ELECTRONOTES

APPLICATION NOTE NO. 382

1016 Hanshaw Road

Ithaca, NY 14850

August 20, 2012

FUN WITH OHM'S LAW

MAXIMUM POWER TRANSFER

Many of us remember the first things we learn about electronics with (perhaps) unjustified respect. Indeed, we even learn to live with (ignore) some apparent contradictions. Perhaps this just means that we never really understood them – and why should we fully understand a perhaps nuanced issue when we first encounter it? In particular here, we all learned first of all about Ohm's Law. Then at some point, we learned we were supposed to pay attention to "matching impedances" principally when connecting loudspeakers to audio amplifiers, and TV antenna "300 Ohm Twin Lead" to antennas and TV receivers (no one remembers that!). So on the one hand, it seemed like you just connected voltage sources to loads and Ohm's Law told you all the rest. On the other side (failure to match impedances) we were to worry about loss of maximum power, reflections ("ghosts" in TV reception) and perhaps even damage to audio amplifiers..

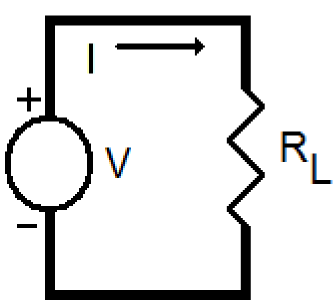
OHM'S LAW – ATTACHING TO A VOLTAGE SOURCE

During the winter I often use an electric heater down here in my "office". It's rated at 1500 watts. Now 1500/117V is about 13 amps, and this is indeed on a 15 amp circuit. Now, the heater puts out heat. That is what I want – in the room. I am not interested in heating the wiring in my home or in the generating and transmission facilities on the way there. Presumably, I'm not doing that very much. So I conclude that the output impedance of the power supply is very low compared to the load resistance of the heater (which must be about 9 ohms). Indeed, we suppose that to a good approximation in this application, there is zero output impedance.

Many applications however are not concerned with the transfer of power to a load – for example, in our music circuits, we merely transfer voltages which represent signals. And when we ARE concerned with the transfer of power (as for heating a room) we should be concerned with efficiency, not with maximizing power.

Fig. 1 is nothing more than Ohm's law. We have a voltage source V connected to a resistive load R_L. We easily calculate the current and the power. This is first the specific model we had in mind for the electric heater. We choose R_L to be small enough that the current is large enough (the power dissipated is large enough) that the resistor gets <u>significantly hot to be a useful room heater</u>. In another instance, the source V might be an op-amp output (known to have a low output impedance) and we are merely <u>transferring a signal V to another stage</u> of processing. In such a case, R_L is chosen to be large, the input impedance of the following stage. In fact, it is probably just the input resistor to an inverting summer, or a very high resistance load that merely keeps the following stage from "floating" when no V is plugged in. Both of these applications have the commonality of assuming a very low, negligible, output impedance. What we are trying to accomplish however is quite different in the two cases.

In the voltage transfer (signal) case, R_o really can be tiny compared to R_L which can be very large. But if we do want to deliver significant power, we want a significant value of $P=I^2R_L$. It would appear that we would want to make both I and R_L larger. But the essential thing is to note that V is a constant, so as R_L is made larger, I decreases. So how does that balance out? Well, in fact, it is much more informative to write the power as: $P = V^2/R_L$. So power is inversely proportional to the load resistance. Consider for example a familiar 1/4 watt resistor. Could you connect such a resistor across a 117 volt line? Suppose it is a 1Meg resistor. The power would be 0.0137 watts, well below the 0.25 watt rating. If the resistor is changed to 10k, the power will be 1.37 watts, and it won't take long for the resistor to burn up. Make the resistor 100 ohms and the power is 137 watts and it will be gone instantly. Consider the heat of more than a 100 watt bulb in that tiny resistor.



$I = V/R_L$

$P = VI = V^2/R_L = I^2R_L$

Fig. 1 Ohm's Law
Power to a Resistive Load

In a more general case, we can suppose that there is a non-zero source impedance, R_o as in Fig. 2.   Note that Fig. 1 is a special case of Fig. 2.

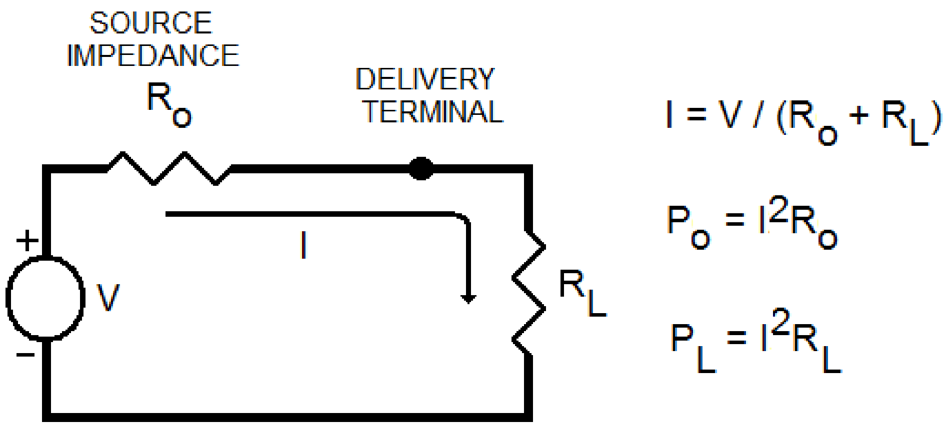


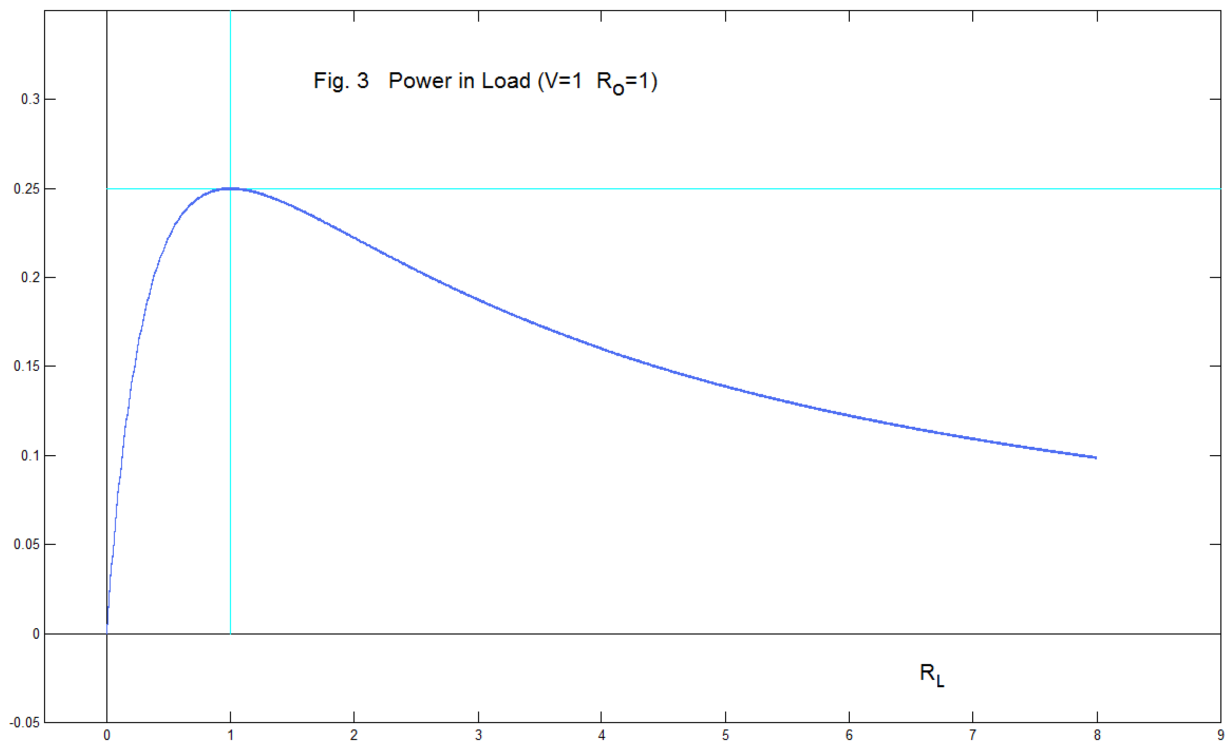
Fig. 2 Non-Zero Output Impedance
Some Power Dissipated in Source

Here some power is dissipated by the source impedance, assumed resistive for this case. There are now two possible performance issues to look at: maximizing power, and efficiency.

Maximizing the power is a classic problem going back about 170 years. Given V and a fixed value of R_o, find the value of R_L that maximizes the power in R_L. It's clearly a calculus problem, and we begin by writing down the equation for the power in R_L. From Fig. 2, we have:

$$P_L = I^2R_L = V^2R_L /(R_o + R_L)^2 = V^2 / [\, R_o^2/R_L + 2R_o + R_L \,] \qquad (1)$$

We can differentiate this (or as we have set it up, the denominator instead) with respect to R_L and set it to zero, solving for R_L. This we will do, but let's cheat first. We can just plot equation (1), setting V=1 and R_o=1, and plot for R_L in a range of say 0 to 8. This we show in Fig. 3. It is seen there that the power has a maximum value of 0.25 when R_L=R_o. Not surprisingly, when R_L goes to zero, the power goes to zero, and for R_L greater than R_o, we see the power decreasing (as the current decreases).



To see that this is the right answer, we can do the calculus. Differentiating the denominator of equation (1) we have:

$$d/dR_L\ [\ R_o^2 R_L^{-1} + 2R_o + R_L\ ] = -R_o^2 R_L^{-2} + 1 = 0 \qquad (2)$$

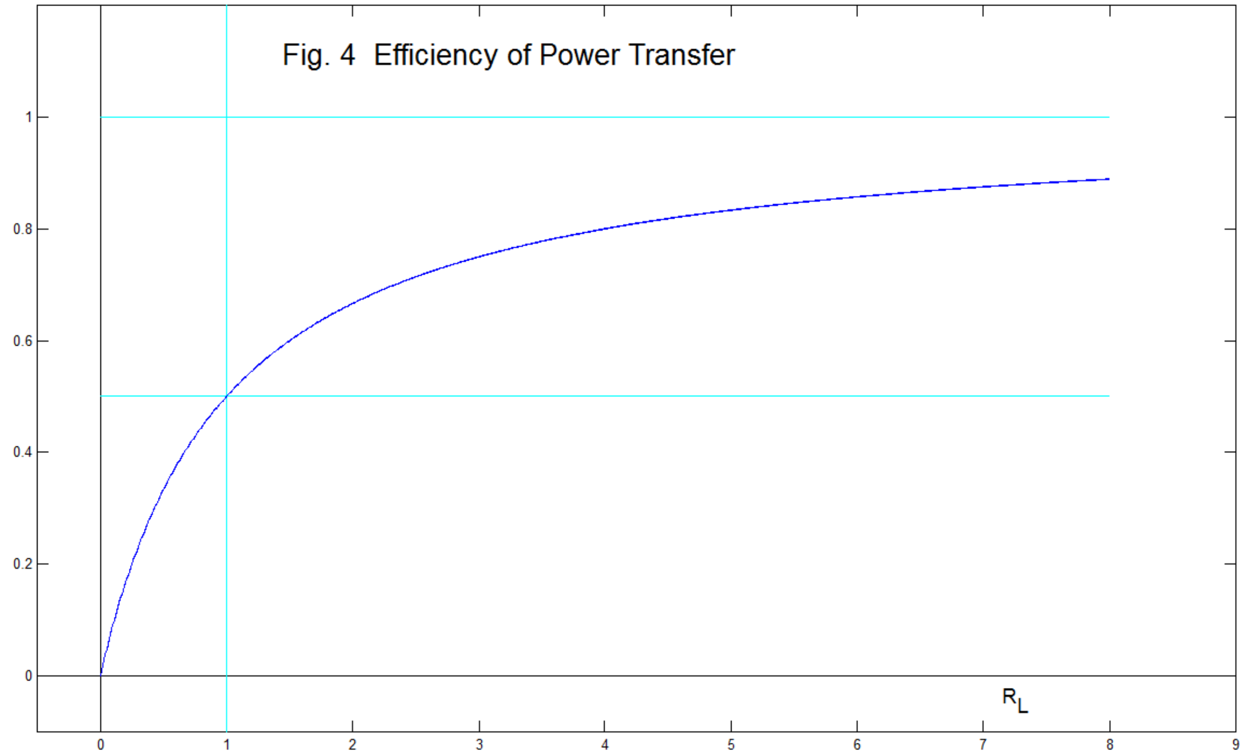
Which gives $R_L = R_o$. Plugging this back into equation (1) with V=1 and R_o=1 we indeed get 1/4. The math verifies the plot and vice versa.

So that's the "maximum power transfer theorem" and is often thought to suggest more than it does. Before commenting on this, we need to consider the second issue – efficiency.

The efficiency E is defined as the power dissipated in the load divided by the total power – the power in the load plus that in the source. From Fig. 2, it is clear that:

$$E = R_L / (R_o + R_L) \qquad (3)$$

since the current I cancels top and bottom in the ratio (Fig. 2). This is pretty simple, and probably obvious. A plot is seen in Fig. 4. Thus at maximum power (R_L=R_o) the efficiency is only 50%. This at first seems to be a severe limitation, but we always have to ask what we are looking to achieve.



Suppose as an example I have some music synthesizer modules available and have mistaken them for a room heater. I turn them on, and wait for the room to warm up. Yes, it's going to be a long wait. We have agreed here that we have the load resistance (the input resistance of the following stage) very high compared to the source resistance. So let's re-plot our Fig. 3 and Fig. 4 for a much larger range of R_L, as seen in Fig. 5 and Fig. 6. These are the same graphs, they just go up further. What happens for the lower values of R_L (the interesting part) is now compressed and hard to see. What we do see is "Low Power" in the case of Fig. 5 and "High Efficiency" in the case of Fig. 6, with red arrows directing our attention to remind us that this is the area where we are working in our suggested applications. Roughly speaking, once R_L is sufficiently larger than R_o, our power is low, and our efficiency is nearing 100%. That's the whole point. The synthesizer modules were not supposed to be room heaters – they were just moving voltage signals around.

So in suggesting that in the synthesizer modules we are <u>unconcerned</u> with power transfer, are we suggesting that we <u>are concerned</u> with power transfer when we actually get out the room heater? **<u>Yes and no</u>**. Yes we are interested in generating heat. We are also interested in efficiency. But we still have no desire, nor ability, to match a load impedance to a source impedance. We are still well inside the red arrow regions of Fig. 5 and Fig 6.

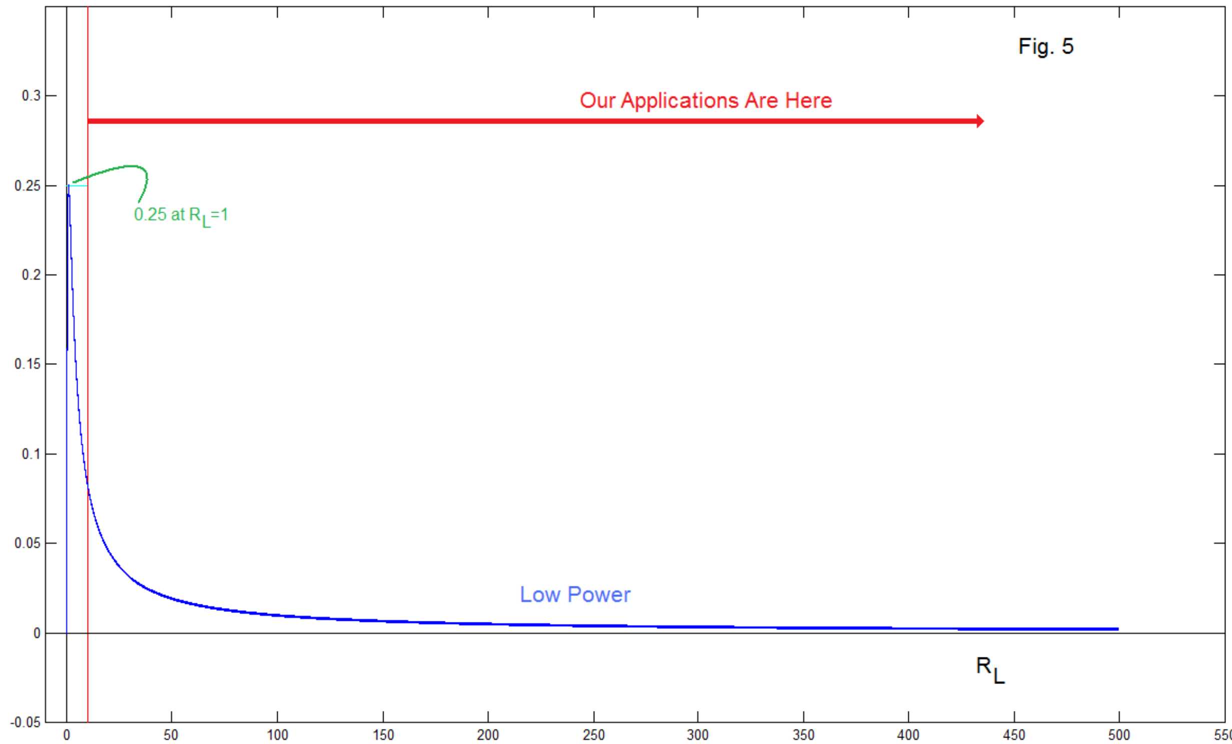


Fig. 5
Our Applications Are Here
0.25 at R_L=1
Low Power
R_L
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0 50 100 150 200 250 300 350 400 450 500 550

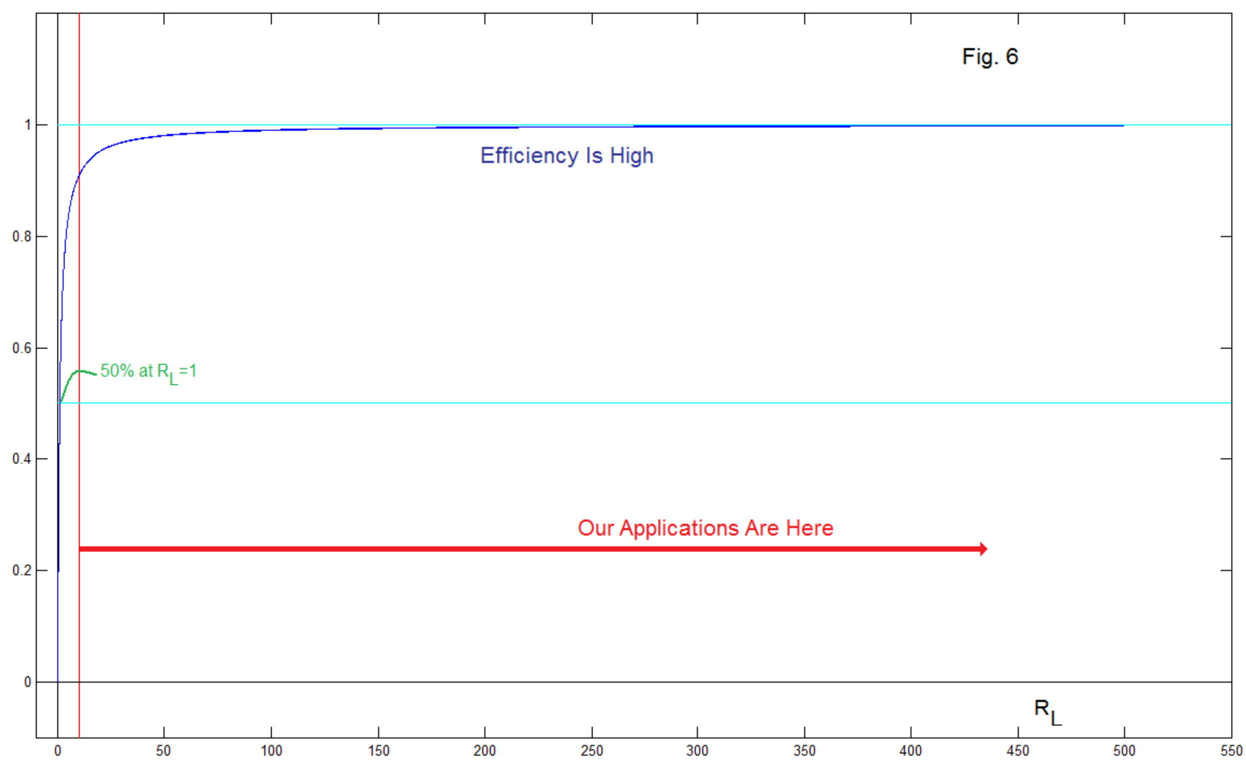


Fig. 6
Efficiency Is High
50% at R_L=1
Our Applications Are Here
R_L
1
0.8
0.6
0.4
0.2
0
0 50 100 150 200 250 300 350 400 450 500 550

Why is this? Well, the internal impedance of our house wiring is very low. If we were to attach an electric heater load with the same very low impedance, first of all, our circuit breakers would blow – the current would be too high. Take out the breakers and your heater will get very hot all right, as will your house wiring. It wasn't so much heating power you wanted, as heating power in the amount you wanted, and efficiently (and safely) delivered.

We can point out that sometimes we may see evidence of the source impedance of our household wiring – when a refrigerator turns on, the light may well dim momentarily. We know that what happens is that the compressor motor in the fridge draws a surge current, all the way back to the transformer on the road. This is a very temporary large current, and the breakers are designed to ignore it (a fuse ignores it through needing time to melt!).

Incidentally, a better way to observe a dimming due to source resistance is to use a flashlight. Choose one that has an incandescent bulb (not a LED) and weak batteries. Such an old flashlight and weak batteries kind of go together at the present time. We want one where the bulb just get orange. Knowing that the batteries when new summed to about 3 volts, you might guess they now summed to only about a volt or two, no longer to three. Measure them. It's still 3 volts! Why isn't the light bright. Well, in measuring the batteries, you took them out and held them in series with your voltmeter across the ends (no load). Now take out the bulb, and rig it with a wire across the batteries, and the voltage will likely drop to about two volts. Accordingly, we might say that a dying flashlight battery is "running out of juice" or we could say that while the open circuit voltage is still OK, the internal resistance is getting too high.

So what if you really do need more heat? Well, you need another heater of course – see how being an EE helps! And if it's another 1500 watts, you are going to need to plug it into another circuit (a different breaker).

So, for the most part, reserve the notions of a need to match impedances to cases where we do this to prevent signal losses and reflections on transmission lines, and that sort of thing.