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APPLICATION NOTE NO. 360

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FLAT AND NOT-SO-FLAT SPECTRA

It is conventional wisdom, even among persons with little engineering training, that "white noise has a flat spectrum." Here we will look at some aspects of this idea. Is the spectrum always flat? What would we need to do to demonstrate a flat spectrum that is convincing? What sort of averaging might be necessary to show this?

It is first of all clear that if we take a random signal, it is unlikely that its spectrum will be flat. This is because when we choose any one example of an arbitrary random signal, there is a good chance that it will have some patterns. We might be so unlucky, for example, as to get, by chance, something very much like a single cycle of a sine wave, and this would of course have a spike-like spectrum, not a flat one.

In Fig. 1, we show an example of the spectrum of a 100-point random signal. This signal was generated in Matlab™ using the ***rand*** function. The distribution is uniform from –1 to +1, and we use the magnitude of the lower half of the FFT for our spectral calculations. The result is the solid line, which is certainly not flat. Would all such random signals have exactly this spectrum? That this is not true is also seen in Fig. 1 where the dotted and dashed examples represent additional examples with different random signals. All these have a crude tendency to hover around some value like 5. Individual points range from near zero to perhaps 12 or so.

It is evident that the tendency to accidentally hit particular patterns (like the sine wave suggested above) would be greatest for short length. For example, any length-two random signal would include a dc term and a first harmonic. Would we expect to get a flatter overall spectrum if the signal were much longer? Perhaps. Yet Fig. 2 shows the spectrum of a length 10000 random signal. Here we see a strong tendency for some flatness centered about 50 perhaps, but the variations are once again extreme, with some samples being near zero, and some about 170.

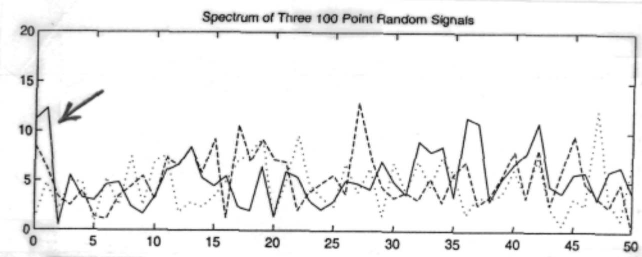

Fig. 1

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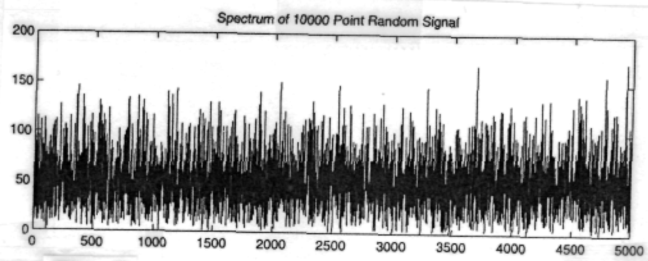

Fig. 2

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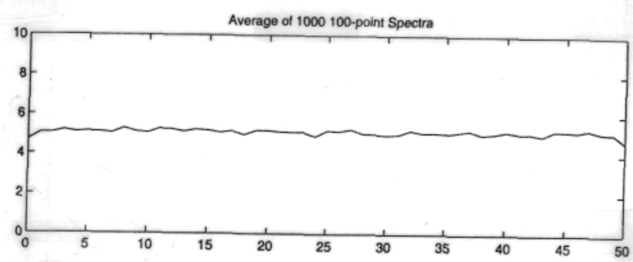

Fig. 3

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Taking a (visual) clue from Fig. 1, we suppose that it might be the case that we do not need to take longer length sequences, but rather, average the spectra of multiple, shorter sequences. Figure 3 shows the average of 1000 spectra of length 100 sequences. The tendency toward a flat value of about 5 is fairly clear.

Another sort of averaging approach would be to average or smoothen the spectrum of a single example. This we show in Fig. 4 where the spectrum of Fig. 2 has been averaged over 100 consecutive frequency points. In practice, this was done by convolving the magnitude spectrum with a length 100 vector with elements 0.01, and removing the end "transients." Again we see a tendency toward a convincing flat spectrum, hovering around 50. We thus see that averaging is effective. This is more or less what we expect to find when discussing the statistics of a random signal. We expect the statistics to be true when we look at enough examples.

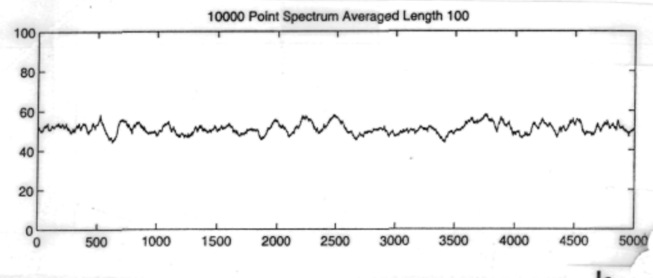

Fig. 4

Any notion that we might do the averaging in the time domain can be set aside by the example of Fig. 5. Here the length 10000 time sequence is averaged by A length 10 convolution as was done for Fig. 4. The resulting spectrum is not flat. In fact, it clearly reflects the shape of the length 10 "moving average" filter. Thus we see two thing of interest. First, the spectrum at the output of a filter which has white noise at the input gives us at least some notion of the frequency response of the filter. Second, the filtering in the time domain "colors" the noise.

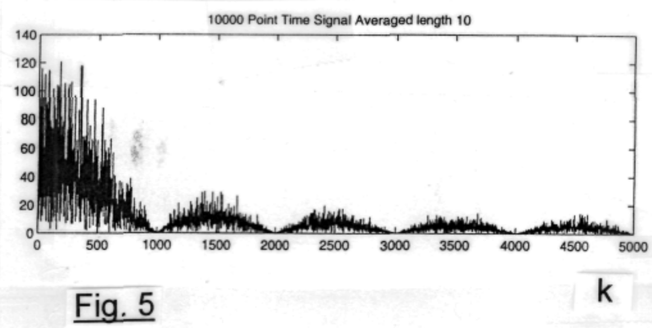

Fig. 5

Now that we know that white noise is only flat on average, another approach to a flat spectrum might to simply invent a flat spectrum and invert it to a time signal. This is easy enough. We choose an FFT consisting of all ones. Inverting this gives us, of course, an impulse at n=0, and nothing else. The connection between an impulse and a random signal is of course, that the autocorrelation of a random signal tends toward a pulse. Further, it is interesting to think of the pulse as a sum of sinusoidal waveforms. In general, an inverse FFT of all ones could be written as a sum of cosines (using the Euler relations). When this sum is evaluated at the integers, we find all the cosines Cancel, except for n=0 where they sum to 1.

We can think of another signal that has a flat spectrum. Rectangular frequency domain descriptions are characteristic of sinc functions. Here we need a spectrum that is flat, so the "rectangle" is always "high." This tells us that our time-domain sinc needs to be infinitely sharp, so once again, our impulse returns as a solution.