ELECTRONOTES

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A FUN ANALYSIS – INSTRUMENTATION AMPLIFIER

We often feel that we must have covered just about every possible simple analog circuit, either in our newsletter and/or application notes (as well as many complicated circuits). Recently I needed to explain the standard "Instrumentation Amplifier" to a group, and to my surprise, discovered that we had apparently not covered it. How hard could it be – I had the circuitry and the "right answer". Well, it is easy, but it can fool you once or twice first. It's fun.

The "instrumentation amplifier" is a controllable gain differential amplifier with very high input impedances. Recall that an op-amp itself is a very high-gain differential amplifier with very high input impedance. The op-amp gain is something like a million to several million, and the inputs draw only very tiny currents. True, the gain is way too high for most applications (hence the use of feedback) and rolls off with frequency. But it is evident that, using op-amps, we can make what we need.

The differential amplifier in Fig. 1 is basic. We often use this where we only want to expend a single op amp, where the gain is fixed (at R_2/R_1) and where the input voltages V_1 and V_2 are supplied by low impedance sources (such as other op-amp outputs).

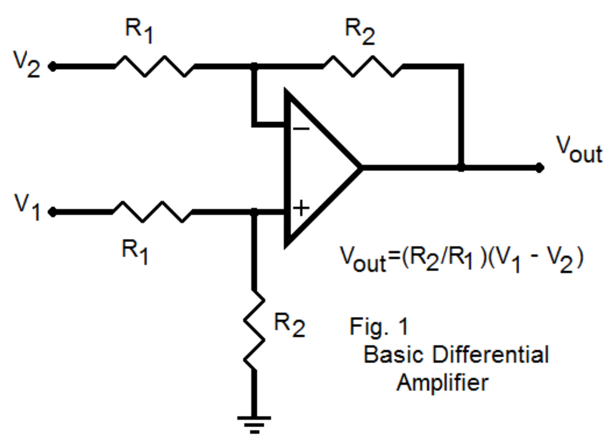


Fig. 1
Basic Differential Amplifier

The input impedances of Fig. 1 for both V_1 and V_2 are low. Clearly the input impedance (input resistance) at the V_1 input is just $R_{in1}=(R_1 + R_2)$. It is just a series connection of the voltage divider resistors, and the connection to the (+) input of the op-amp does not change anything. The input impedance at the other input (V_2) is different (but not generally high either), and strangely depends on the input voltages. Why! Well, we are interested in the current that flows into the upper R_1 resistor when a voltage V_2 is applied. This depends on the voltage on the (-) input of the op-amp, which is the same as that on the (+) input, and thus is:

$$V_{(-)} = V_{(+)} = V_1[R_2/(R_1+R_2)] \qquad (1)$$

So the current is:

$$i_2 = (V_2 - V_{(-)}) / R_1 = [\, V_2 - V_1R_2/(R_1+R_2) \,] \; / R_1 \qquad (2)$$

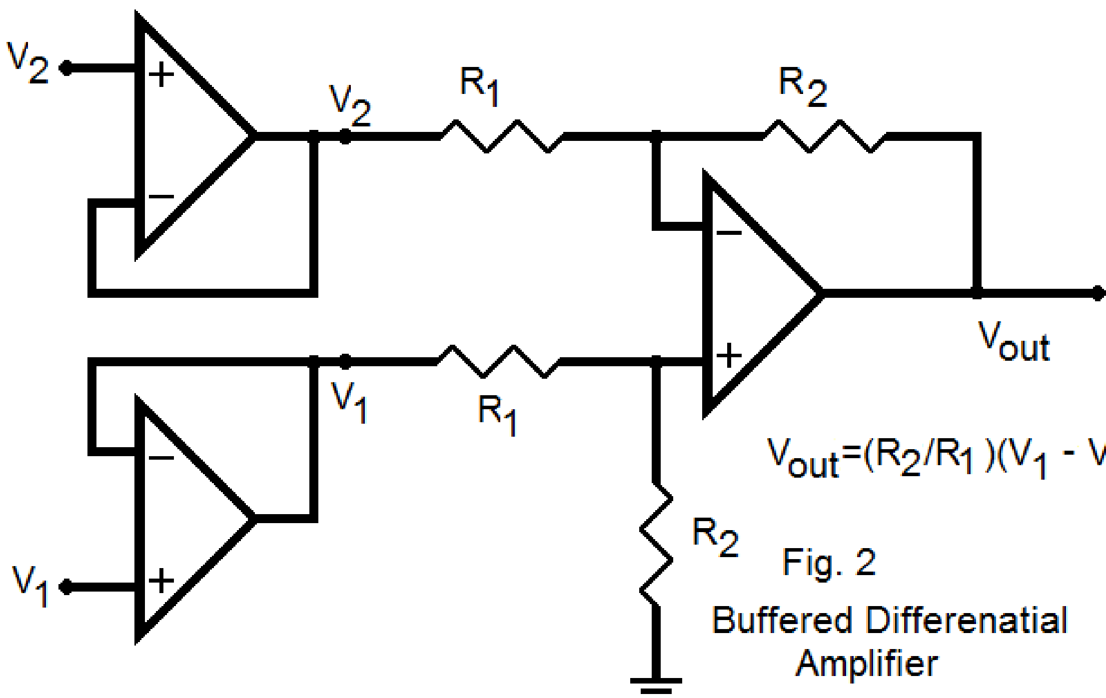
so the input impedance (input resistance) is

$$R_{in2} = V_2/i_2 = V_2R_1 / [\, V_2 - V_1R_2/(R_1+R_2) \,] \qquad (3)$$

This is a curious equation, so we should make some checks. Notice that for the case where V_1=0, the $V_{(+)}$ terminal of the op-amp is effectively grounded, and what remains is just an op-amp inverting amplifier (with input V_2) with input resistance R_1 as it should be (a resistor R_1 to virtual ground). Equation (3) gives this result. R_{in2} can't be smaller than R_1 ever. Notice also that when $V_1=V_2$ (thus V_{out} = 0), equation (3) gives $R_{in2}=R_1+R_2$, just the same as R_{in1}. This is correct as here we would have the upper R_1 and R_2 series connecting to ground (the zeroed output).

Note further the curious fact that the denominator of equation (3) has a minus sign and thus can become zero or negative! The condition where the denominator becomes zero is that the input V_2 is equal to the attenuated V_1, thus to $V_{(+)}$, in which case the $V_{(-)}$ input also has this voltage, and no current flows. Hence the infinite values of R_{in2} is justified as no current flows. The values in which R_{in2} becomes negative really are the consequence of the voltage (and thus the current) reversing sign. Perhaps we should be using magnitude bars on equation (3) although it is likely clear what is happening.

It is clear however that the input impedances are low, very roughly on the order of value of the resistors used. R_{in1} is always the sum of R_1 and R_2, and R_{in2} could be as low as R_1 itself. In many cases where a differential amplifier is being used it is precisely because we want to cancel a common-mode signal such as an AC hum which is picked up by the high-impedance source (like an electrode on the skin).



$V_{out}=(R_2/R_1)(V_1 - V_2)$

Fig. 2

Buffered Differenatial Amplifier

Since the input impedances in Fig. 1 are too low, the use of additional op-amps as buffers (voltage followers) is the next logical step. Fig. 2 shows the two added followers. The equation for the output voltage remains the same.

Using the same approach to buffering, and adding three additional resistors we have the instrumentation amplifier structure shown in Fig. 3. It is not difficult to get this analysis wrong.

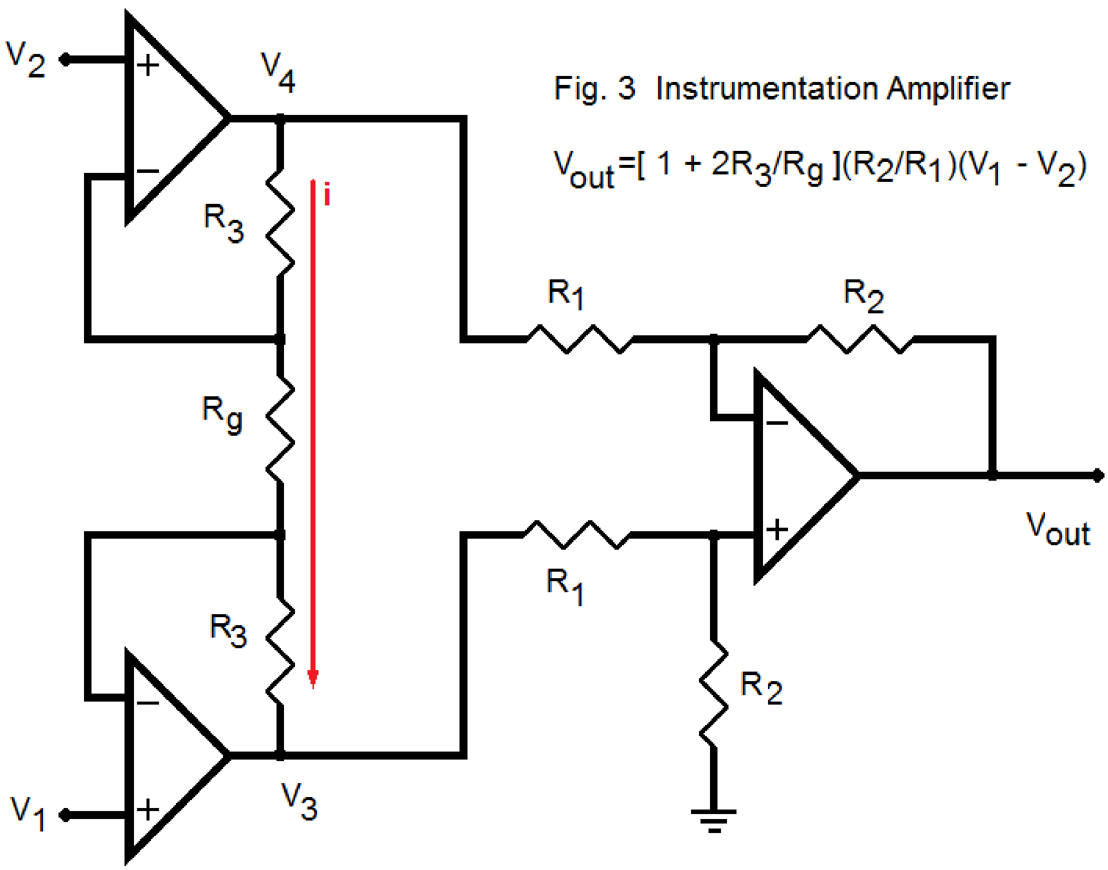


Fig. 3 Instrumentation Amplifier

As suggested, there are plenty of ways to make this more complicated than it needs to be. But as with nearly every circuit of practical purpose, there is a "trick" to a much easier solution. Here it is to note that there is a single current i passing through the series resistors $R_3 \rightarrow R_g \rightarrow R_3$ from V_4

to V_3. Further, these are op-amps with negative feedback, so the (-) inputs are the same as the (+) inputs. The point below the upper R_3 resistor is thus at V_2, and the one above the lower R_3 resistor is V_1. To the right of this current we just have a standard differential amplifier (Fig 1) taking the difference now between V_3 and V_4.

Thus we can observe that:

$$i = (V_2 - V_1)/R_g \tag{4}$$

and the drops of this current along the series resistors give:

$$V_4 = V_2 + iR_3 \tag{5a}$$

$$V_3 = V_1 - iR_3 \tag{5b}$$

while the ordinary differential amplifier is:

$$V_{out} = (R_2/R_1)(V_3 - V_4) \tag{6a}$$

$$= (R_2/R_1)\,[\; V_1 - V_2 - iR_3 - iR_3 \;] \tag{6b}$$

$$= (R_2/R_1)\,[\,(V_1 - V_2) + 2(V_1 - V_2)R_3/R_g\,] \tag{6c}$$

$$= (R_2/R_1)\,[\,1 + 2R_3/R_g\,]\,(V_1 - V_2) \tag{6d}$$

So this analysis is simple enough. It is also perhaps simple to actually see what is going on. Consider that the original differential input voltage is forced across the resistor R_g, producing a proportional current. This same current is forced (by the usual op-amp feedback) to also go through the entire series $R_3 \rightarrow R_g \rightarrow R_3$, so the total voltage between V_4 and V_3 (that will be the input to the ordinary differential amplifier) is “spread” or amplified by $(R_3 + R_g + R_3)/R_g$, which is the gain factor $[\,1 + 2R_3/R_g\,]$ inserted in equation (6d).

The most useful feature of equation (6d), and thus of the circuit, is likely that the gain can be controlled by one resistor, R_g. Note that with differential amplifiers we in general think in terms of the need to set resistors precisely and/or to match them (or trim them). This is because we often use a differential amplifier for subtracting a common mode signal, and this is a matter of balancing positive gains with negative. With Fig. 1, the circuit will work as long as the R_2/R_1 ratios in the two legs are the same, exact matching of R_1 to R_1 and R_2 to R_2 is not what is required. If the ratios match exactly, the "common mode" gain is zero. That is, if we connect the two inputs V_1 and V_2 together, we always get zero out. Trimming any one of the four resistors in Fig. 1 can optimize the "common mode rejection ratio" (usually the most important – not the exact gain).

With Fig. 3 the need to set an exact gain is relegated to the choice of a single resistor, R_g. The need to match positive and negative gains is the same – except that we have more choices. In one sense, we seem to have more requirements with Fig. 3, the matching of the R_1, R_2, and now the R_3 resistors. The situation with regard to R_1 and R_2 is the same as in Fig. 1. If we were to obtain perfect matching of the R_2/R_1 ratios, a mismatch of the R_3 resistors would upset this. Looked at the other way, this same intertwining of the overall gain convinces us that we need to trim only one of six resistors involved. Note for example that the (-) input to the top buffer is restricted to be V_2. If the R_3 in the upper leg increases, the gain with regard to V_2 is increased, continuing in that branch only, and so on. The gain is adjusted by R_g independently.

When we speak of trimming, we are thinking of reducing the value of a resistor to be trimmed to perhaps 95% its nominal value. Then we put in series with it a "trim pot" of perhaps 10% its nominal value. This allows us to adjust the value from 95% to 105%. Common mode gains is generally monitored by connecting the two signal inputs together and applying an AC signal. The output should be small to start with, and we adjust the trimmer to zero.