THE DESIGN OF WIDEBAND ANALOG 90° PHASE DIFFERENCING NETWORKS WITHOUT A LARGE SPREAD OF CAPACITOR VALUES:

-by Bernie Hutchins

INTRODUCTION:

The design of wideband 90° phase differencing networks is fundamental to the design of audio frequency shifters [1-4], of certain pitch extraction devices [5,6] and of other quadrature signal and Hilbert transform schemes. Fortunately, the basic design theory for these networks is a well-studied and well-reported art [7,8,3] and is therefore at most a matter of simple reference, or of simple calculations.

One feature of the more-or-less standard approach to 90° phase differencer design is that an actual realization must involve a wide spread of capacitor values. This is a direct result of the wideband nature, with pole frequencies staggered throughout the band. A corresponding range is required in the RC time constants of the network, and only just so much of this can be taken up by spreading resistor values. [If one tries to accommodate the entire spread with resistors, some resistors will be too small and will load op-amp circuits, while others will be too large, and will cause offsets and noise pickup.] This large spread of capacitor values is something every builder of these networks has experienced first hand [4]. This is perhaps at most an inconvenience to the individual builder working at the circuit board level. It is however a much more serious problem for any designer considering monolithic realization of 90° phase differencing networks. At the same time, the unavailability of small ready-made 90° phase differencing networks is some hinderance to their wider application in many useful schemes.

The approach offered here is one of designing with second-order sections rather than first-order ones. The use of second-order phase shifters is certainly not new [9-11] and it has been applied to 90° shifter design [9,15]. However, the use in these shifters would seem to have been mainly motivated by economy of op-amp use, and not to manipulate the values of passive components in any useful way. Note that when applied to the 90° phase shifter problem, this use of second-order sections is unusual in that the goal is not to achieve complex conjugate pole pairs (which makes the use of second-order sections manditory in many cases). In this case, all the poles (and mirror zeros) are real and separate (first-order).

For our study, we first can look at the way in which poles can be grouped so as to reduce component spread. [Obviously it is logical to start by grouping the lowest frequency with the highest, the second lowest with the second highest, and so on.] In addition, a state-variable approach is useful here since it becomes necessary to have a low, controllable Q, which can be obtained with state-variable, and because the linear blocks of state-variable filters can be easily compensated for frequency dependent effects of the op-amps [12].

THE LOCATION OF THE POLE/ZERO PAIRS:

As mentioned above, the determination of the pole frequencies for the phase shift networks involved in the 90° phase differencing scheme is readily available [7,8,3,13,14] so we could begin our realization discussion from that data. However, for the sake of completeness, we will take a bit of space to review the necessary calculations and the overall scheme. Probably the best known approximation method is that of Weaver [7] and we will use this here.

The design of a 90° phase differencing network begins with the choice of three parameters: the bandwidth, the number of poles, and the allowable error. In fact, the choice of the last two parameters is not independent. If we want a certain bandwidth, then there is a trade-off between the number of poles we are willing to invest in the networks, and the maximum error value that we must settle for. This is illustrated in Fig. 1 where we recall that the 90° phase shift desired is presented rather as an approximation to the phase difference between two parallel networks. We can see that the result is a bandwidth region between f_ℓ and f_u where the phase is 90°±E, where E is the error. Note that the cycling of the phase about 90° is a matter of placement of pole/zero pairs. If more poles are added, the error can be made smaller, but at the expense of the need for more filter sections. Another point not always mentioned is that more sections also result in a longer total delay.

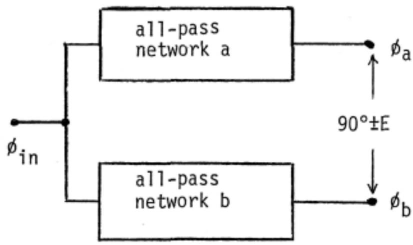
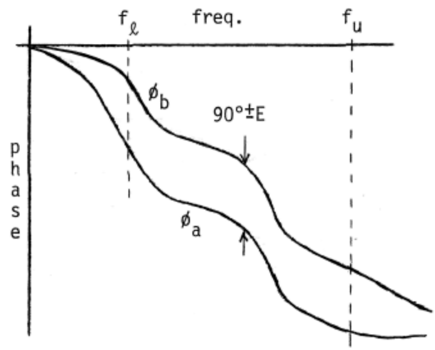


Fig. 1



The usual starting point is to choose a working bandwidth. For many purposes, this will be something like the audio bandwidth from 15 Hz to 15 kHz, or a 1000:1 ratio. Once this is selected, the error can be traded-off with the number of poles. The calculations go as follows:

First, choose the bandwidth as a ratio $B = f_u/f_\ell$. Calculate then a constant A by the following steps:

$$k = \sqrt{1-1/B^2} \qquad (1)$$

$$L = \frac{1}{2}\frac{1-\sqrt{k}}{1+\sqrt{k}} \qquad (2)$$

$$A' = L + 2L^5 + 15L^9 \qquad (3)$$

$$A = e^{\pi^2/\text{Log}_e A'} \qquad (4)$$

From the value A, and the maximum error allowed (assumed here to be in degrees), the number of poles n that are required (for both networks) is given by [13]:

$$n > \ell n(E \cdot \pi/720) / \ell n\, A \qquad (5)$$

where E is the error and A comes from equation (4) above. Equivalently, the error E is given by:

$$E = 720A^n/\pi \qquad (6)$$

However, we can actually set aside the question of the error for a certain case as it is only the bandwidth B and the number of poles n that comes into our calculation of the pole positions. The poles are divided between the two networks, which we can denote as "a" and "b" by finding angles ϕ_a and ϕ_a' for network a and angles ϕ_b and ϕ_b' for network b, as follows:

$$\phi_a(r) = (45°/n)(4r-3) \qquad \text{for } r = 1,2,\ldots.(n/2) \qquad (7)$$

$$\phi_a'(r) = \text{Tan}^{-1}\frac{(A^2-A^6)\text{Sin}(4\phi_a)}{1+(A^2+A^6)\text{Cos}(4\phi_a)} \qquad (8)$$

$$p_a(r) = \sqrt{B}\,\text{Tan}[\phi_a - \phi_a'] \qquad \text{(poles - network a)} \qquad (9)$$

while the corresponding equations for the B network are:

$$\phi_b(r) = (45°/n)(4r-1) \qquad \text{for } r=1,2,....(n/2) \tag{10}$$

$$\phi_b'(r) = \mathrm{Tan}^{-1} \frac{(A^2-A^6)\mathrm{Sin}(4\phi b)}{1 + (A^2+A^6)\mathrm{Cos}(4\phi_b)} \tag{11}$$

$$p_b(r) = \sqrt{B}\ \mathrm{Tan}[\phi_b - \phi_b'] \qquad \text{(poles - network b)} \tag{12}$$

Here we have assumed that n is even, which is the usual choice so that there are the same number of sections in each network. If n is odd, then equation (7) will run up to (n+1)/2 while equation (10) will run up to (n-1)/2, or vice versa. The poles as given in equations (9) and (12) are relative to f_ℓ, so for the actual pole positions, multiply by f_ℓ. Note that the poles are of course negative and real. This completes the review of the standard calculations. Of course, it would be the usual practice to program these, even though they are rather simple in theory.

COMBINING POLES FOR SECOND-ORDER

As mentioned above, it is the usual practice to consider a second-order network when poles in complex conjugate pairs are needed. Here we are using second-order networks to try to distribute a range of time constant in a favorable way. We begin by considering how two real poles are combined to a second-order denominator. If the poles are at $s = p_1$ and $s = p_2$, then the denominator term is:

$$(s-p_1)(s-p_2) = s^2 - s(p_1+p_2) + p_1p_2 \tag{13}$$

which can be put in a standard form of:

$$D(s) = s^2 + (\omega_0/Q)s + \omega_0{}^2 \tag{14}$$

Since we are concerned here only with stable networks, the poles are negative, and Q is given by:

$$Q = \frac{-\sqrt{p_1p_2}}{p_1 + p_2} \tag{15}$$

which can't be greater than 1/2. Later we will see how this restricts our selection of network configuration.

Perhaps of more interest is the ω_0 term:

$$\omega_0 = \sqrt{p_1p_2} \tag{16}$$

This will translate into the spread of time constant that we will need to be concerned with, and at this point, it will be convenient to look at some actual data. We can look at, for example, network a for the case of an audio bandwidth (B = 1500:1) for n=12, which places six poles in network a as [14]:

r	Pa
1	−0.3846
2	−3.0076
3	−12.977
4	−55.782
5	−239.10
6	−1112.9

TABLE 1

Data for Network a

If we were to realize this as first-order sections, we would have about a 3000:1 spread of time constant. In going over to second-order sections, we have our choice about grouping the poles together. If we group, for example, r=1 with r=2, r=3 with r=4, and r=5 with r=6 (serial ordering) then using equation (16) we have a spread of time constant that is still about 150:1. However, if we group r=1 with r=6, r=2 with r=5, and r=3 with r=4, we would get:

Values of r Combined	ω_o	
1, 6	20.7	TABLE 2
2, 5	26.8	
3, 4	26.9	

which is remarkably small in spread. Additional study of other useful cases will likewise reveal spreads of only 1.5 to 1 or so.

Thus we find the following useful design step at this point. First, we choose a value of n that is a multiple of 4. The usefulness of this is that it places an even number of poles in both network a and network b, so that we can make the best use of our second-order networks. Then we combine the poles, for each of the two networks, by grouping the highest frequency with the lowest, the next highest with the next lowest, and so on. We will find the time constant spread within either network to be less than 2:1, and even between network a and network b, to not exceed 4:1.

SELECTING A CONFIGURATION:

We see above that a proper grouping of poles makes possible second-order network time constants with reduced spread. We next need to consider the practicalities of realization. Note that while we have been mentioning only "poles" above, we are also aware that we will need to realize mirror image zeros for every pole. Thus our study converges on configurations for second-order all-pass networks, and to the particulars of realizing the real pole pairs needed.

In considering the various networks possible, we must first of all ask if the network is capable of realizing real poles (can we get the Q low enough?), and if so, can we usefully employ the grouping of poles suggested above. In particular, we may be able to reduce the spread of capacitor values, but will this result in an equally undesirable spread of resistor values somewhere else? In addition to studying the Q and the resistor spread, we are also interested in the gain of the all-pass, and whether or not the network can be easily compensated for non-ideal op-amp effects if necessary.

Here we will consider five possible networks, three of them in some detail. The first network is a useful circuit of Lloyd [9,10] which is shown in Fig. 2. The network is all-pass when R_3 and R_4 are set so that $K=R_4/(R_3+R_4) = 1/(2R_1/R_2 + 2C_2/C_1 + 1)$, in which case the transfer function is:

$$T_L(s) = \frac{K(1 - sC_1R_1)(1 - sC_2R_2)}{(1 + sC_1R_1)(1 + sC_2R_2)} \qquad (17)$$

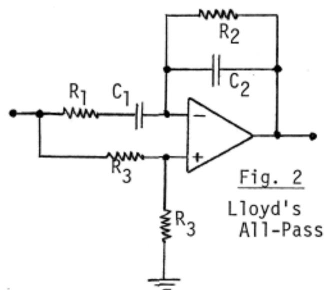


Fig. 2
Lloyd's All-Pass

which can be put in the form:

$$T_L(s) = \frac{K[s^2 - (s/C)\frac{(R_1+R_2)}{R_1R_2} + \frac{1}{R_1R_2C^2}]}{s^2 + (s/C)\frac{(R_1+R_2)}{R_1R_2} + \frac{1}{R_1R_2C^2}} \qquad (18)$$

where we have set $C_1=C_2=C$, which is in line with our general ideas of reducing the spread of capacitor values. From equation (18) we get the design equations for ω_0 and for Q as:

$$\omega_0 = 1/(\sqrt{R_1R_2}\,C) \qquad (19)$$

$$Q = \sqrt{R_1R_2}/(R_1+R_2) \qquad (20)$$

We can of course set ω_0 to any desired values with both C's equal, and also R_1 and R_2 equal, as can be seen in equation (19). However, we also have to set the proper Q according to equation (20). In order to see what sort of spread we need between R_1 and R_2, we need to look at some examples. This we can do by using the data of Table 1 with the grouping of Table 2, and finding the Q from equation (15).

TABLE 3

Values of r Combined	Q
1,6	0.0186
2,5	0.1108
3,4	0.3913

According to our discussion above, these Q's are less than 1/2, and the lowest Q occurs in the case where the real poles are furthest apart.

To see what effect the Q has on the R_2/R_1 ratio, we can solve equation (20) for R_2/R_1, and we get:

$$R_2/R_1 = \left[\frac{1 \pm \sqrt{1 - 4Q^2}}{2Q}\right]^2 \tag{21}$$

where the ± sign in the solution only implies that R_2 and R_1 may be reversed. Here if we look at the lowest Q, we find that the resistor spread is $R_2/R_1 = 2893.65$. Whether or not this is too large to conveniently realize, we should note that it is exactly the ratio of the pole frequencies at the extremes of Table 1. In short, while this network is very nice for many cases, it does not help us here. The result is the same as using two first-order networks, a fact that can be seen somewhat more directly from equation (17), and from equation (20), we can see that the spread must be the same by comparing with equation (15).

Our second network is that of Budak [11,16] which is a generalization of a popular multiple-feedback infinite-gain bandpass structure. This network is shown in Fig. 3, and is all-pass when $K = R_4/(R_3+R_4) = R_2/(R_2+4R_1)$, and has transfer function:

$$T_B(s) = \frac{K[s^2 - (s/C)\frac{2}{R_2} + \frac{1}{R_1 R_2 C^2}]}{s^2 + (s/C)\frac{2}{R_2} + \frac{1}{R_1 R_2 C^2}} \tag{22}$$

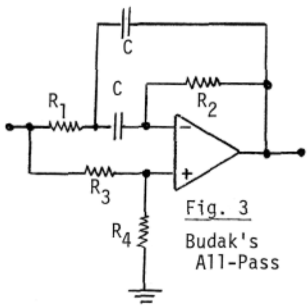


Fig. 3
Budak's All-Pass

Superficially this seems to resemble equation (18), but here we find the ω_0 and Q to be:

$$\omega_0 = 1/(\sqrt{R_1 R_2}\, C) \tag{23}$$

$$Q = (1/2)\sqrt{R_2/R_1} \qquad (24)$$

which is a different equation for Q, and we get the ratio:

$$R_2/R_1 = 4Q^2 \qquad (24)$$

In the case of the minimum value of Q from Table 3, this gives a ratio of 723:1 which is an improvement over the Lloyd circuit (and the equivalent first-order case), but is still fairly large.

Our third and fourth configurations will be taken to be the two popular versions of the three-op-amp state-variable filter [17, for example]. These would require a fourth op-amp to sum the low-pass and high-pass outputs to form the all-pass. However, these two networks are unsuitable for our purposes here because they can not reach a low enough value of Q. One of them is limited to Q greater than 1/2, while the other is limited to Q greater than 1/3 [18 with details left to the reader.].

Faced with the unusual problem of trying to get the Q down, we can now look at a fifth network, which is also state-variable. This network, shown in Fig. 4, is a basic four-op-amp state-variable with added summer for all-pass, giving five op-amps total. This network has the fourth op-amp added in the Q-determining feedback loop from the bandpass to the input. This network has transfer function:

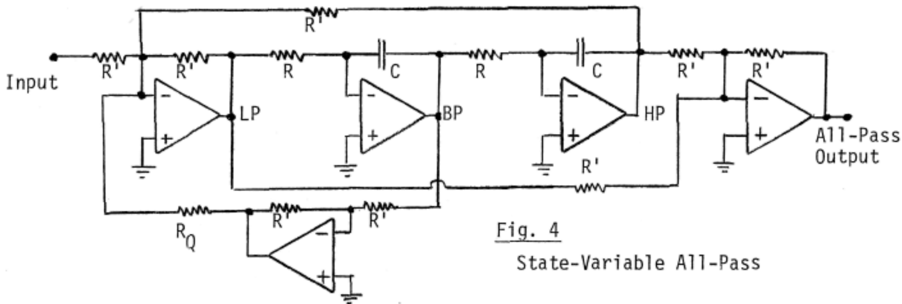


Fig. 4

State-Variable All-Pass

$$T_S(s) = \frac{s^2 - (R'/R_Q)(s/RC) + 1/R^2C^2}{s^2 + (R'/R_Q)(s/RC) + 1/R^2C^2} \tag{25}$$

which has ω_0 and Q given by:

$$\omega_0 = 1/RC \tag{26}$$

$$Q = R_Q/R' \tag{27}$$

This network is very attractive (except for the number of op-amps) in that the equations for frequency and for Q are decoupled. In setting the frequency, there is no spread of resistance to consider, other than that between different second-order sections as impled by Table 2. The only resistor spread within the second-order network itself is that implied by equation (27), where we see that the spread is equal to the Q itself. The smallest Q in Table 3, which we used to compare the other networks, is Q = 0.0186, which imples a spread of about 54:1, a considerable improvement over the 3000:1 ratio we started with. We note that for small values of Q, R_Q is to be small, and accordingly, we might want R' to be as large as is practical, if it turned out that the lowest op-amp might have trouble driving a resistor this low.

Accordingly, the state-variable approach would seem to be about the best of the common networks. Our final considerations are the gain of the networks, and the ease with which they can be compensated for non-ideal op-amps. We can see that the gain of the first two networks is their respective value of K. For the Budak circuit, Fig. 3, we can solve for K, getting $K = Q^2/(1+Q^2)$ which gets very small as Q gets small. Note that this further imples a spread in the resistors R_3 and R_4 which we may need to contend with. [In fact, it is easy to show that the spread in R_3/R_4 is approximately four times worse than the spread in R_1/R_2]. Since there was no spread improvement in the Lloyd circuit, we do not need to calculate the value of K there (it is a more complicated calculation anyway). This leaves the state variable, which has a nice gain of 1 for all cases. Again, state-variable is the clear winner.

In regard to compensation for the effects of non-ideal op-amps, again we know how to handle the state-variable [12], compensating both integrators and summers, when this becomes necessary. In contrast, the compensation of single-op-amp networks has been traditionally a matter of an "overdesign", which is fairly complicated in theory and in practice.

<u>SUMMARY:</u>

Two things have been established in this study. First, it is shown that the time constant spread in 90° phase differencing networks can be reduced by proper grouping of poles into second-order networks. Secondly, it has been shown that a state-variable approach can be used to exploit this grouping, and that the state-variable approach has some otherwise useful properties to offer.

For the most part, the reduction in capacitor spread has major implications only for those doing monolithic realizations of 90° phase differencing networks. However, it should be pointed out that the design also works well at the circuit board level. A number of practical 90° shifters were constructed for a frequency shifter application, specifically to test the design theory presented here, and these worked as well as standard first-order approaches.

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