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APPLICATION NOTE NO. 71
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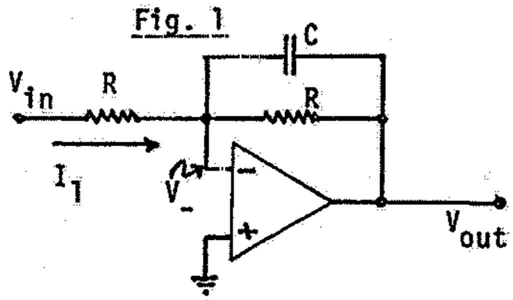
MULTI-MODE FILTER BASED ON FIRST-ORDER LOW-PASS

A first-order low-pass filter with a Gaussian (no overshoot) characteristic is shown in Fig. 1. The transfer function of the filter is derived as follows:

Fig. 1



$$V_- = 0$$

$$I_1 = V_{in}/R$$

This same current I_1 must be flowing out through the parallel combination of R and C in the feedback loop. The impedance of this loop is:

$$Z = \frac{R(1/sC)}{1 + 1/sC} = \frac{R}{1 + sCR}$$

The current I_1 flows out through Z just as in a standard op-amp inverting amplifier. The output voltage is thus:

$$V_{out} = -I \cdot Z = \frac{-V_{in}}{R} \cdot \frac{R}{1 + sCR} = \frac{-V_{in}}{1 + sCR}$$

The transfer function T(s) is thus:

$$T(s) = V_{out}/V_{in} = \frac{-1}{1 + sCR}$$

This T(s) has a pole at s = -1/RC. If we cascade two such sections, the overall transfer function is:

$$T_2(s) = [T(s)]^2 = \frac{1}{(1 + sCR)(1 + sCR)} = \frac{1/R^2C^2}{s^2 + 2s/RC + 1/R^2C^2}$$

which is a second-order Gaussian low-pass function.

If we let RC=1 to keep the math simple, we can consider doing a weighted sum of the outputs of a cascaded pair of first-order sections. The basic setup is shown in Fig. 2.

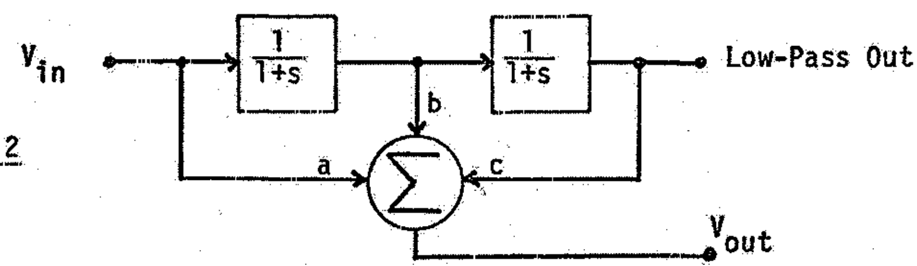


Fig. 2

For a weighted sum we have the output $V_{out} = aV_{in} + bV_{in}/(1+s) + cV_{in}/(1+s)^2$ and we can form the transfer function:

$$T(s) = V_{out}/V_{in} = \frac{as^2 + (2a + b)s + (a + b + c)}{s^2 + 2s + 1}$$

For a low-pass function, the numerator of T(s) should be 1, for a bandpass, it should be s, and for a high-pass, it should be s^2. A notch response can also be obtained by placing zeros at +j and -j, which is possible with a numerator $s^2 + 1$. It is then a matter of simple algebra to get:

Function	Numerator	a	b	c
Low-Pass	1	0	0	1
Bandpass	s	0	1	-1
High-Pass	s^2	1	-2	1
Notch	s^2+1	1	-2	2

A typical realization is shown in Fig. 3. Note that the -1 in the transfer function of the circuit of Fig. 1 automatically takes care of the alternating + and - signs in the summing coefficients.

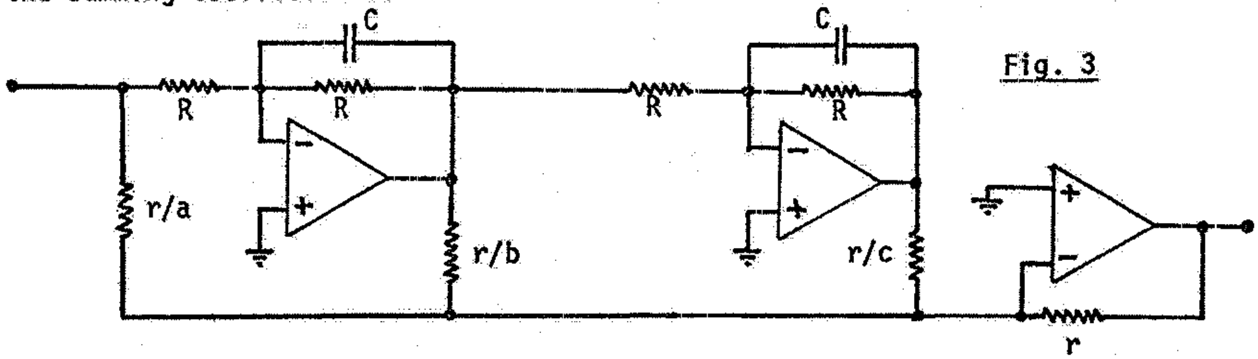


Fig. 3

Experimental results with R = 470k and C = 1000pf are shown in Fig. 4.

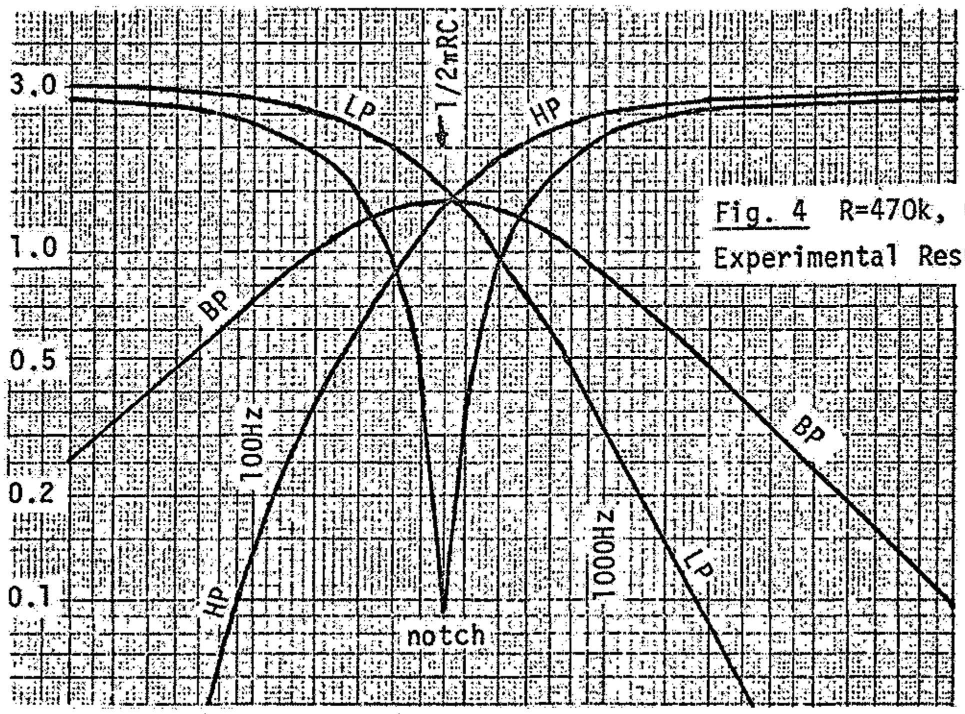


Fig. 4 R=470k, C=1000p, r=200k
Experimental Results

The results in Fig. 4 bear out the theory above. The circuit uses the same number of op-amps as the state variable, except for the notch which uses one less op-amp. The method can be extended to higher order, but component sensitivity becomes more of a problem at third-order, and at fourth-order, resistors and capacitors with a tolerance of 1% are suggested.