ELECTRONOTES

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APPLICATION NOTE NO. 167

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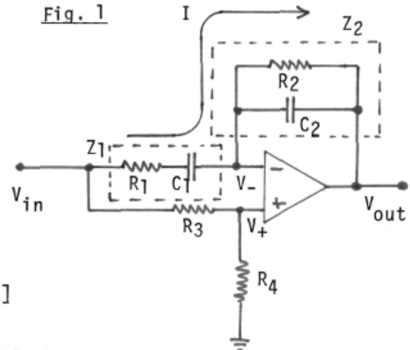
90° PHASE-DIFFERENCE NETWORKS

SECOND-ORDER ALL-PASS NETWORKS - 1

For many purposes, the first-order all-pass network (AN-65) serves well, and even when a higher order all-pass is needed, cascaded first-order sections are often the most convenient. Here we will look at a second-order network that is also useful from A. G. Lloyd, "Here's a Better Way to Design a 90° Phase-Difference Network," Electronic Design, #15, pg. 78 (1971).

The Lloyd circuit is shown in Fig. 1 and can be seen to be a basic differential amplifier structure with series and parallel RC structures replacing two of the resistors. It is convenient to consider these RC structures as equivalent impedances, Z_1 and Z_2, as indicated in the figure. The network can be easily solved then by the following relationships:

Fig. 1



$$V_+ = V_- = V_{in}\frac{R_4}{R_3+R_4} = K\,V_{in}$$

$$I = [V_{in}-KV_{in}]/Z_1$$

$$V_{out} = KV_{in} - I\,Z_2 = KV_{in} - [1-K]V_{in}[Z_2/Z_1]$$

$$T(s) = V_{out}/V_{in} = K - [1-K][Z_2/Z_1]$$

Then, substituting in for Z_1 and Z_2, we have

$$Z_1 = R_1 + 1/sC_1\,, \quad Z_2 = \frac{R_2}{1 + sC_2R_2}$$

$$T(s) = \frac{K(1-sC_1R_1)(1-sC_2R_2) + \{2sKC_1R_1 + 2sKC_2R_2 - (1-K)sC_1R_2\}}{(1 + sC_1R_1)(1 + sC_2R_2)}$$

where T(s) is now a second-order all-pass if we can get the quantity in the { } to vanish, which is accomplished if $2sK(C_1R_1 + C_2R_2) = (1-K)sC_1R_2$. Thus, the "final" results are given by:

$$T(s) = \frac{K(1-sC_1R_1)(1-sC_2R_2)}{(1+sC_1R_1)(1+sC_2R_2)}$$

$$K = \frac{1}{2(R_1/R_2) + 2(C_2/C_1) + 1}$$

This essentially solves the problem, but there are a number of design variables that can be tied down better on further examination.

First we note that K is the gain of the all-pass; not one in this case, but K. This is of some concern, but by no means what need be considered serious. The other factor important with the all-pass is the 90° frequencies of the poles, given by $f_1 = 1/2\pi R_1C_1$ and $f_2 = 1/2\pi R_2C_2$ in this case. The important thing to note is that we cannot choose all parameters independently. Equation (2) shows us that K will always be less than one, and depends on the ratios of capacitors and resistors, which in turn depend on the 90° frequencies. To look at this further, let's assume that the 90° frequencies are in the ratio $f_1/f_2 = r$, where r could be less than, equal to, or greater than 1. Let's further assume that capacitors are harder to get in any good assortment, so we will choose these first, and calculate resistors. From (2), we see that we should not choose $C_1 = C_2$, unless we can accept a value of K less than 1/3. So let's keep this ratio variable, and call $C_2/C_1 = h$. This done, we can write equation (2) in a new form:

$$K = \frac{1}{2h(1/r + 1) + 1} \tag{3}$$

Since it is probably a reasonable goal to keep K fairly large, say above 0.25 if possible, we can look at equation (3) to see how this might be accomplished. We see that K is large if the quantity $2h(1/r + 1)$ is small, which happens if h is small to begin with, and r is large, if possible.

In practical design terms, we have no real choice of $r = f_1/f_2$, as these two 90° frequencies are given to us by the needs of our application. Our real choice is in which one we make f_1 and which one we make f_2. To keep K large, we will make f_1 the larger of the two 90° frequencies. That is, the higher of the two frequencies is associated with the R_1C_1 part of the circuit. [Of course, if $f_1=f_2$ then we have no preferred selection, but things need not be bad if we choose C_2 larger than C_1, as we see below.] We do have a choice of values for $h = C_2/C_1$, as we have seen, and while we often like to choose these equal in other filters, just to keep things simple, here there is a real advantage to making C_2 much smaller than C_1 if we wish to keep K large. Thus we would tend to keep a respectable spread (but not a ridiculous one) between the values, with something like h = 1/5 to h = 1/100 being reasonable. Note that we would do this even if $f_1 = f_2$, in which case we would spread the capacitors as $C_2/C_1 = h$, and adjust the resistors to keep the time constants the same (i.e., $R_1/R_2 = h$ for this case where the 90° frequencies are the same).

It is of course of interest to see what range of K we can expect in a practical case. In a typical application, a 90° phase differencing network, pole frequencies are typically spaced about 10:1, so we can take r = 10. We can also spread capacitors $h = C_2/C_1$ to give a value of 0.01 without much trouble. The result from equation (3) is K = 0.978, a respectable value not all that far from 1.00.

Fig. 2 shows the use of the Lloyd network in an 8-pole, 10Hz - 15kHz bandwidth, 2.5° error, 90°PDN (see AN-165 for data). Here h = 10 and r is also on the order of 10. The characteristic frequencies associated with the RC elements next to them are indicated in ( ). The through gain of the circuit is about 0.67 for each output. As a practical matter, we found that larger values of h, or of r by changing frequencies between the left/right sections, leads to too much component spread. The network was constructed with closest 5% values, and found to work well, although some tuning by measuring or selecting high accuracy components would be a good idea. Note that associated RC pairs appear a bit strange, a small resistor associated with a large capacitor (or vice versa). This is a result of our desire to keep the gain up.

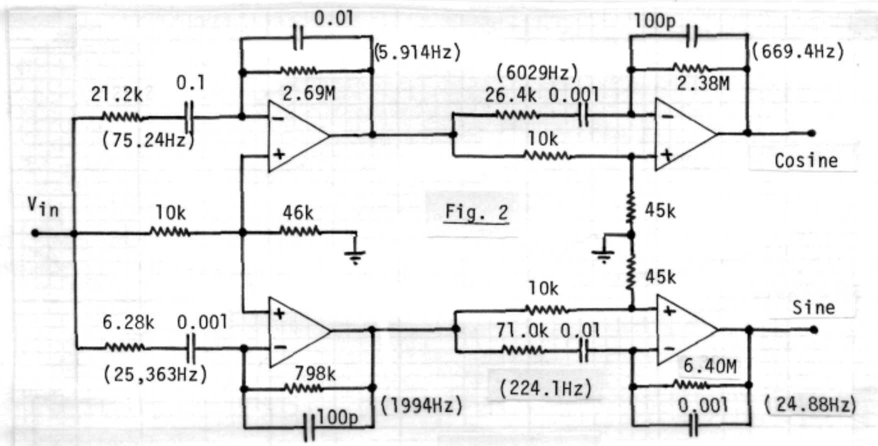


Fig. 2

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SECOND-ORDER ALL-PASS NETWORKS - 2

The previous note has served as a bridge between a discussion of 90° phase-difference networks and second-order all-pass networks. In that note, the second-order network was used in a very natural way - in place of two first-order networks, thus saving one op-amp. Here we want to take a more general look at second-order all-pass networks. We need to see that even an all-pass can have a meaningful "Q" and that we can become concerned with the actual shape of the phase response curve.

The general form of a second-order all-pass filter is:

$$T(s) = \frac{s^2 - (\omega_0/Q)s + \omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2} \tag{1}$$

The denominator of this transfer function is probably quite familiar to the reader, as it appears in all sorts of low-pass, band-pass, and high-pass functions. The parameter of the term in s is taken as Q (actually as 1/Q) according to the usual custom. It is sometimes said that Q has no meaning except for the band-pass case. This is not true. Q always has a meaning in terms of the frequency range over which a response changes. Here, a high-Q all-pass corresponds to a case where most of the phase shift of 360° occurs over a very narrow bandwidth, similar to the 3db bandwidth that defines the Q of the corresponding bandpass.

The reader has noted in equation (1) above that the numerator is identical to the denominator, except for the sign in front of the term in s. To see what this means, we can factor the numerator and denominator using the quadratic equation. Thus the zeros from the numerator are at:

$$s = z_{1,2} = \omega_0/2Q \pm (1/2)\sqrt{(\omega_0^2/Q^2) - 4\omega_0^2} \tag{2}$$

while the poles are at:

$$s = p_{1,2} = -\omega_0/2Q \pm (1/2)\sqrt{(\omega_0^2/Q^2) - 4\omega_0^2} \tag{3}$$

As always with such a factorization, there are two cases of interest: the one where the square root term yields a real number, and the one where it yields an imaginary number. The square root term gives a real number as long as Q is less than 1/2, zero when Q = 1/2, and an imaginary number when Q is greater than 1/2. This is exactly the same as it is with low-pass, band-pass, etc. What is important here is that there is also a real term present, $\omega_0/2Q$, which places the poles or zeros initially before the square root term comes into play. This displacement is $-\omega_0/2Q$ for the poles (into the negative half-plane) and $+\omega_0/2Q$ for the zeros (into the positive half-plane). This displacement is symmetric about the $j\omega$-axis. Now, note that the square root term has two solutions (the ± sign), and that this term moves the poles and places them symmetrically, either along the real axis, or vertically perpendicular to it, relative to the initial positioning. The result is that for each pole in the left (negative) half-plane, there is a zero in the right (positive) half-plane as a mirror image across the $j\omega$-axis. Fig. 1 shows possible pole-zero positions for all-pass networks.

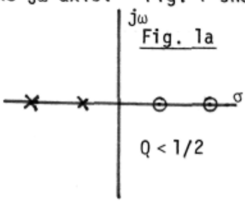


Fig. 1a

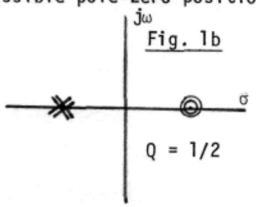


Fig. 1b

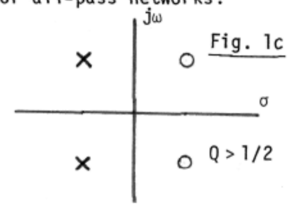
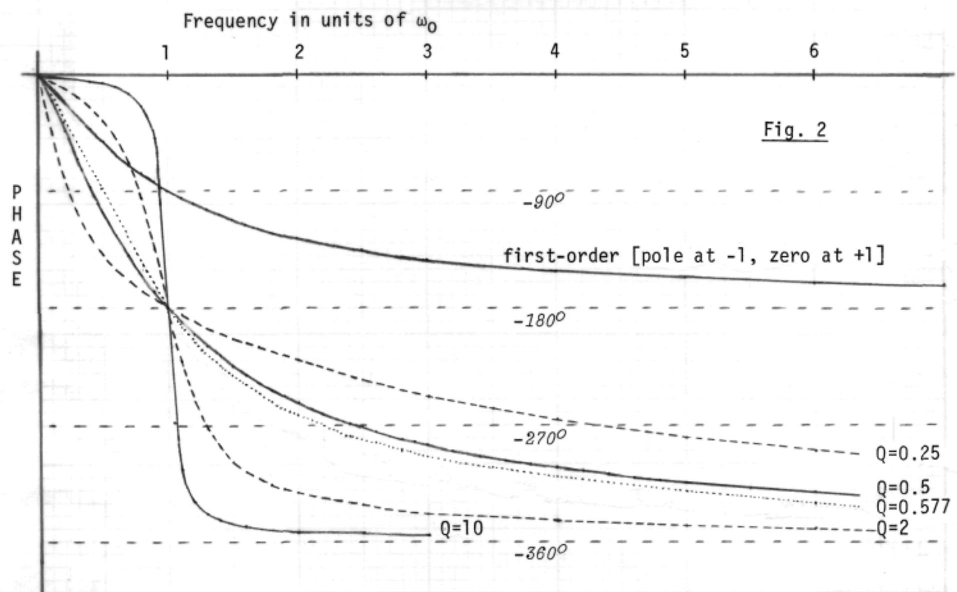


Fig. 1c

Fig. 1a shows the case where there are two pair of real poles and zeros. This is the case that we get from cascading two first-order sections, since first-order can give only real poles. Keep this in mind when designing 90° phase-difference networks, since the data for these is in terms of <u>real</u> pole-zero pairs. Thus if you are going to use a second-order section for these networks, the Q must be less than 1/2. Fig. 1b shows the case of Q = 1/2, giving second-order poles and zeros. Fig. 1c shows the case where Q is greater than 1/2, resulting in complex poles and zeros

In order to understand the effects of changing these pole positions, it is necessary to look at the phase response. Given a value of Q, it is possible to use equations (2) and (3) to arrive at the poles and zeros. To determine the phase, you can derive the formulas, but a graphical interpretation is much more natural here. You first select a frequency point at which you need to know the phase. You then move to each of the s-plane features (the poles and zeros) in turn, and measure the angle from that feature to the frequency point, measured counter-clockwise from a line parallel to the real axis. Start with zero phase angle. If the feature is a zero, add the angle you find. If it is a pole, subtract the angle. The final figure (for four features, two poles and two zeros, in this case) is the total phase. Since the total phase is really a phase lag (you don't get something out before it goes in!), it is reasonable to interpret the total phase angle as a negative number. The result is somewhat arbitrary however for periodic waveforms, so you may see different interpretations.

The results for several Q values of interest are shown in Fig. 2, with a first-order network included for reference. Note that all networks pass through 180° phase at ω_0. However, there is a wide variety of curves for different Q values. The Q=0.577 is the "linear phase" or "Bessel" curve between 0° and 180° (dotted line). Note that the other curves bend to either side of this linear phase, and are therefore useful as possible (approximate) corrections to networks that are not linear phase.



<u>Fig. 2</u>

Q=0.25	Poles at -0.268, -3.464	Zeros at +0.268, +3.464
Q=0.50	Poles at -1.000, -1.000	Zeros at +1.000, +1.000
Q=0.577	Poles at -0.865 ± 0.500j	Zeros at +0.865 ± 0.500j (Linear Phase)
Q=2.00	Poles at -0.250 ± 0.968j	Zeros at +0.250 ± 0.968j
Q=10.0	Poles at -0.050 ± 0.999j	Zeros at +0.050 ± 0.999j

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APPLICATION NOTE NO. 169

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SECOND-ORDER ALL-PASS NETWORKS - 3

The past two application notes have given a feeling for the nature of second-order all-pass networks. In AN-169 and AN-170 we will be taking a look at some actual networks (we have already looked at Lloyd's network) to see how well they work out. Some things we must keep in mind are:

(1) Is the network capable of an all-pass response?

(2) If so, what are the conditions on its parameters for all-pass?

(3) Does the network produce poles for Q's on both sides of Q = 1/2 (that is, imaginary poles/zeros for Q greater than 1/2)?

(4) Are the frequency parameters easy to set (are they tuned by a simple RC time constant involving easy to change R's and C's)?

(5) Are there any limitations due to gain or sensitivity, etc.?

For the moment, let's return to the Lloyd all-pass of AN-167, where it was found that the transfer function is:

$$T(s) = K\frac{(1-sC_1R_1)(1-sC_2R_2)}{(1+sC_1R_1)(1+sC_2R_2)} \tag{1}$$

which we recognize as the same as the cascade of two first-order all-pass sections. We want to determine if this is capable of producing imaginary poles/zeros. We can compare this with the general form of a second-order all-pass from AN-168:

$$T(s) = A\frac{s^2-(\omega_0/Q)s+\omega_0^2}{s^2+(\omega_0/Q)s+\omega_0^2} \tag{2}$$

where A is a constant, which we will assume takes care of K in equation (1). If we multiply out the factors in equation (1), and equate powers of s in equation (2), we arrive at the formula for Q (after some algebra):

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2} \tag{3}$$

Is it possible for Q to be greater than 1/2? (Clearly it is 1/2 if $R_1C_1 = R_2C_2$). To test this, suppose we have selected R_2C_2 and Q, and want to solve for R_1C_1. If we square equation (3), we get a quadratic equation in R_1C_1:

$$(R_1C_1)^2 + R_2C_2(2-1/Q^2)(R_1C_1) + (R_2C_2)^2 = 0 \tag{4}$$

which, using the quadratic formula, has solution:

$$(R_1C_1) = \frac{-R_2C_2(2-1/Q^2)}{2} \pm \frac{R_2C_2}{2Q}\sqrt{1/Q^2-4} \tag{5}$$

The square root term gives an imaginary result if Q is greater than 1/2, which would require us to have an imaginary valued resistor or capacitor. Hence, Q is always equal to or less than 1/2 for the Lloyd circuit. Just great for 90°PDN's.

The Lloyd circuit uses a single op-amp, is very easy to tune to the proper pole frequencies, and is very useful where only real poles/zeros are needed. What about the cases where you need complex poles/zeros in your design (Q's greater than 1/2)? As we saw in the previous note, this can lead to some very useful phase responses. There are numerous approaches to this problem, and we will look first at some relatively straightforward ones, since for any limited production run (i.e., one unit), you probably want to get done as easily as possible, and the reduction to a one op-amp circuit is not particularly imperative.

▷ The absolutely most straightforward design scheme is to construct the required numerator using a biquadratic approach (state-variable filter), (see AN-113). To do this, we first construct the state-variable filter, setting it for the needed frequency and needed Q. This gives us the proper denominator that we need. We then sum various nodes in the filter to give us the proper numerator (the same as the denominator, except the sign of the term in s is changed). A useful circuit is shown in Fig. 1 below:

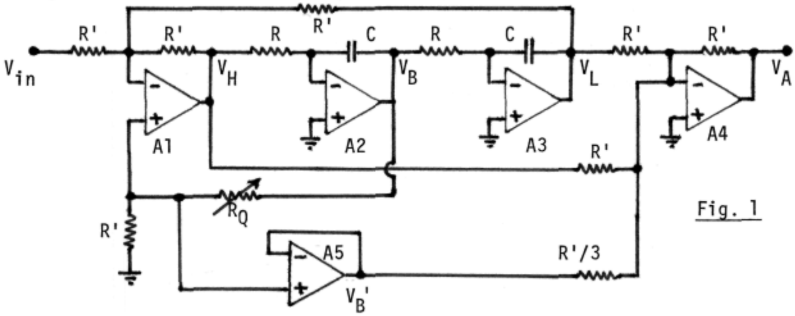

Fig. 1

The state variable filter (A1, A2, and A3) has been studied well before (AN-11, AN-91) so we need only give the most basic equations of analysis here. They are:

$$V_- = \frac{V_{in} + V_L + V_H}{3} = V_+ = \frac{V_B\ R'}{R' + R_Q} \text{ (for op-amp A1)},\ V_B = -V_H/sCR,\quad V_L = V_H/s^2R^2C^2$$

These equations can be solved to give transfer functions for the V_H, V_B, and V_L nodes:

$$T_H(s) = V_H/V_{in} = -s^2/D(s),$$

$$T_B(s) = V_B/V_{in} = (s/RC)/D(s)$$

$$T_L(s) = V_L/V_{in} = (-1/R^2C^2)/D(s)$$

where $D(s) = s^2 + [3R'/(R'+R_Q)]\frac{s}{RC} + 1/R^2C^2$

which can be compared with the general form we have used: $s^2 + (\omega_o/Q)\,s + \omega_o^2$

showing that $Q = (R'+R_Q)/3R'$.

Note next how nicely A4, an inverting summer, adds up the proper voltages to give the correct denominator. The change of sign of the middle term, due to the inverting integrator, is certainly fortuitous. We have also added buffer A5, since that scales V_B by the needed ratio $R'/(R_Q+R')$, and then we pick up the needed factor of 3 by making the summing resistor into A4 take on the value R'/3. Note that this is most useful when you don't know the Q you need ahead of time. You can then change R_Q as you wish, and the network remains all-pass. If you know the Q you need, you can take out A5, driving from V_B instead of V_B' with a resistor of value $(R'+R_Q)/3$. The two integrators determine the characteristic frequency ω_o, as usual, and $1/2\pi RC$ is the frequency at which the phase reaches -180°.

▷ The above biquadradic method, while straightforward, is not elegant in the efficient use of op-amps. We want here to describe another straightforward method, the subtraction of a bandpass response from unity. Recall that a bandpass function is of the form:

$$T_B(s) = \frac{As}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

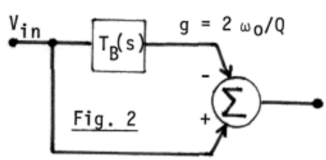

Fig. 2

If we subtract this from 1 (see Fig. 2), we get:

$$T_A(s) = 1 - T_B(s) = \frac{gs - s^2 - (\omega_o/Q)s - \omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2} = -\frac{s^2 - (\omega_o/Q)s + \omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

where the last equality in the equation above holds for $g = 2\omega_o/Q$, and shows that an all-pass does result.

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APPLICATION NOTE NO. 170

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SECOND-ORDER ALL-PASS NETWORKS - 4

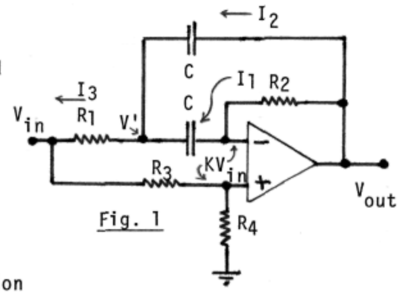

Fig. 1

In the last AN, we ended by showing how a bandpass response and unity may be weighted and summed out of phase to give an all-pass response. You can do this, using a separate summer, with any bandpass filter you have in mind. A circuit which does both the bandpass and the summation (subtraction) is shown in Fig. 1. We found this circuit in A. Budak, Passive and Active Network Analysis and Synthesis, Houghton Mifflin Company. A quick glance at the circuit will give the reader the impression that he has seen it before. It looks a bit like the Lloyd circuit of AN-167. It also looks a bit like the "Deliyannis" bandpass filter (AN-145) except the R3-R4 loop connects to the input here, not the output. The top portion (R1, R2, C, and C) looks exactly like the standard bandpass (AN-25), and this is the best thought. Since the standard bandpass is inverting, we can see that the R3-R4 divider going to the input and feeding to the (+) input of the op-amp provides the type of subtraction we are looking for.

Analysis of the circuit is standard, but takes about three pages of algebra, so we will leave some of it to the reader. We start by noting that the divider R3-R4 puts a voltage KV_{in} on the (+) input of the op-amp, where $K = R_4/(R_3+R_4)$, and this same voltage will be on the (-) input. We then have four unknowns: I_1, I_2, I_3, and the unknown node voltage V'. We can immediately write I_1 as:

$$I_1 = (V_{out}-KV_{in})/R_2 \tag{1}$$

Since I_1 must be flowing out through the lower C, we can get V' as:

$$V' = KV_{in} - I_1/sC = \frac{K(1+sCR_2)V_{in} - V_{out}}{sCR_2} \tag{2}$$

and V' thus identified gives us I_2 and I_3:

$$I_2 = (V_{out}-V')sC = \frac{V_{out}(1+sCR_2) - V_{in}K(1+sCR_2)}{R_2} \tag{3}$$

$$I_3 = (V'-V_{in})/R_1 = \frac{V_{in}[K(1+sCR_2)-sCR_2] - V_{out}}{sCR_2R_1} \tag{4}$$

We can then set $I_1 + I_2 = I_3$, and a page of algebra then gives the transfer function:

$$T(s) = K\frac{s^2 + \left[\frac{2}{CR_2} + \frac{(K-1)}{K}\frac{1}{CR_1}\right]s + \frac{1}{C^2R_1R_2}}{s^2 + (2/CR_2)\,s + 1/C^2R_1R_2} \tag{5}$$

We will want to make equation (5) an all-pass, but first we can work with the denominator, putting it in the standard form:

$$s^2 + (\omega_0/Q)s + \omega_0{}^2 \tag{6}$$

from which we can identify the characteristic frequency ω_0 and the Q as:

$$\omega_0 = 1/C\sqrt{R_1R_2} \qquad \text{and} \qquad Q = \frac{1}{2}\sqrt{R_2/R_1} \tag{7,8}$$

which should look familiar - they are exactly the same as they were for the standard bandpass (AN-26). Thus we are able to use familiar equations for the Q and characteristic frequency, and by grounding the (+) input, we can check this bandpass separately for the proper frequency response, which is easier than checking the phase later. Once these are set right, we know that the denominator is correct, and that as long as we can match numerator and denominator terms (with the coefficients of s of opposite sign), we will get our needed all-pass. Thus we must now find the required value of K.

Setting the coefficient of s in the numerator equal to the negative of the coefficient of s in the denominator, we get:

$$\left[\frac{2}{CR_2} + \frac{(K-1)}{K}\frac{1}{CR_1}\right] = -\frac{2}{CR_2} \tag{9}$$

which is easily solved for K as:

$$K = \frac{1}{1 + 4R_1/R_2} \tag{10}$$

It is necessary to ask how the value of K that we select will effect the overall response of the network. In particular, the factor K is an overall multiplier of the transfer function, as can be seen by the updated equation:

$$T(s) = K\,\frac{s^2 - (2/CR_2)s + 1/C^2R_1R_2}{s^2 + (2/CR_2)s + 1/C^2R_1R_2} \tag{11}$$

To see the value of K for different values of Q, we can write K as a function of Q, which is easy since $1/Q^2 = 4R_1/R_2$, hence:

$$K = \frac{Q^2}{1 + Q^2} \tag{12}$$

which has the attractive property of approaching unity quite well for moderate and large values of Q. Thus, this circuit is an excellent choice when we need a large value of Q in our all-pass.

Having worked out a satisfactory low-Q circuit (Lloyd's of AN-167) and a satisfactory high-Q circuit (this note), both of which use only one op-amp, we can consider our job pretty well done. Thus, we will want to close out this series for now (other circuits are still under study) with a few additional comments.

First, it may have occured to the reader somewhere along the line that the subtraction of a bandpass from unity should give a notch, not an all-pass. In fact, it can give a notch, if you subtract half the amount you needed for the all-pass. Remember that for the all-pass, we needed to cancel a term which was there, and then replace it with a negative. For the notch, we just have to cancel it. The reader can work out the details easily.

Finally, we need to say something about the need for all-pass filters for phase correction. Suppose we have nice square wave data, for a digital system say, which we need to transmit. It is received, but with high frequency noise and some phase distortion due to the transmission line. How do we clean it up? Well, a low-pass filter will clean out the high frequency noise (separating the main low harmonics of the square data from the junk), but it will increase the phase distortion. The result is that the various harmonics will be all smeared about, and this is not the sort of thing we like to feed a computer, for example, which likes and probably must have clean data into its input format. A second order phase equalizer, has some capability of correcting for phase distortion, both from the line and from the low-pass filtering, as long as the Q can be varied to bend the phase inward or outward to achieve a more linear phase. Linear phase corresponds to a constant time delay, and thus does not involve phase distortion. Phase correction helps put things back together, although the total correction is usually not exact.