APPLICATION NOTE NO. 424

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"CYCLE-11": IN TIME, FREQUENCY, AND PERIOD

INTRODUCTION:

A lot of earth science data at least peripherally discusses an "11-year sunspot cycle" and whether or not that observed cycle manifest itself in <u>other</u> environmental/geological time signals. As such we need to be aware of how various analysis tools detect and display any <u>related</u> cyclic behavior. In particular, how do our displays appear in the time domain, the frequency domain, and as a plot of components by period – the latter being of lesser use in signal processing? No actual sunspot data is shown here. We need to look at toy data to form a useful baseline for reference.

THE FAMILIAR NOTION OF A "SUNSPOT CYCLE"

Likely most of us learned about sunspots and an 11-year "sunspot cycle" in some sort of "Earth Science" course in high school. (It was one more fact to be memorized.) Perhaps eventually we came to suppose that it meant that dark spots on the sun <u>could</u> be counted (if we didn't think too hard about it!) and graphed year by year. Apparently we would see the number of spots go up and down over a cycle of 11 years. Being a natural phenomenon, we perhaps did not expect a perfect cycling – or perhaps we <u>did</u> expect that at first!

At some point we learned that the "cycle" is very imperfect – neither exactly 11 years, neither perfectly periodic, nor is it a constant amplitude, and can even go missing for many decades. It is perhaps at best a rough pattern that does not surprise us. "Ham Radio" operators found it quite influential – for example.

One problem with any pattern is that humans tend to want to <u>match</u> patterns: correlations, and, seemingly perforce, extend to potential (unjustified) causation. The mistake is in pressing the search without extreme caution. Indeed, involving "tools" such as a Fourier Transform (in some sense) as a seeker-of-periodicity is usually useful, but only part of the effort.

What is first? You look at the data! You always look at the data first.

TIME DOMAIN

Emphasizing that we have no actual sunspot data we will nonetheless choose sunspotlike examples. As a first look, suppose we generated a test signal in Matlab as:

```
n=0:32
ss0 = 50 + 50*cos(2*pi*n/11)
```

This produces exactly three length-11 cycles off a raised cosine reaching a peak of 100 and dips to 0 – very much a toy sunspot signal. Note that this is already special mathematically: (A) an exact number of full cycles (3), (B) cosine phase making an anticipated FFT purely real, and (C) a DC term (sunspot "counts" can't be negative). (The reader is encouraged to also run a more general start.) Two additional test signals are shown.

```
ss1 = (0.96.^n).*(50 + 50*cos(2*pi*n/(10+0.5*rand)) + 0.4*50*rand(1,33))
```

and

```
ss2=70*rand(1,33)+0.3*ss0
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with ss1 showing variations in amplitude, frequency, and some noise and ss2 showing the original signal (30%) inside considerable noise (70%). [This is hard to see. In the appendix (out of easy sight, we show the cosine green component. Look there later.] Fig. 1 is a plot of these three signals.



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FEQUENCY DOMAIN

We have an immense amount of experience taking FFT's of time sequences, and interpreting the result as being in a "frequency domain". Indeed, the FFT is just a fast computation of the DFT, which we usually name X(k) corresponding to a time-domain signals x(n) such as those in Fig. 1.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j(\frac{2\pi}{N})nk} \qquad k = 0, 1, 2, \dots (N-1)$$
(1a)

and:

$$x(n) = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} X(k) e^{j\left(\frac{2\pi}{N}\right)nk} \qquad n = 0, 1, 2, \dots (N-1)$$
(1b)

Since most of us here are audio oriented, we tend to always think of the index k as relating to frequency ($f=kf_s/N$) where f_s is the sampling frequency. Fig. 2 shows the FFT's corresponding to Fig. 1. [Because we chose cosine phase, these are automatically real.] We see exactly what we expect. For the perfect sampling of three full cycles (Fig. 2a), we get the one spike at k=3 (and at k=33-3=30) of course, and the DC term at k=0 (which we included so the counts are non-negative – see DC case removed below).



Fig. 2b and Fig. 2c shows the expected scattering of spectral energy into side frequencies, exactly as we expect (because of the random noise, additional runs will vary). Note that our cycles were length-11, and this is seen in the FFT frequency of k=3. The corresponding frequency is $f = kf_s/N = 3/33 = f_s/11$. The period is thus 11 times the sample time (yearly in our example) or 11 years. Often in audio (as we suggest) we think mainly in terms of frequency – not period – that's the way the ear works.

A PERIODICITY DOMAIN ?

One interesting thing in using a computer interactively in the sense of iterating code for a particular task is that we are usually iterating familiar code and procedures (and correcting familiar blunders). Sometimes things that we suspect we know well HOW to do we just ignore in favor of what we well-remember worked last time. On occasion, we are struck by the notion that we may NEVER have ACTUALLY done something before! Yikes! Did I ever plot FFT data with a period axis? Possibly not. This is a simple matter of plotting the FFT values from right to left (dropping k=0) at the reciprocal positions. Looks unfamiliar at first. For the same results from Fig. 1 and Fig. 2, we get Fig. 3. Note the apparent compression of the plotted points on the left side. The periodicity of length-11 is clearly shown. Note that there is no periodicity (nor should there be at length-22). The length-8.25 would correspond to 4 full cycles (also missing from Fig. 3a). In Fig. 3b and Fig. 3c, we have additional components because of the imperfect periodicity (just "leakage" in the FFT).



This is the basic code for plotting period:

k=0:32 freq=k/33 per= (1./freq) per(1)=0 stem(per(32:-1:1),SS0(32:-1:1))

Where SS0 is the FFT of the samples of Fig. 1a. THAT'S IT.

REDUCING "CLUTTER"-GETTING RID OF THE DC TERM, ETC.

Here we have seen the not-so-surprising example of getting "clean" indications of a strong frequency component (k=3 in Fig. 2a) or of periodicity=11 in Fig. 3a. Possibly the next three figures (4-6) are not necessary, but they are easy to generate and insert. They correspond exactly to Fig. 1-3 above but for cases where the test signals are modified as indicated:

```
ss0 = 50*cos(2*pi*n/11) - just removal of DC
ss1 = (0.96.^n).*(50*cos(2*pi*n/(10+0.5*rand)) + 0.4*50*(rand(1,33)-.5))
- imperfect cycles with DC removed from signal and random components
ss2=35*(rand(1,33)-.5) +0.3*ss0
- noisy case with DC removed from random
```

Basically these are cases where there is no constant (DC) term in the input data. We remind you that these constant offsets were the result of the decision to make the data input look like non-negative "count" integers. Now suppose that this really is sunspot data. That is, we suppose that the length-11 sunspot cycle has some correlation, even possibly some causal relationship; with some other observable (perhaps surface temperatures of the earth, mosquito counts, tree ring widths, etc.).

<u>If</u> there is a potential connection between sunspots numbers (some measure of solar activity presumably) and some observable, quite likely it is driven by the periodic component (length-11 cycle) that by some means induces a signal with corresponding frequency. Should there be a corresponding constant term as a result, this would likely not be a detectable shift and would be "swamped". Thus we take the space here to show just the cyclic component. Below are the figures.

Aside from the obvious disappearance of the DC terms in Fig. 5a and Fig. 6a, we don't gain much. On the other hand, in the cases where we did not have our "clean" cases, we see what look to be significant improvements relative to a useful "detection"



of the length-11 cycle (see Fig. 5b, 5c, 6b, and 6c). Compare for example to Fig. 3c.



At this point, we have seen that we can plot periods with an ease similar to that with which we have been plotting FFT frequencies. In fact, we note that as with frequency, there are ideal conditions (an integer number of full cycles) where the results are, essentially a perfect resolution. Attempts to "do better" really aren't going to get us anywhere. In a moment, we will see how what such things as windowing and zero-padding change our picture. But we need to realize that achieving a perfect setup is going to be fruitless except within our setup of toys.

WINDOWING

We know that any actual sunspot cycle itself, or possible feed-through, can't possibly be exactly 11 years. In the first place, a "year" is some very very exact number corresponding to the motion of the EARTH. There is nothing that would sinc a process of solar activity to this cycle, either exactly or even approximately (some argue that the larger planets have <u>some</u> influence). In consequence, when we use an FFT to look for frequencies or period, we necessarily are artificially fitting some sequence to an available FFT length. This is WINDOWING both in the case were a specific window (perhaps a Hamming window) is brought in, OR IN THE CASE where self-windowing is

automatically in place [rectangular windowing, as is already a result of the summation of equation (1a)]. It's already involved here. Any such thing as "leakage" is already messing up any really clean result that is already confounded by noise or imperfect periodicity.



Fig. 7 in fact reminds us of familiar windowing. Fig.7a is the baseline case where we have perfect rectangular windowing. Here we have taken 14 full length-11 cycles of a cosine (154 samples). Because there are exactly 14 cycles, the FFT (not shown) has all the energy at k=14 (and k=154-14=140). The period is thus 154/14 or 11 (Fig. 7a). In the case where we take, say, 159 samples of the original repeating length-11 cycle (14.4545 cycles), as in Fig. 7b, we have imperfect periodicity and thus "leakage", and while we see period 11, the result is smeared (exactly as with the FFT frequencies). A classic way of reducing leakage is to use a window such as a Hamming window. This is seen in Fig. 7c, where it is not too helpful.

WHAT ABOUT ZERO PADDING ?

We often have suggestions that some sort of "improvement" in using a FFT as a "Fourier Transform" is afforded by "zero padding" an input time sequence. In the few cases where this can be contended, it is largely a matter that the thus interpolated

output has features that are easier for the eye to see in a plot. Since interpolation results from including made up samples that are derived from zeros, nothing new (at least nothing reliable) is availed.

By way of completeness, we offer the study summarized in Fig. 8. Fig. 8a shows the by now tedious case where 14 perfect cycles are analyzed by FFT and the perfect period if length 11 is detected. Note again that this peak is at FFT frequency k=14, the sequence length is $14 \times 11 = 154$, so the period is 154/14 = 11. Now we can make a length-154 sequence by taking 3 full length-11 cycles and zero-padding. In our example, we took samples 17-49 of the sequence used in Fig. 8a and put 61 zeros on front and 60 zeros on the end (thus also length 154). The resulting output (Fig. 8b) shows the actual period of 11, but smeared in what is likely a non-useful way.



CONCLUSION

Nothing much new here. Just a "raising of awareness" about the alternate way in which a FFT frequency can be exhibited as a period. More to the point, because we do our plots in this way at least once as periods we know the ones we see elsewhere are no longer perfect strangers.

Here is the promised version of Fig. 1c indicating some notion (green) of the 'signal".



"REFERENCES"

No numbered references are inserted above. Many possible references are in the Electronotes literature and indeed, generally in the DSP literature. Below are a very limited selection of interesting references that are online.

(a) Here is a reference that more-or-less prompted this note: <u>http://wattsupwiththat.com/2015/08/19/is-the-signal-detectable/</u>

(b) ELECTRONOTES 222 Volume 23, Number 222 June 2014 FFT – INTERPOLATION IN TIME AND FREQUENCY http://electronotes.netfirms.com/EN222.pdf

(c) ELECTRONOTES APPLICATION NOTE NO. 398 July 20, 2013, A SHORT PRESENTATION OF FFT (DFT) INTERPOLATION http://electronotes.netfirms.com/AN398.pdf

(d) ELECTRONOTES APPLICATION NOTE NO. 410 May 6, 2014 FOURIER MAP For Reference: The first map was published in EN#188 (1993). The second map (with comments, pages (2)-(4) here, was produced in 2006 and used for teaching

http://electronotes.netfirms.com/AN410.pdf