ELECTRONOTES
1016 Hanshaw Rd
Ithaca, NY 14850
(607)-257-8010

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CASCADING DISCRETE AND CONTINUOUS HOLDS

We are familiar with the sample-and-hold effect, by which terminology we usually mean that a high-frequency roll-off results as a consequence of samples being stretched (held constant) for a non-zero period of time. Typically the hold time is the same as the time between samples. For example, if samples (numbers) are fed to a D/A converter, we expect the output of the converter to rapidly stabilize and hold a value (usually a voltage) that corresponds to the number. This voltage does not change until the number at the input of the D/A changes (Fig. 1).

In some sense, which may be mathematically involved, we can consider this sort of sample-and-hold (S&H) as the convolution of a pulse of width T with discrete samples spaced T apart (Fig. 2). This in turn means that we are obliged to think in the frequency domain in terms of multiplication by a sinc function. In fact, specifically, if we have a pulse of width T, the Continuous-Time Fourier transform is:

$$P(\Omega) = \int_{\infty}^{\infty} p(t)\, e^{-j\Omega t}\, dt = \int_{0}^{T} e^{-j\Omega t}\, dt = T\; e^{-j\Omega T/2}\left[\sin(\Omega T/2) / (\Omega T/2)\right] \qquad (1)$$

which is, in terms of ordinary (physical) frequencies, f:

$$P(f) = (1/f_s)\; e^{-j\pi f/fs}\left[\sin(\pi f/f_s) / (\pi f/f_s)\right] \qquad (2)$$

Where $f_s = 1/T$ is our ordinary sense of sampling frequency. For example, if f_s is 16,000 Hz, the sampling interval (duration of hold pulse) would be T =1/16,000. As a second example, T=1/8000 would correspond to f_s=8000. In this second case, the pulse would be twice as long.

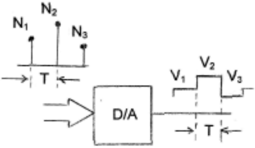

Fig. 1 A D/A converter has an inherent Sample-and-Hold.

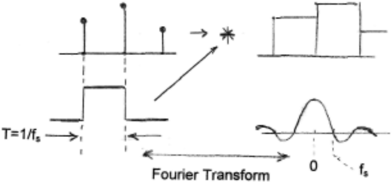

Fig. 2 Sample-and-Hold as a convolution with a pulse.

Consider two ways in which a width of T=1/8000 could occur. In the first, we would simply use a sampling rate of 8000 Hz. In a second, we would use a rate of 16,000 Hz, but would hold samples with a length-two discrete time hold. That is, every sample would be held for T=1/16,000, but each sample would be output twice (Fig. 3).

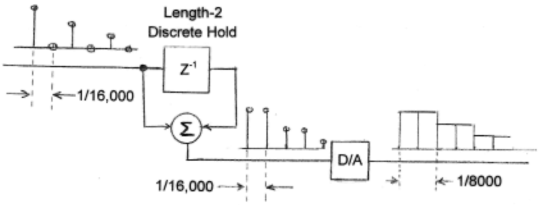

Fig. 3 Discrete-Time Hold Cascaded into a Continuous-Time Hold

The discrete-time hold has an impulse response h(n) that is equal to 1 for n=0 and n=1, and is zero for all other times. It's Discrete Time Fourier Transform, written for physical frequencies, is:

$$H(f)= \sum_{N=-\infty}^{\infty} h(n)\ e^{-j2\pi nf/fs} = 1 + e^{-j2\pi f/fs} = 2\ e^{-j\pi f/fs} \cos(\pi f/f_s) \qquad (3)$$

It must be true that if we plug the value of f_s=16,000 into equation (3) and equation (2), and multiply these two terms, we get equation (2) for a value of f_s=8000:

$$P(8000) = P(16{,}000)H(16{,}000) \qquad (4)$$

This is easily established using the trig identity sin(2x)=2cos(x)sin(x).

We note that this result almost had to be true based on intuition. One troubling factor might have been the periodic nature of equation (3). Indeed, the result of equation (3) is the simplest form of a "periodic sinc." However, it is because it is multiplied by the non-periodic P(f) that the product of equation (4) is just an ordinary sinc.