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APPLICATION NOTE NO. 349

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TWO POLE VS. FOUR-POLE FILTERS - THE ISSUES

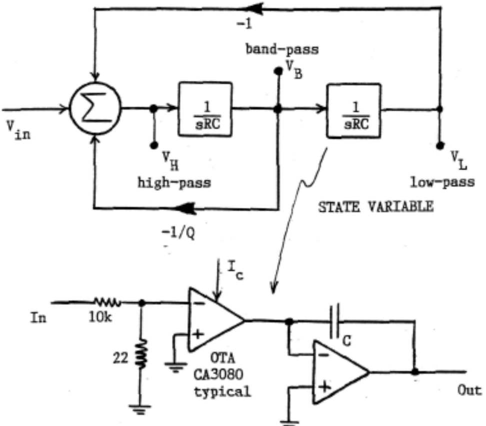
BACKGROUND

Voltage-Controlled Filters (VCF's) used in electronic music have taken many forms, but by far the most popular response type has been the low-pass. This low-pass in turn has appeared, generally, either as a two-pole (so-called "state-variable") form or as a four-pole (often referred to as a "Moog Ladder" filter). At one time, any purely technical comparison between the two was decorated by the additional elements of a lively contest (marketing, patenting, publishing, etc.) between Moog and ARP (and some others). Today, the technical (i.e., network) issues are well understood, and the original commercial issues have largely vanished, so the two filters can be compared on a flat playing field. Either filter can be, for the most part, realized with equal ease (see Fig. 1a and Fig. 1b, and the data sheets on the CES and SSM chips).

So the original issues are somewhat irrelevant, but not uninteresting, and the technical history as usual offers a perspective from which today's designers and users can usefully benefit. Further, some of the original issues may have "grain of truth" aspects which we can't afford to ignore, even today. Let me address two of these concerns before going on to the technical history.

First, one of the touted advantages of the two-pole (ARP, state-variable) filter was its ability to offer band-pass, high-pass, and notch functions in addition to the popular low-pass. Accordingly, the user was offered many additional, different filtering schemes for unique new sounds. Sadly, for the most part, no one was much interested in the unique new sounds possible from this filter option (or other such engineering delights). Synthesizer users were primarily interested in low-pass. The world of acoustical musical instruments is primarily a low-pass world (due to issues relating to physics, and the properties of materials).

What we found was something like: "Gee - that was a neat sound. Please make that sound again - <u>but not right now</u>. Can you make a clarinet?" This musical (acoustic) conservatism was certainly disconcerting to ARP in the case of the state-variable filter. But the same elements of conservatism were evident (for example) when Moog offered its unpatchable "mini" series.



$R = R_{eq} = 23.7/I_c$ (I_c in ma.)

Typical Integrator Stage Using Operational Transconductance Amplifier

$V_H(s)/V_{in}(s) = s^2/[s^2 + (1/Q)(s/RC) + 1/R^2C^2]$ etc. etc.

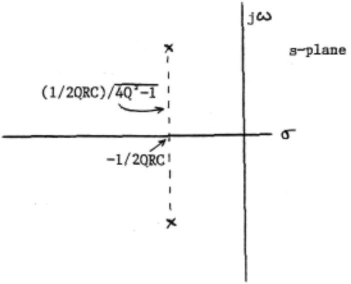
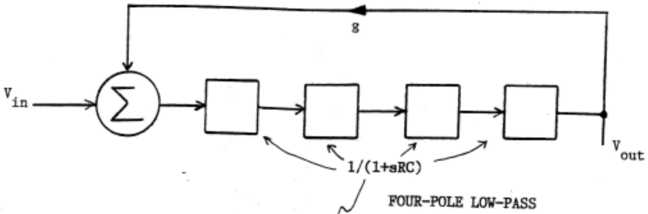
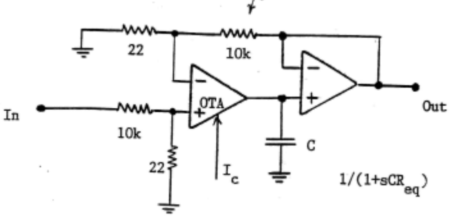
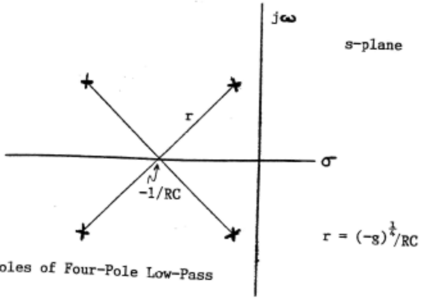


Fig. 1a The state-variable or "multi-mode" two-pole VCF. The poles, obtained by setting the denominator of the transfer function to zero, are in a complex-conjugate pairing for $Q > 1/2$.





Typical First-Order Low-Pass Using OTA



Poles of Four-Pole Low-Pass

Fig. 1b The 4-pole low-pass or "Moog Ladder" type of VCF. The four poles are on the corners of a square centered at $-1/RC$. Normally g ranges from 0 to -4, all poles being at $-1/RC$ for $g=0$, and two poles on the $j\omega$-axis for $g=-4$.

We all liked the low-pass. This comes from our listening experiences, both to music and to other real-world sound sources. As such, this is a somewhat blameless recalcitrance, perhaps akin to purchasing a new toy for a child and finding the child prefers to play with the box it came in.

The second issue for which there may be relevant analogs today relates to the fact that there is a (perhaps supposed) almost "mystical" nature to the "Moog Ladder." To the extent that a more skeptical person might insist on a real mechanism, the supposition is generally that any difference is due to aspects relating to the particular realization with semiconductor devices (generally supposed to be some mild non-linearity), and not to any network (linear time-invariant) aspects. This is because, without doubt, the original Moog ladder structure and the now more familiar transconductor realization have the same transfer function. As for listening aspects, I can only relate that one of Moog's engineers heard my OTA version of the 4-pole and without hesitation proclaimed it to be the same filter. I know of no blind tests however. For myself, and likely for many others, it would be difficult to knowingly listen to, or even better, be in the presence of a "real Moog Synthesizer," and not be at least subconsciously influenced into perceiving a superior sound. If this is mysticism, it is one we can afford to indulge.

THE TECHNICAL HISTORY

Everyone is familiar with the situation of an elder contending that life in the past was much more difficult (accompanied by at least an implicit contention that, none the less, the elder was able to buck up and get the job done, unlike today's soft generation!). Accordingly, while factoring a fourth-order polynomial is nothing today, in the past, it was extremely difficult. [It is still true that a first-order is trivial, a second-order falls trivially to the quadratic formula, there is a closed form solution for third order which virtually no one knows or uses, and a fourth-order (or higher) can not be done (provably) except by numerical methods. Alas, the numerical power is readily available today.]

In 1977, I became so curious about the positions of the poles of Moog's four-pole filter that I actually figured out where they were by using what was essentially an educated guess [1]. This was likely the first time the pole positions became generally known. (I recall that when I told Bob Moog that I had located the poles, he asked where they were. The fact that he was able to develop his filter without resort to network calculations is indeed a tribute to his intuitive understanding of feedback. I think he said he had "just put feedback around the loop" looking for resonance, the so-called "regeneration" known to radio engineers). In any event, my elation at finding the poles by a trick would have been short lived, as the available programs for hand-held calculators were not far off [2]. However, even more humbling was a note submitted by Richard Bjorkman [3] where he showed how simply a substitution of variable could blow the problem asunder. (Richard was charitable enough to characterize my prior efforts as "unnecessarily cumbersome!")

The most interesting question associated with knowing the position of the fourth-order poles is that they are not, in many cases of interest, radically different from second-order poles. To be more exact, nearly everyone has heard the term "dominant pole." This reference is (often) actually to a pair of poles - poles generally coming in complex conjugate pairs. When we say that a pole (pair) is dominant, we mean that, at least for a certain range of frequencies, the overall response of the filter is much the same as it would be if only that pair of poles were present. That means that other poles are either effective in other ranges of frequency (for example, as asymptotic roll-off), or perhaps, only effective in relatively subtle tuning aspects such as having a very flat pass-band.

Typically filters with many poles are designed with a rational strategy in mind: one that makes efficient use of each pole available. We do not just pile up poles in an ad hock manner. Accordingly, we often expect to find fourth-order filters with s-plane poles in Butterworth (Fig. 2a) or Chebyshev (Fig. 2b) arrangements. In contrast, the Moog's four-pole low-pass would have poles at the corners of a square (Fig. 2c) with the poles closest to the $j\omega$-axis dominating, and therefore resembling a two-pole filter (Fig. 2d). The fact that the four-pole seems inefficient, at least from a conventional network viewpoint, can be attributed to the fact that the paramount interest was achieving voltage-control. That is, no matter how nice a particular, conventional, filter characteristic appeared, it was not useful for a VCF unless a corresponding voltage-controllable network could be implemented.

To make the point about a dominant pole more precisely, we can consider Fig. 3, which is a semilog plot to better show the asymptotic behavior of the frequency response magnitudes shown. The particular case we start from is the Moog ladder $g=-1.5$ case. This response, the lowest curve in Fig. 3, has a bump (ripple) at the passband edge, and then rolls off asymptotically at 24db/octave. If we then simply remove the pair of poles that are furthest from the $j\omega$-axis (and renormalize to 1 at dc) we obtain the upper curve in Fig. 3. This is the response of the dominant pole pair. We note immediately that the two curves are very similar in the passband (frequencies from 0 to 1), but differ in the asymptotic region, where the dominant pole pair case (a second-order low-pass) only rolls off at 12db/octave. This establishes that the passband region for the four-pole filter is essentially a two-pole type response.

Finally, also on Fig. 3 we have plotted a third curve which essentially matches the four-pole response in the frequency range of 0 to 1, and then moves over to the dominant pole response for higher frequencies. This curve just represents a small manipulation of the positions of the dominant poles to even better illustrate that the four-pole response is well approximated in this region by just two poles.

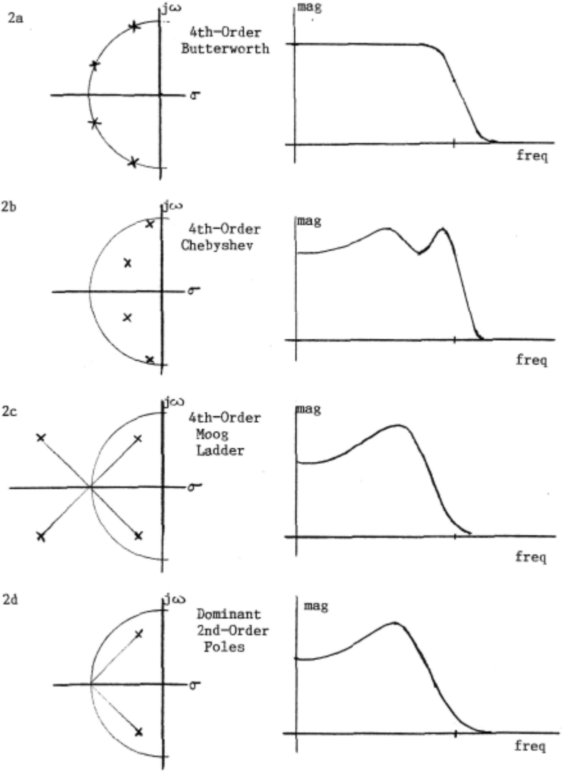
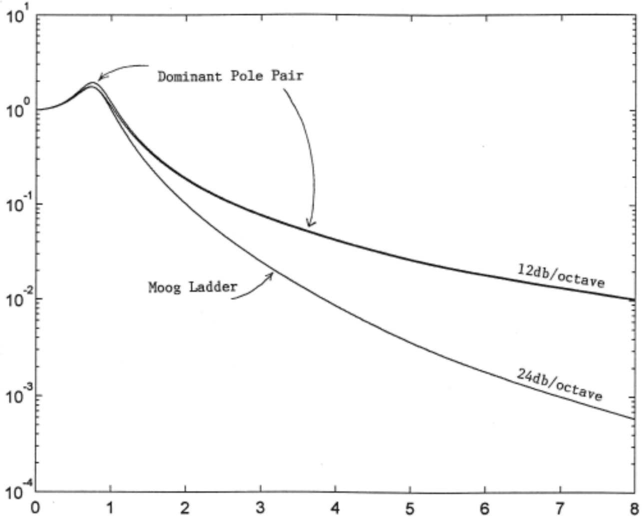


Fig. 2 In 2a, we have a standard 4th-Order Butterworth low-pass, which is a "maximally flat" response. By reducing the real part of the Butterworth poles, all by the same factor, we achieve an equiripple or Chebyshev type of response (2b). In 2c we have an example of a 4th-order "Moog ladder" which is not a standard response, since the network was chosen for its voltage-controllable aspects. In fact, the response is largely determined by the two "dominant poles" (2d) which are the two that are closest to the jω-axis. The two responses, 2c and 2d, differ in the asymptotic roll-off rate which is much better seen in Fig. 3.



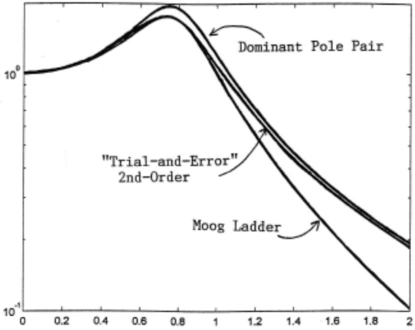


Fig. 3 Comparison of Moog Ladder response and that of its two dominant poles. Above we see the responses similar in the passband (normalized to 1/RC=1) from 0 to 1, but reaching different asymptotic values for higher frequencies. Between the two responses we have a trial-and-error fitting of a second-order response to the 4th-order passband (a slight maneuvering of the dominant poles). A detail of the passband above is shown at the right.

SOME DESIGN POSSIBILITIES AND TESTS

We close with two related topics. (1) Can we make a VCF circuit that switches easily from 2-pole to 4-pole? (2) Can we do a hearing test to see how important the different asymptotic roll-off rates are?

The first question has a simple answer, at least for a start. Yes. Let's suppose you have one of the VCF chips that has four transconductors on it. You might suppose that you then need to choose a state-variable or a four-pole circuit. But, suppose you decide to use your four transconductors to make two state-variable filters in parallel, giving you the option of using just one. The four-pole option is then a matter of fixing the Q of the two state-variable filters at a wimpy 1/2 (two real poles at -1/RC), putting them in series (now four real poles at -1/RC), and then implementing the g loop around them.

The answer to the second question may be more difficult, but would perhaps be easier if you do implement a VCF in response to question 1. There are two difficulties associated with this test. The first is that we would need to be able to switch rapidly between comparable cases. In particular, we would like the two-pole comparison case to become available at the flick of a switch, and correspond to the dominant pole pair of the four-pole case. This is doable, but not exactly trivial. Secondly, there is the always present problem of running valid hearing comparison tests. That is, do we hear a difference that is the result of an artifact that we are not supposed to be testing? Is the test valid if not done blind? And so on!

References

[1] B. Hutchins, "Additional Design Ideas for Voltage-Controlled Filters," Electronotes, Vol. 10, No. 85, pp 5-12, Jan 1978

[2] W. Luke & B. Hutchins, "TI-59 Program for Roots of a Polynomial," Electronotes Supplement S-018, August 1979

[3] Richard Bjorkman, "A Brief Note on Polygon Filters," Electronotes, Vol. 11, No. 97, January 1979, pp 7-9

-Bernie