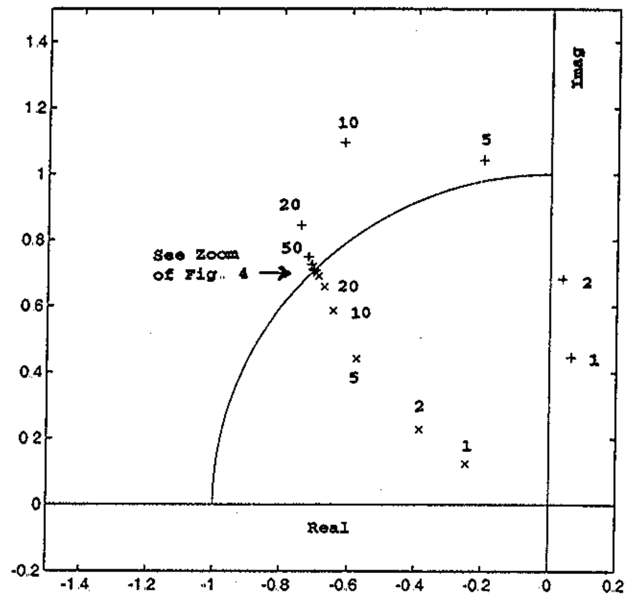


Fig. 4 Zoom of Fig. 3 about the nominal Butterworth poles (-0.7071± 0.7071j). The compensated case is better, but something only on the order of twice as good.

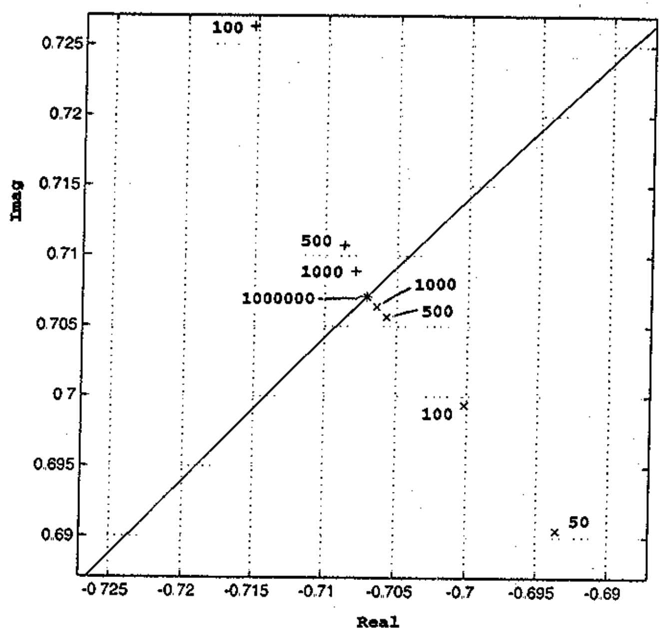


Table 1 Q=0.7071 (Butterworth), RC=1, g_n=GRC

g_n	Uncompensated	Compensated
1000000	-0.7071 + 0.7071j	-0.7071 + 0.7071j
1000	-0.7079 + 0.7089j	-0.7064 + 0.7064j
500	-0.7087 + 0.7107j	-0.7057 + 0.7057j
100	-0.7153 + 0.7263j	-0.7002 + 0.6994j
50	-0.7239 + 0.7486j	-0.6936 + 0.6905j
20	-0.7468 + 0.8443j *	-0.6757 + 0.6572j *
10	-0.6189 + 1.0950j **	-0.6485 + 0.5862j *
5	-0.2006 - 1.0429j **	-0.5779 + 0.4402j **
2	0.0374 - 0.6821j ***	-0.3870 + 0.2282j **
1	0.0652 - 0.4454j ***	-0.2469 + 0.1233j **

* non-nominal behavior of some concern
** significant non-nominal behavior
*** unstable

Fig. 5 Uncompensated case poles (+) and compensated case poles (x) for Q=10 and values of g_n as shown. See zoom of Fig. 6 for a much better view of what is happening.

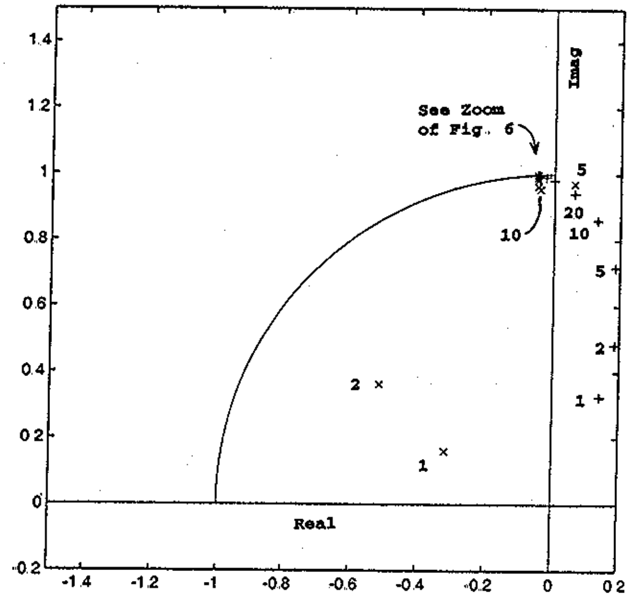


Fig. 6 Zoom of Fig. 5. Note uncompensated poles becoming rapidly unstable while compensated poles move inward toward origin, more slowly, and approximately radially.

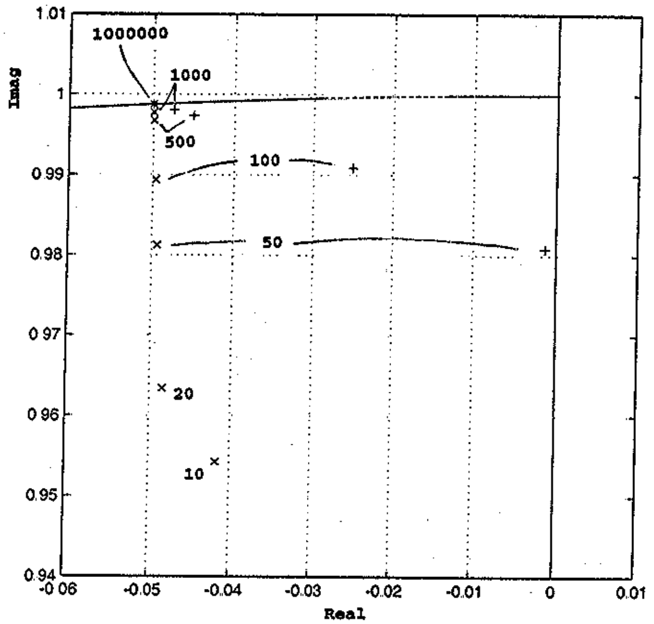


Table 2 Q=10, RC=1, g_n=GRC

g_n	Uncompensated	Compensated
1000000	-0.0500 + 0.9987j	-0.0500 + 0.9987j
1000	-0.0475 + 0.9981j	-0.0500 + 0.9978j
500	-0.0450 + 0.9974j *	-0.0499 + 0.9968j
100	-0.0251 + 0.9909j **	-0.0496 + 0.9894j
50	-0.0013 + 0.9809j **	-0.0493 + 0.9812j
20	0.0610 + 0.9411j ***	-0.0484 + 0.9634j *
10	0.1309 + 0.8608j ***	-0.0417 + 0.9543j **
5	0.1870 + 0.7173j ***	0.0609 + 0.9706j ***
2	0.1861 + 0.4827j ***	-0.5129 + 0.3623j **
1	0.1427 + 0.3250j ***	-0.3160 + 0.1603j **

* non-nominal behavior of some concern
** significant non-nominal behavior
*** unstable

ALL THE POLES AND ZEROS

Above we have been discussing the "nominal" poles. Yet in the uncompensated case there were five poles total (two nominal, and one from each of three op-amps), and in the compensated case, there are eight poles total (two nominal, and one from each of six op-amps.) Where are they all? Consider the example of a Q of 10 with g_n=1000. The poles are listed in Table 3

In the ideal op-amp case, this high-pass response has two poles (at s= -0.05 ± 0.99875j) and two zeros (at s=0). A high-pass filter needs to have the same number of zeros as it has poles. That is, as frequency approaches infinity, we need to be moving away from no net poles or net zeros, or else the response would continue to fall or rise. However, we also note that in a real network, we can not have a true high-pass, since the response is available at the output of an op-amp, which must have a finite bandwidth. This we see in the data of Table 3. Both the uncompensated and the compensated cases involve real op-amps, and both show one net pole.

Notable also in Table 3 is the near perfect cancellation of many of the poles and zeros. Of course, most of our interest is in the near-nominal poles - how far from nominal they happen to be, and if we can live with that. But we do note that even in the uncompensated case there is a near cancellation of a pair of poles by a pair of zeros, leaving only the extra pole at -333.1903. In the compensated case, we end up with near cancellation of two pairs of poles and zeros, and the net pole is the result of the remaining poles at -166.61 ± 288.71j with a zero at -333.33, which are associated almost exactly with the compensated summer stage (A1 and A2) of Fig. 2. These are also the favorable condition, equation (7-109) of Chapter 7, for an initially flat phase characteristic.

So much for algebraphobia. Did I get it right? Could the numbers in equation (13) possibly be correct, and could I have possibly typed them in correctly? Well, they might well be. First of all, there was the usual checking procedures - do the algebra once just to see how "big" the problem is, and then do it correctly, and a couple of more times to check. Not very reassuring - we usually make the same errors over and over. But there is the dimensional checks - the physical units of the terms agree. This does not prove that the results are correct, but it is very reassuring. More to the point, if we do dimensional checking as we do algebra, certain errors shout at us before we go any further. We almost always find some in-progress errors by checking dimensions. By far the best indication that things are right is that a complicated system does get down to a simple, familiar, expected result - the favorable pole/zero condition for zero incremental phase. Still - if anyone wants to check one more time, please let me know, one way or another.

TABLE 3 All the Poles and Zeros

	Uncompensated		Compensated	
POLES:				
	-0.0475 + 0.9981j	(near nominal)	-0.0500 + 0.9978j	(near nominal)
	-0.0475 - 0.9981j	(near nominal)	-0.0500 - 0.9978j	(near nominal)
	-1000.3	(real integrator)	-500.31 + 866.60j	(comp. int.)
	-1001.7	(real integrator)	-500.31 - 866.60j	(comp. int.)
	-333.1903	(real summer)	-500.69 + 866.01j	(comp. int.)
			-500.69 - 866.01j	(comp. int.)
			-166.61 + 288.71j	(comp sum.)*
			-166.61 + 288.71j	(comp sum.)*
ZEROS:				
	0	(nominal)	0	(nominal)
	0	(nominal)	0	(nominal)
	-1001.0	(real integrator)	-500.50 + 866.31j	(comp. int.)
	-1001.0	(real integrator)	-500.50 - 866.31j	(comp. int.)
			-500.50 + 866.31j	(comp. int.)
			-500.50 - 866.31j	(comp. int.)
			-333.33	(comp. sum.)*

* Non-nominal un-cancelled poles/zeros, but the poles are on a circle, centered about the zero, passing through s=0 - our favorable condition for zero initial incremental phase.

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