ELECTRONOTES 133-A

PERSPECTIVES

Volume 4, Number 37 January 1982*

PERCUSSION INSTRUMENTS - 1

In the past few Perspectives we have been looking at the physics of some musical instruments. These have been instruments which produce a fairly well-defined pitch. Even in such cases, we found that the available pitches and natural modes of the instrument were interrelated in a complex way. It was found in particular that the present designs of instruments are the result of much craftsmanship, and of relatively little science.

Percussion instruments are of still a different nature. In fact, it is a bit difficult to define the term "percussive instrument" very exactly. In general, it should be something we beat on or strike in some manner to produce a sound. This sound may or may not have a pitch, and if it does have a pitch, the pitch may be strong or weak, or even ambiguous. A bass drum for example has very little or nothing at all that we might call pitch. Some orchestral chimes, on the other hand, have very well defined pitches and we can play a melody on them as easily as on some wind or string instrument. It should perhaps be noted, that some writers even would include the piano as a percussive instrument, on the grounds that the strings are struck with felt hammers.

This view of the piano as a percussive instrument is strengthened a bit if we look at another view of a percussive instrument as one which produces a sound that is not sustained, but rather which decays from the moment of excitation, in an exponential-like manner if possible. This somewhat vague definition is in line with our idea of a "convolution type" of instrument (see Perspectives No. 131A) if we assume that the excitation is impulsive. [In simpler terms, you strike it, it does its thing, and the energy damps away.]

Continuing this idea a bit further, we can consider the resonant system associated with a percussive instrument to be multi-mode. When we strike it, we excite a number of modes in general. The exact modes excited, and their relative response strengths, may well be a function of the striking method. In general, those modes which are pitched will have frequencies that are not related to each other in a simple way. In particular, they need not be integer multiples (harmonics). The pitches of the modes may have different decay rates, and they may be strong or weak. Percussive sounds may also contain a good deal or broadband noise.

PERCUSSION INSTRUMENT STRUCTURES

Since nearly anything can be (and frequently is) a percussion instrument, we have quite a large number of structures that can be considered. Two approaches to their analysis are common. The first is to simply take measurements on the actual instruments and try to make sense out of them. The second is to look at the structure in terms of some idealized simpler structure that can be handled theoretically. (In general, the actual instrument structures are of too complex a shape, or the properties of the materials are too poorly known, to allow an exact analysis). Thus, we might look at a marimba bar as an ideal bar-like structure of such and such length, thickness, width, and material. We can then often calculate its characteristic modes of vibration. We can see if the actual frequencies of vibration correspond to any of the

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theoretical ones. We can see if different theoretical modes can be excited in practice, particularly by knowing the vibration pattern. Perhaps of most interest, we can admire the craftsmanship. For example, why is the marimba bar hollowed out (undercut in an arch) near the center. Is it just for decoration? No, actually it raises the second partial from a theoretical 2.76 times the fundamental to 4.00 times the fundamental. Was this figured out theoretically years ago? That's very unlikely of course - it was a matter of trial and error, the instrument maker's art.

The idea of craftsmanchip goes even further when we consider such percussive instruments as large church bells. It is one thing to consider a wooden bar which you could work in your hands, shaving a bit here and a bit there until it has partials that are harmonic. It is quite another thing to think about a large church bell, of very complex shape, which is difficult just to move, and which must have been cast from molten metal. This is something the average person, even the person who is handy, can not even think about doing. That there is any success with such imposing structures is something of a wonder which must be attributed to the number of years that have gone into the production of bells. Perhaps in consequence, large bells are less precise in pitch and each one has some order of individual characteristic sound. Most have a pitch that is reasonably close to some fundamental frequency, and then perhaps two partials that are fairly close to harmonics of this fundamental. And then there are also partials that just don't fit in, are totally out of place from a harmonic viewpoint, and seem to be unavoidable as well. Perhaps these are responsible for the somewhat "uneasy" sound of the bells. We are accustomed to having bells somewhat out of tune and somewhat unpleasant to listen to, relatively speaking. We are not surprised to find that the individual sound of a bell is "etched" into our memory, even after some years of not hearing it. In fact, we may find the sound of a well matched set of bells to be very strange, and very intriguing.

Still other percussion instruments involve not relatively solid structures such as those of metal or wood, but rather flexible materials, typically menbranes supported in part by volumes of air. In particular, the drums, which everyone would group into the percussion group. Again, we find a large variety even in the drums, most of which involve a stretched circular membrane. We have pitched instruments (the timpani) and unpitched ones (such as the snare drum, where the snares actually add unpitched noise). The kettledrum (one of the drums of a timpani, timpani being always plural) is a very interesting structure. On the one hand, we have in mind a theoretical structure, which is the ideal membrane. This structure, a two dimensional analog to the vibrating string, if you will, is well understood. The string vibrates (ideally) producing harmonic overtones. The membrane is not so nice. Its vibrational pattern (see EN#131 for a mathematical description) produces non-harmonic overtones. In fact, the partials of the ideal membrane are at 1, 1.59, 2.14, 2.30, 2.65, 2.92, etc., a series which depends on Bessel functions where sine waves were good enough in the case of the string. In an actual kettledrum, the first mode (1 in series above) is not excited. In addition, the kettle itself is a volume of air which in effect "stiffens" the membrane, as the resistance of the air to compression is some sort of restoring force. As a result, the partials are shifted upward, compared to the ideal membrane (and probably to a real membrane without the kettle). As a result, the partials of a well-tuned kettledrum may be placed at a fifth above the fundamental, at a sharp sixth, and at an octave. It is thus the case that the craft of instrument making has again taken an inherently untuned structure and made it tunable, vastly increasing its musical versatility.

In most all instruments, there is of course playing technique involved along with the instrument design. A skillful player not only plays the right note at the right time, but has additional control over his instrument, allowing it to do different and often subtle things. This is also true of percussion instruments, although sometimes things are not subtle. For example, there is a right way and a wrong way to strike a kettledrum. In a chime, we may get different timbres depending on where we strike the bars. There may also be wholly different ways of exciting the structures. The "stroke rods" are excellent examples here. Here the metal rods are excited by stroking them along their length with a rosined glove, thus exciting longitudinal modes (along the length). The sound is different and in a sense non-percussive, because it is, at least for a time, continuously rather than impulsively excited.

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PERSPECTIVES

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PERCUSSION INSTRUMENTS - 2

In the last Perspectives, we began a look at percussion instruments. We will continue this here. Our interest in percussion instruments is first the same as we would have in any other acoustical instruments, and then special because of the increased use of percussive sounds in modern acoustic music. In addition, there is a good deal of interest in the electronic synthesis of percussive sounds.

We have looked at the acoustics of air columns (EN#128A, EN#129A, EN#130A) and at strings (EN#131A). In general, our results showed overtone relationships that were harmonic. Yet air columns and strings are two of the relatively few acoustic systems that come at all close to being ideal in this sense. Most percussion instruments produce non-harmonic overtones, or produce harmonic overtones only as a consequence of the craftsmanship and the art of their design. Here we will be looking at some of the idealized model structures for percussive instruments. Our interest is first in the mathematical predictions, and then in how well a real instrument structure follows these predictions, or how a modification has brought about a response that is judged superior for musical reasons.

BARS

One structure of interest is your basic "bar" which is often rectangular and made of metal or wood. [Many results for "bars" also apply to what might also be called "rods", having a circular cross section.] Thus our bar will be in general a rectangular slab of metal with a length L, a width W, and a thickness T, with $L > W > T$, as suggested in Fig. 1. In addition to the physical dimensions and properties of the bar itself, it is important to know how the bar is mounted. We could have a bar end free, it could be "supported" or "hinged", or it might be clamped. A free end is what it says, no support. A supported or hinged bar suggests that an end point or some intermediate point is to be fixed, but that bending is allowed at that point. A clamped bar suggests that the end is rigidly fixed, as it might be if set in cement. This means that not only is its position fixed, but also no bending is allowed at the end. If you wish, the position of the bar is fixed at zero, and its first derivative must also be zero. Examples of various bar terminations are shown in Fig. 2. In a vibration, it is the usual case that certain points are "nodes" or points that do not move for a certain mode of vibration. Thus support at a node point is always a possibility, although it may restrict the excitation of some other modes. This is also illustrated in Fig. 2.

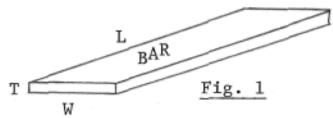

Fig. 1

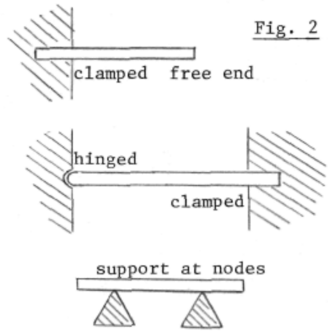

Fig. 2

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With three different end conditions (clamped, hinged, and free) there are six different combinations of end conditions, given the two ends of a bar. Nearly all common percussive instruments are of the free-free mounting, being supported at nodes. We would like however to review the consequences of all possible end conditions, and to take note of the lowest frequency mode, and the relative positioning of overtones. These are summarized below:

Clamp-Clamp	*Free-Free	Hinge-Hinge	Clamp-Free	Clamp-Hinge	Hinge-Free
$f_1 = 3.56\kappa$	$f_1=3.56\kappa$	$f_1 = 1.57\kappa$	$f_1 = 0.56\kappa$	$f_1 =2.45\kappa$	$f_1 =2.45\kappa$
$f_2 = 2.756f_1$	$f_2=2.756f_1$	$f_2 = 4f_1$	$f_2 = 6.27f_1$	$f_2 =3.25f_1$	$f_2 = 3.25f_1$
$f_3 = 5.404f_1$	$f_3=5.404f_1$	$f_3 = 9f_1$	$f_3 = 17.55f_1$	$f_3 =6.75f_1$	$f_3 = 6.75f_1$
$f_4 = 8.933f_1$	$f_4=8.933f_1$	$f_4 = 16f_1$	$f_4 = 34.39f_1$	$f_4 =11.5f_1$	$f_4 = 11.5f_1$

*a "twisting" mode may also be present between f_2 and f_3.

Here κ is a constant depending only on the dimensions and material of the bar, so for the same bar, κ is the same. Thus we see, for example, that a clamped free bar has the lowest frequency of all mountings, while a hinged-hinged bar has a frequency about three times higher. This you can demonstrate for yourself with a plastic ruler. Hold one end on the edge of a desk, and "flick" the free end. This is a clamp-free mount, and you can almost see the individual vibrations. Now press one end against the edge of the desk and hold the other end lightly in your fingers, with enough pressure to secure the desk end. Flick this in the middle. The frequency is clearly higher, and you may be able to convince yourself that three time higher is about right.

As for the overtones, there is more order here than may first appear. There is a pattern to the Hinge-Hinge case of course. Not so obvious is the fact that in the Free-Free (and Clamp-Clamp) case, the partials are quite well approximated by placing the ratios of $f_1:f_2:f_3:f_4$...... as $3^2:5^2:7^2:9^2$..... . This fact, known empirically to E.F.F. Chladni (who died in 1827, the same year Beethoven died) can also be shown mathematically. [It is sometimes stated that the ratios are $3.0112^2:5^2:7^2:9^2$... It would be quite remarkable if the first number 3.0112 were not a whole number and the rest were exactly whole numbers. It is in fact the case that the numbers 5, 7, 9,... etc. are not exactly whole numbers, but are so close, that it is not practical to carry additional decimal places.] Another pattern appears in that in the Clamp-Free case, the partials after the first are very close to being in the ratio of $3^2:5^2:7^2:9^2$.... etc.

To look at some actual examples, the glockenspiel or the bell lyra is composed of free-free bars with nodal supports of felt. We are mainly interested in the fundamental frequency of these bars. Although the first overtone is at 2.756 times the fundamental, it is of little importance as far as pitch goes (it is important to timbre). The reason is that it damps rapidly and is very high in frequency anyway (as the instruments themselves are of high frequency). In part, the rapid damping is due to the felt supports at the nodes of the fundamental mode (0.22 and 0.78 of the total length), which are not the nodes of other modes. Another remarkable bar is the "orchestral chime" of "tubular bell". In this case the "bar" is actually a pipe, but the math is the same. This is a free-free bar, and has overtones going as $3^2:5^2:7^2:9^2:11^2:13^2$:... which is 9:25:49:81:121:169:..., highly non-harmonic, except as overtones of 1, which is very low in pitch. Instead what is heard is a pitch of about 41. This is apparently generated by the overtones 81, 121, and 169, which the ear apparently accepts as the second, third, and fourth harmonic of 41! Other interesting bars are those of the xylaphone and the marimba, also free-free bars, with nodal mountings. Unlike the glockenspiel, the overtones of these are fairly low in frequency and would sound non-harmonic. Thus these bars are partly hollowed out or undercut near the middle, raising the 2.76 overtone to 3 for the xylaphone, and to 4 for the marimba.

In addition to the transversal modes of vibration of the bars considered above, longitudional and torsional modes can be excited. These are interesting for their higher frequencies, and in particular, because the overtones are harmonics, and thus join the string and the air column as somewhat unique structures.

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PERSPECTIVES

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PERCUSSION INSTRUMENTS - 3

Here we will be continuing our discussion of percussion instruments. One of the general features of instrument analysis that will appear here, as we take up the discussion of drums, is that of modal patterns. We know how to solve mathematically for the vibrational modes of various simple structures. This solution tells us rather exactly where nodal lines occur, what the characteristic frequencies are, and what the rates of decay are. In an actual instrument, we expect correctly that certain non-ideal properties will shift these results at least a bit. This is useful, but perhaps equally useful is the fact that even some quite non-ideal structures will still exhibit modes that can be recognized as corresponding to ideal modes of ideal structures. A cymbal for example (or a garbage can lid, for that matter) is distinctly different from a flat circular plate. Yet we can look for at least some of the same modes to appear in both. Certainly they will have different frequencies from an ideal circular plate, and the exact dimensions of the nodal boundaries will be different, but the general pattern may still be there. Thus we might look for a (0,1) mode of vibration in a grabage can lid, find its frequency, its strength, and its nodes. If it's not there, can we find why not? We fully expect to locate at least some of the low-order modes. We also expect some quite different ones later on higher up. Yet it is a good place to start. Consider the alternative of being asked to set up the proper equations and solve for the vibrations of a cymbal or a garbage can lid from scratch. In a similar manner, the vibrational modes of an ideal membrane are fundamental to a study of drums.

DRUMS

The interested reader can find a discussion of the ideal membrane in the regular newsletter (EN#131). Here we want to just draw patterns for the more important modes, and then to relate the results to drumheads. Fig. 1 below shows the patterns.

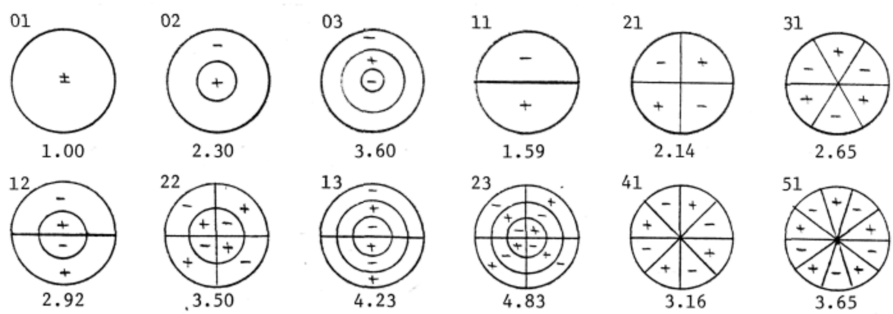

Fig. 1 Mode Numbers (above and to left) and relative frequencies (below) for some modes of membrane. Mode numbers MN where M=number of nodal diameters and N=number of nodal circles (outside mounting always counted).

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In the drawings, all the lines shown are nodal lines, lines where the membrane does not move. This includes the outside rim, since the membrane is mounted here and can not move. The modes are numbered by counting the number of nodal diameters and the number of nodal circles, as indicated in the caption below the figure. The + and - signs on various parts of the membrane indicate that these sections are always moving in opposite directions relative to the plane of the membrane (of the paper). Note that whenever a nodal boundary is crossed, the sign changes, indicating that the membrane is stretched through the nodal line - it can not simply go down to the line and return in the same direction.

The head of a drum is basically a real membrane (of calfskin, or better, of plastic mylar) stretched uniformly across some circular enclosure, usually of a cylindrical form. There may be more than one head, a second one being on the other end of the cylinder, thus forming an enclosure of air which in some way couples the heads. Various drums are commonly used, each for different purposes. Usually the distinction between drum types has to do with the degree to which the vibration is pitched. It should also be noted that the same drum head can respond in quite different ways according to the position at which it is struck, the manner of striking, and the material of the striking mallet (or hand, etc.).

As can be seen, the frequencies of the modes are not harmonically related at all. In some drums, this is an advantage in that a relatively unpitched sound is desired. In the kettledrum however, we are looking for a pitched sound. As we discussed briefly in EN#133A, it is the function of the kettle of the drum to bring some of the higher modes into tune. In fact, referring to Fig. 1, it is not the lowest frequency mode (01) that is the pitch, but rather the (11) or "sloshing" mode. Because of the enclosed air and the conventional manner of striking the kettledrum, the (01) mode is only weakly excited. Also, the 21 mode, which in the ideal membrane is at a ratio of 1.34:1 with the 11 mode, comes to a ratio of nearly 1.5:1, or a musical fifth above the main pitch. At the same time, a third strong mode, the 31 or the 12 (depending on the source of the data, and very probably the individual drum) comes in about 2:1, at the octave of the main pitch. Thus we hear a pitch which is not the lowest frequency present, and is not even the missing fundamental of the (2:3:4) ratio. Apparently it is the relative strength of the 11 mode that dominates over other factors in pitch determination. The tuned partials apparently contribute to the timbre more than to the pitch.

While the timpani are drums intended to give a definite pitch, it is the more usual case that a drum is intended to give an indefinite pitch or no pitch at all. Perhaps the most familiar type of drum of all is the so-called "snare drum." This is a two headed drum which has snare wires touching the lower (unstruck) head, and as this head starts to vibrate in response to the upper head, the wires produce additional noise. Some small sized, heavy membrane type of drums may have a prominent fundamental mode (01) if struck in the center, and yet the sound is still basically unpitched. Drums of this type, of which the "bongo" may be typical, are still of relatively low Q, as is evidenced by the relative ease with which their basic sound can be synthesized (ringing of a medium-Q bandpass filter). The snare drum sound is probably familiar to the reader as the military "parade drum" or "marching drum", and bongos are also well known. Another of the more familiar drums is the "tom-tom" which is the one used by the American Indians in most all old western movies. It is usually a two-headed drum of indefinite pitch, but without snares.

Speaking of Indian drums, we call attention to the drums of the Indian Indians, as this music has become popular in recent years. Such drums, of which the "tabla" is familiar (in Ravi Shankar's groups, etc.), are of a fairly standard drum structure, but they have a head that is loaded. This loading is a deposit of some gum-like substance at the center of the head, and has the effect of bringing overtones to harmonic positions. This is another example of art, craftsmanship, and tradition beating science to the correct answer. Still another example is found in the "steel drums" of Trinidad. This tradition cannot be hundreds of years old, as these drums are formed from hammered out sections of 55 gallon oil drums. Yet the musicians using them have developed a tuned drum in the time available.

An excellent source of information on all percussive instruments is Tom Rossing's "Acoustics of Percussion Instruments" in Physics Teacher, Dec. 1976 and May 1977.