ELECTRONOTES
1016 Hanshaw Rd
Ithaca, NY 14850
(607)-266-8492

APPLICATION NOTE NO. 353

March 2003

A NOTE ON THE GAIN AND TIME CONSTANT OF INTEGRATORS

Few analog circuits are as frequently used as the inverting integrator (Fig. 1) which finds wide application in signal processing and in many other circuit schemes. Treating the op-amp as ideal, we find that the transfer function of the integrator is:

$$T_i(s) = -1/sRC \qquad (1)$$

This is an integrator (the 1/s), which is inverting (the - sign), and which is said to have a time-constant of RC. It has a pole at s=0, and provides a phase shift of 90°. Note that the magnitude of the frequency response is:

$$|T_i(s)| = 1/\Omega RC \qquad (2)$$

which falls off as the reciprocal of frequency. On a log-log plot, this is a 45° angle (Fig. 2). Note that the DC gain is infinite! In fact, the integrator by itself (undamped, or not a part of a loop – such as in a state-variable filter) is unstable. It is further significant that the integrator has no separable notions of "gain" and "time-constant." This we can see by assuming that a gain α is imposed on the integrator. In such a case, we see that the gain can be thought of as a change of time-constant by a factor of $1/\alpha$.

$$T_\alpha (s) = -\alpha/sRC = -1/s(RC/\alpha) \qquad (3)$$

Another way of seeing this is that the 45° slope on the log-log plot can be moved up or down (change of gain) or left or right (change of time constant) to the same effect (Fig. 2).

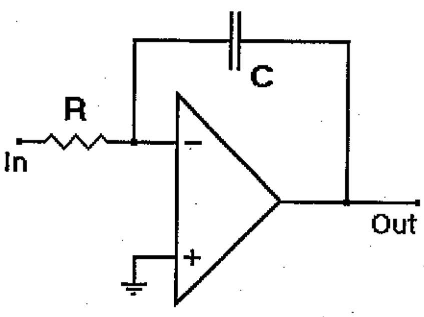

Fig. 1 Inverting Integrator

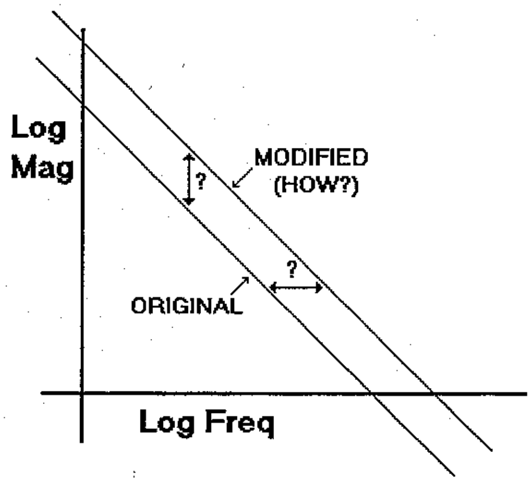

Fig. 2 Equivalence of change of gain or of time-constant

Is there anything strange here? Yes – in general the gain of a network is separated from its frequency properties. For example, the closely related first-order low-pass filter of Fig 3 (also a damped integrator) has transfer function:

$$T_{LP}(s) = -(R/r)/(1+ sRC) \quad (4)$$

This network has a pole at -1/RC, and accordingly a 3db low-pass cutoff frequency of Ω=1/RC. That is, at DC the network has a magnitude R/r, and at Ω=1/RC it has a gain of $(R/r)/\sqrt{2}$. If we impose a gain change α on this network, at DC the gain is αR/r and at Ω=1/RC, it is $(\alpha R/r)/\sqrt{2}$. That is, the cutoff frequency is independent of gain (Fig. 4). The curve just moves up or down. Further, if we do change the RC product, the curve moves left or right. The difference is seen to be the lack of a "kink in the curve" in the case of the integrator. We can't tell how the integrator curve moves because there is no "marker" on it.

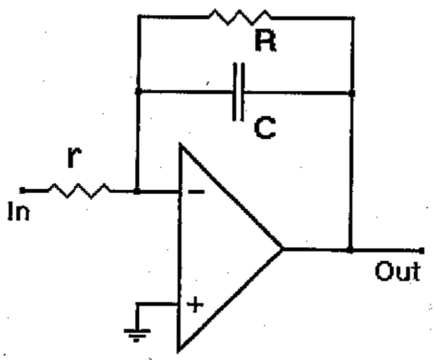

Fig. 3 Simple Low-Pass

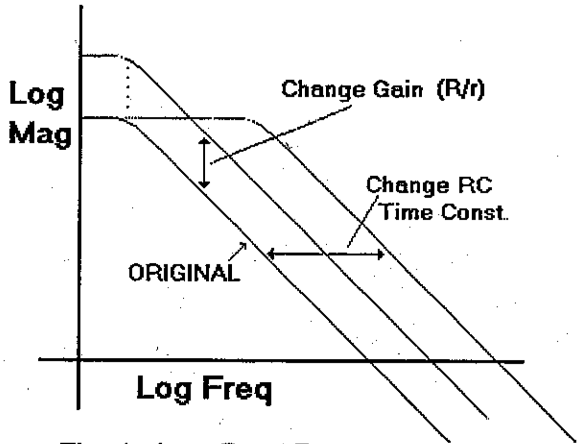

Fig. 4 Low-Pass Responses

Another way to look at this is to consider the number of free parameters we have in the two circuits. In the case of the integrator, there is only the RC product. We only get to choose one thing - the product. (It does not matter for example if we choose R=100k and C=0.01 microfarads or R=10k and C=0.1 microfarads, the product is a millisecond.) In the case of the low-pass filter, we choose two things, the RC time constant (the cutoff frequency) and we also get to control the gain ratio (R/r).

We often suppose that we can define a filter by relating the positions of all the poles and zeros of the network. If we know that a filter has L zeros $z_1, z_2, \ldots z_L$ and M poles $p_1, p_2, \ldots P_M$, we might suppose that the transfer function must be:

$$T(s) = [\, (s-z_1)(s-z_2)\ldots(s-z_L) \,] \,/\, [(s-p_1)(s-p_2)\ldots(s-p_M)\,] \quad (5)$$

and this is almost right. The reason it is not exactly right is because we can apply an arbitrary multiplicative constant to equation (5), (an arbitrary constant gain), and we have the exact same poles and zeros. So the poles/zeros define the transfer function only up to an arbitrary multiplicative factor. Is this important? Only when we forget it. Many times we do get to adjust the gain to a desired value with no additional consequences for a particular application. But at other times, we can get bit.

Here is why this may concern us in the case of the integrator. Above we talked about an ideal op-amp integrator. We know that many times we need to worry about integrators made with real, non-ideal op-amps [1]. In particular, when we acknowledge the realness of the op-amp, the integrator has a second pole. We still have the pole at s=0, but there is a second pole, that is often negligibly far away (on the negative real axis). In cases where we need to worry about this pole, we may attempt to cancel it by putting a zero right on top of it [2]. In as much as we can suppose that we can cancel this second pole exactly (in practice we get approximate cancellation), we have only the one pole at s=0. Do we have our ideal integrator? No. At least – don't count on it!

If we wanted a network (an integrator) with only one pole, at s=0, and this is what we got, how could we have the wrong result? Because as we just said, the poles/zeros only give the network to within a multiplicative constant. In the pole cancellation scheme, we may have changed the gain of the integrator. Equivalently, we may have changed the time constant. Our scheme for creating a zero that cancels the extra pole may well have changed the gain.

We have frequently noted that many active filter networks have embedded integrators. These obviously include such filter configurations as the state-variable and active flow-graph [3], but also such networks as the multiple feedback infinite gain low-pass [4]. When we propose to "just fix the integrators" we need to be careful to see if we are just changing the gain (the time constant) slightly (changing the frequency slightly), or if we are changing the filter's characteristic.

REFERENCES:

[1] B. Hutchins, Analog Signal Processing, Chapter 7, Passive and Active Sensitivity, Electronotes, Vol. 20, No. 195, July 2000

[2] Reference [1], Section 7-7, Compensation of Linear Circuit Blocks for the Effects of Real Op-Amps

[3] B. Hutchins, Analog Signal Processing, Chapter 6, Integrator Based Designs, Electronotes, Vol. 20, No. 194, April 2000

[4] B. Hutchins, Analog Signal Processing, Chapter 5, Additional Configurations, Section 5-1, Electronotes, Vol. 20, No. 194, April 2000