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PERSPECTIVES

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THE RESONANT FREQUENCY OF A SOFT DRINK BOTTLE AND OTHER ACOUSTIC PROBLEMS OF THE SAME GREAT IMPORTANCE:

We think it will be useful if we spend a number of issues on some acoustic problems and some basic discussions of acoustic musical instruments. We will begin here with some very elementary musical instruments.

The first "instrument" is the empty soft drink bottle. It is doubtful that there is anyone who has not finished a bottle at some time, and then given a playful blow across the top, causing it to hum out. Probably you also observed that as you drank the contents, you could get a series of tones of lowering pitch - the more air space in the bottle, the lower the pitch.

The bottle on which I did my experiment was one of those "no refill" 16 oz glass bottles. This type has a fairly simple shape, which helps the problem. The bottle can be considered to be a "Helmholtz Resonator", the basic form of which is an empty cavity with a circular opening, as indicated in Fig. 1. The solution to the resonant frequency of such a cavity is a well known physics problem, and we will use here the equations given by H.F. Olson in his book Music, Physics and Engineering, Dover, 1967, pg. 71. According to these equations, the resonant frequency is given by (combining Olson's results):

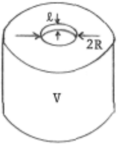

Fig. 1 Helmholtz Resonator

$$f = \frac{1}{2\pi}\left[\frac{\pi R^2 c^2}{(\ell + 1.7R)\,V}\right]^{\frac{1}{2}} \qquad (1)$$

where:

R = radius of circular opening (in cm)

c = speed of sound (in cm/sec)

ℓ = thickness of boundary walls (in cm)

V = volume of the cavity (in cm^3)

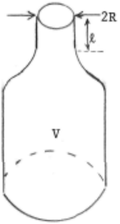

Fig. 2 Soft Drink Bottle

Most of these are quantities which we can easily obtain. There is one tricky point however. As stated in the formulas, ℓ is the thickness of the boundary walls. It is clear however that this is not important as such. The walls are rigid compared to air, and we could add an inch of cement around the outside, and it would not make any difference. Thus what is really of interest is the thickness at the circular opening. This is the length of the neck (see Fig. 2). Thus we have $R = 0.9$ cm, $c = 35{,}000$ cm/sec., $\ell = 3$ cm, and the volume is 1 pint = 1/2 quart = 473 cm^3. If we plug these in, we get $f = 192$ Hz. If we measure the pitch, it comes out at the $F^{\#}$ below middle C on a piano (185 Hz), and more accurately, by beating with an electronic oscillator, at 181 Hz.

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The agreement is quite good. As we have said above, the most common error in the calculation has to do with the neck length/wall thickness interpretation. In cases where the neck is very long compared to the radius of the opening, the 1.7R term can be ignored, reducing equation (1) to the following:

$$f = \frac{Ac}{2\pi}\left[1/\sqrt{vV}\right] = 5573\frac{A}{\sqrt{vV}} \qquad (2)$$

where A is the area of the opening and v is the volume of the neck, with the other symbols as defined for equation (1). White and White, Physics and Music, Saunders College/Holt, Rinehart and Winston (1980) give a similar equation, except the constant we give as 5573 is instead, 5380 there (see pg. 62). Probably their equation is more accurate, because the effect of the 1.7R term would be to lower the frequency if it were included.

Another interesting resonant system which we have all played with is the "pipe". Our experience with these begins usually in grade school when we find that by blowing across a soda straw or a ball-point pen tubing, we can obtain a whistle or tone. (It is a surprise that fourth grade teachers don't capitalize on this enthusiasm of a room full of whisteling youngsters, and take the opportunity to teach them the acoustics of air columns in pipes! We shall try to correct this missed opportunity, even at this late date.)

A typical pipe, open at one and and closed at the other is shown in Fig. 3. If we blow across the open end (actually, across the open end for the most part), we excite a standing wave in the pipe, such that there is a maximum variation of pressure at the closed end. The open end remains pretty much at constant pressure, since it is open to the atmosphere. Thus a pressure wave wave as indicated in Fig. 3 occurs. The dots are meant to represent the density of air molecules at any one moment. Note that any cycle of the tone produced has both a pressure maximum and a pressure minimum (3a and 3b). The similar case for a pipe open at both ends is shown in Fig. 4.

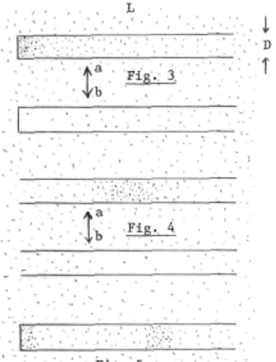

Fig. 3

Fig. 4

Fig. 5

From Fig. 3, we can see that the length L contains 1/4 of a wavelength, while the length L of the open-open pipe in Fig. 4 contains half a wavelength. [You may need to think about this a bit.] Thus the frequencies of the pipes are given by:

$$f_{\text{closed-open}} = \frac{c}{4L} \qquad (3)$$

$$f_{\text{open-open}} = \frac{c}{2L} \qquad (4)$$

In practice, it is useful to make an end correction: the effective length depends on the diameter as L' = L + 0.3D. Some results of experiments are shown below:

Device	L	D	L'	f_{c-o}=35,000/L'	$f_{experimental}$
Pencase	12.6cm	0.7cm	12.8cm	683 Hz	647 Hz
Straw	6.85cm	0.6cm	7.03cm	1244 Hz	1176 Hz
Straw	5.35cm	0.6cm	5.53cm	1582 Hz	1435 Hz
Straw	20.7cm	0.6cm	20.9cm	419 Hz	1188 Hz

The results are about 6%-10% low in general. The longest straw is too long and narrow, and friction prevents the excitation of the fundamental. Instead, we got the third harmonic (see Fig. 5). Open-open pipes produce only weak tones in these cases and were not measured.