ELECTRONOTES

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APPLICATION NOTE NO. 194*

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SIMPLE SHELVING EQUALIZER

The circuit of Fig. 1 is about as simple as a circuit can get consisting of an op-amp and two resistors and two capacitors. The circuit has one gain at low frequencies, and a different gain at high frequencies. Inbetween, the gain changes smoothly from one to the other. Since the gain is level except in the transition region, the flat regions look something like shelves, and you can push something up to a higher shelf, or down to a lower shelf. The levels of the shelves can be chosen for whatever values you need. Some typical examples are sketched in Fig. 2.

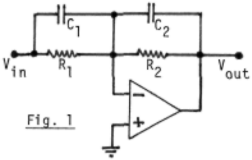
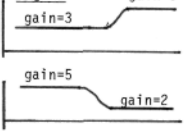


Fig. 1

Fig. 2


To get some idea as to how the circuit works, first observe that at very low frequencies, DC let's say, the capacitors are out of the circuit and we have only an inverting amplifier with gain R_2/R_1. At very high frequencies, the capacitive reactance of the capacitors will be much less than the resistances, and the capacitors will dominate. Since the reactances of the capacitors go as 1/C, the high frequency gain goes as C_1/C_2.

To go much further, we need the transfer function of the network. This is quite easy. The parallel combination of R_1-C_1 has an impedance $R_1/(1+sC_1R_1)$ while the parallel combination R_2-C_2 is similarily $R_2/(1+sC_2R_2)$. The transfer function is then done much as the inverting amplifier, using the impedances just determined:

$$T(s) = \frac{V_{out}}{V_{in}} = -\frac{R_2/(1+sC_2R_2)}{R_1/(1+sC_1R_1)} = -\frac{R_2(1+sC_1R_1)}{R_1(1+sC_2R_2)} \qquad (1)$$

This is a simple equation which has a zero at $s = -1/R_1C_1$ and a pole at $s = -1/R_2C_2$. We also see that in the limits of s going to zero, we are left with a gain of $-R_2/R_1$, and when s is very large, we get a gain of $-C_1/C_2$, just as we argued above. To get the exact shape of the response, we need to take the magnitude of T(s), which is done by taking $|T(s)| = [T(j\omega)\cdot T(-j\omega)]^{1/2}$, which gives us:

$$|T(s)| = \frac{R_2}{R_1}\left[\frac{1+\omega^2 R_1{}^2C_1{}^2}{1+\omega^2R_2{}^2C_2{}^2}\right]^{1/2} = \frac{R_2}{R_1}\left[\frac{1+39.478\,f^2\,R_1{}^2C_1{}^2}{1+39.478\,f^2\,R_2{}^2C_2{}^2}\right]^{1/2} \qquad (2)$$

where in the rightmost term of equation (2) we have substituted $2\pi f$ for ω. Note that given equation (2) with the values of R_1, R_2, C_1, and C_2, we can calculate the frequency response at any point.

It should be kept in mind that the rate of transition between one shelf and the other is quite gradual, and never more than 6db octave. In general, it will take several octaves for the transition to take place. Secondly, the equations for the design of the network are straightforward, but not simple in their final form in general. The types of formulas we need are those that will give us 3db frequencies, frequencies for average gains, and gains at the pole and zero frequencies. In all we do below, a frequency may be given in terms of ω or in terms of f, with $\omega = 2\pi f$.

We begin by identifying the pole and zero frequencies as follows:

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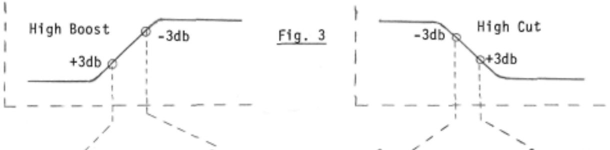
$$f_p = 1/2\pi R_2C_2 \qquad \text{and} \qquad f_z = 1/2\pi R_1C_1 \qquad (3,4)$$

By substituting $\omega_p = 1/R_2C_2$ and $\omega_z = 1/R_1C_1$ into equation (2) we get the gain at the pole frequency G_p and the gain at the zero frequency G_z as:

$$G_p = \frac{R_2}{R_1}\left[\frac{R_2^2C_2^2 + R_1^2C_1^2}{2R_2^2C_2^2}\right]^{\frac{1}{2}} \qquad G_z = \frac{R_2}{R_1}\left[\frac{2R_1^2C_1^2}{R_2^2C_2^2 + R_1^2C_1^2}\right]^{\frac{1}{2}} \qquad (5,6)$$

Next it is useful to derive the 3db frequencies. These must be done for the separate cases of higher or lower gain at high frequency relative to low frequency. The equations are shown below the corresponding diagrams in Fig. 3 below:



$$f_{\ell+3db} = \frac{1}{2\pi\sqrt{R_1^2C_1^2 - 2R_2^2C_2^2}} \qquad f_{h-3db} = \frac{\sqrt{R_1^2C_1^2-2R_2^2C_2^2}}{2\pi R_1R_2C_1C_2} \qquad f_{\ell-3db} = \frac{1}{2\pi\sqrt{R_2^2C_2^2-2R_1^2C_1^2}} \qquad f_{h+3db} = \frac{\sqrt{R_2^2C_2^2-2R_1^2C_1^2}}{2\pi R_1R_2C_1C_2}$$

(7,8,9,10)

Also of interest are frequencies that are somehow associated with the center of the transition region. We can consider the center of a linear plot, or the center of a log plot. For the center of a linear plot (for example, a gain of 6 where the shelves are at gains of 2 and 10), we set equation (2) equal to the average of R_2/R_1 and C_1/C_2, which is $(R_1C_1+R_2C_2)/2R_1C_2$. Then solving for the frequency we get:

$$f_a = \frac{1}{2\pi}\sqrt{\frac{R_1^2C_1^2 + 2R_1R_2C_1C_2 - 3R_2^2C_2^2}{R_2^2C_2^2(3R_1^2C_1^2-2R_1R_2C_1C_2-R_2^2C_2^2)}} \qquad (11)$$

The center gain on a log-log plot would be the square root of the product of the gains at low and high frequency, thus the center gain is $\sqrt{(C_1/C_2)(R_2/R_1)}$. Equating this to equation (2) and solving for frequency, we get:

$$f_o = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \qquad (12)$$

This is a useful result since it is just the square root of the product of the pole and zero frequencies. Thus:

$$f_o = \sqrt{f_zf_p} \qquad (13)$$

Fig. 4 shows an actual example, a boost of four on the high frequency end.

Fig. 4 corresponds to $R_1=R_2=100k$, $C_1=0.01$, and $C_2=0.0025$. Note that the 3db frequencies approach the pole and zero frequencies, and this is even more true as the shelves spread more and more apart. The average gain is $(4+1)/2 = 2.5$ and occurs at f_a, while the center of the log plot is at a gain of 2 at f_o since the ratio of 1:2 is the same as the ratio 2:4. Note the symmetry (except for f_a) about f_o on the log-log plot.